# FIN 6160
# Investment Theory

Lecture 9-11

Managing Bond Portfolios

# Bonds Characteristics

- Bonds represent long term debt securities that are issued by government agencies or corporations. The issuer of bond is obligated to pay interest (coupon) payments periodically such as (annually or semi-annually) and the par value (principal or face value) at maturity. The coupon rate, maturity date, and par value of the bond are part of the bond indenture, which is the contract between the issuer and the bondholder.
- Bonds are often classified according to the type of issuer. Treasury bond are issued by the treasury, federal agency bonds are issued by federal agencies, municipal bonds are issued by state and local governments, and corporate bonds are issued by corporations.
- Most bonds have maturities between 10 to 30 years. Bonds can also be classified by the ownership structure as either bearer bonds or registered bonds. Bearer bonds require the owner to clip coupons attached to the bonds and send them to the issuer to receive coupon payments. Registered bonds require the issuer to maintain records of who owns the bond and automatically send coupon payments to the owners.

# Different types of Bonds

- **Treasury Bonds and Notes**: The federal govt Treasury commonly issues treasury notes or treasury bonds to finance federal govt expenditures. The minimum denomination for treasury notes or bonds is $1,000. The key difference between a note an a bond is that note maturities are usually less than 10 years, whereas bond maturities are 10 years or more. An active over the counter secondary market allows investors to sell treasury notes or bonds prior to maturity. Investors receive semi-annual interest payments from the treasury. Although the interest is taxed by the federal govt as ordinary income, it is exempt from state and local taxes.

- **Striped Treasury Bonds**: The cash flows of bonds are commonly transformed (striped) by securities firms so that one security represents the principal payment only while a second security represents the interest payments. For example, consider a 10 year treasury bond with a par value of $100,000 that has a 12% semi-annual coupon rate. This bond could be striped into a principal only (PO) security that will provide $100,000 upon maturity and an interest only (IO) security that will provide 20 semi-annual payments of $6,000 each.

- Investors who desire a lump-sum payment in the distant future can choose the PO part, and investors desiring periodic cash inflows can select the IO part.

# Different types of Bonds

- **Inflation-Indexed Treasury Bonds**: Inflation-indexed bonds provide returns tied to the inflation rate. These bonds commonly referred to as TIPS ( Treasury inflation protected securities) are intended for investors who wish to ensure that the returns on their investments keep up with the increase in prices over time. The coupon rate offered on TIPS is lower than the rate on typical Treasury bonds, but the principal value is increased by the amount of the inflation rate every six months.
- For example, consider a 10 year inflation indexed bond that has a par value of $10,000 and a coupon rate of 4%. Assume that during the first 6 months since the bond was issued, the inflation rate was 1%. The principal of the bond is increased by $100 (1% of $10,000). Thus the coupon payment after 6 month will be 2% of the new par value ($10,100) = $202.
- **Savings Bonds**: Savings bonds are issued by the treasury, but can be purchased from many financial institutions. They are attractive to small investors because they can be purchased with as little as $25. Large denominations are available as well. Savings bonds have a 30 year maturity and do not have a secondary market.
- **Federal Agency Bonds**: Federal agency bonds are issued by federal agencies. The bonds are backed both by the mortgages that are purchased with the proceeds and by the federal government.

# Different types of Bonds

- **Municipal Bonds**: State and local govt often issue municipal bonds to finance their budget deficit. Payments on general obligation bonds are supported by the municipal govt's ability to tax, whereas payments on revenue bonds must be generated by revenues of the projects (toll way, toll bridge, etc.). Municipal bonds typically promise semi-annual payments. Common purchasers of these bonds include financial and non-financial institutions as well as individuals. The minimum denomination of municipal bonds is typically $5,000. A secondary market exists for them, although it is less active than the one for treasury bonds. Most municipal bonds contain a call provision, which allows the issuer to repurchase the bonds at a specified price before the bonds mature. A municipality may exercise the call option to repurchase the bonds it interest rates decline substantially because it can reissue bonds at the lower interest rate and reduce its cost of financing.
- **Variable Rate Municipal Bonds**: Variable rate municipal bonds have a floating interest rate based on a benchmark interest rate. The coupon payment adjusts to movements in the benchmark interest rate (LIBOR). Some variable rate bonds are convertible to a fixed rate bond until maturity under specified conditions. In general, these bonds are desirable to investors who expect that interest rate will rise.

# Different types of Bonds

- **Corporate Bonds**: When corporations need to borrow for longer term periods, they issue corporate bonds which usually promise the owner interest on a semi-annual basis. The minimum denomination is $1,000. Larger bond offerings are normally achieved through public offerings, which must first be registered with the SEC. The bonds issued by smaller corporations tend to be less liquid because their trading volume is relatively low.
- Although most corporate bonds have maturities between 10 to 30 years. Corporations such as Boeing, Ford and Chevron have recently issued 50-year bond. These bonds can be attractive to insurance companies that are attempting to match their long term policy obligations.
- The bond indenture is a legal document specifying the rights and obligations of both issuing firm and the bondholders. Federal law requires that for each bond issue of significant size a trustee be appointed to represent the bondholders in all matters concerning the bond issue. The trustee's duties include monitoring the issuing firm's activities to ensure compliance with the terms of the indenture. If the terms are violated, the trustee initiates legal action against the issuing firm and represents the bondholders in that action.

# Corporate Bonds

- **Sinking fund provision**: Bond indentures frequently include a sinking fund provision or a requirement that the firm retire a certain amount of the bond issue each year. This provision is considered to be an advantage to the remaining bondholders because it reduces the payments necessary at maturity. For example, a bond with 20 years maturity could have a provision to retire 5% of the bond issue each year.
- **Protective covenants**: Bond indentures normally place restrictions on the issuing firm that are designed to protect the bondholders from being exposed to increasing risk during the investment period. These so called protective covenants frequently limit the amount of dividends and corporate officer's salaries the firm can pay and also restrict the amount of additional debt the firm can issue.
- **Call provision**: Some corporate bonds are issued with call provisions allowing the issuer to repurchase the bond at a specified call price before the maturity date. For example, if a company issues a bond with higher coupon rate when market interest rates are high, and interest rates later fall, the firm might like to retire the high coupon bond and issue new bonds with lower coupon rate to reduce coupon payments. This is called refunding. Callable bonds typically comes with a period of call protection, an initial time during which the bonds are not callable. Such bonds are referred to as deferred callable bonds. A call provision normally requires the firm to pay a price above par value when it calls its bonds. The difference between the bond's call price and par value is the call premium. Bondholders normally view a call provision as a disadvantage because it can disrupt their investment plans and reduce their investment returns. As a result, firms must pay slightly higher rates of interest on bonds that are callable, other things being equal.

# Corporate Bonds

- **Bond Collateral**: Bonds can be classified according to whether they are secured by collateral and by the nature of that collateral. Usually the collateral is a mortgage on real property (land and buildings).
- **A first mortgage bond** has first claim on the specified assets. **A chattel mortgage bond** is secured by personal property.
- Bonds unsecured by specific property are called **debentures** (backed only by the general credit of the issuing firm). These bonds are normally issued by large, financially sound firms whose ability to service the debt is not in question.
- **Subordinated debentures** have claims against the firms assets that are junior to the claims of both mortgage bods and regular debentures. Owners of subordinated debentures receive nothing until the claims of mortgage bondholders, regular debenture owners, and secured short term creditors has been satisfied. The main purchasers of subordinated debt are pension funds and insurance companies.
- **Convertibility**: **Convertible bond** allows investors to exchange the bond for a stated number of shares of the firm's common stock. This conversion feature offers investors the potential for high returns if the price of the firm's common stock rises. Investors are therefore willing to accept a lower rate of interest on these bonds, which allows the firm to obtain financing at a lower cost.

# Corporate Bonds

- **Puttable bonds**: While the callable bond gives the issuer the option to extend or retire the bond at the call date, the extendable or put bond gives this option to the bondholder. If the bond's coupon rate exceeds current market interest rate, the bondholder may choose to extend the bond's life. If the bond's coupon rate is too low, it will be optimal not to extend; the bondholder instead claims principal which can be invested at current yields.
- **Inverse Floaters**: These bonds are opposite to floating rate bonds; the coupon rate on these bonds falls when the general level of interest rates rises.
- **Junk Bonds**: Credit rating agencies assign credit ratings to corporate bonds based on their perceived degree of credit risk. Those bonds that are perceived to have high risk are referred to as speculative grade or junk bond. Junk bonds are also known as high-yield bonds.
- On the contrary, bonds with good credit rating are called investment-grade bond.

# Preferred Stock

- Although preferred stock is considered to be equity, it is often included in the fixed income securities like bond. This is because, like bonds, preferred stock promises to pay a fixed stream of dividends. However, unlike bonds, the failure to pay the promised dividend does not result in corporate bankruptcy. Instead, the dividends owed simply accumulates, and the common stockholder may not receive any dividends until the preferred stockholders have been paid in full.
- In the event of bankruptcy, preferred stockholders’ claim to the firm’s assets have lower priority than those of bondholders, but higher priority than those of common stockholders.

# The Term Structure of Interest Rates

- **Yield to Maturity (YTM)**: Yield to Maturity is defined as the interest rate that makes the present value of future cash flows from the bond equals to its market price. In other words it is the internal rate of return to the bond.

$$P = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots\ldots\ldots\ldots + \frac{(C_n + M)}{(1+y)^n}$$

- **Yield Curve**: Yield curve is a plot of yield to maturity as a function of time to maturity. The yield curve is also called the term structure of interest rates and it helps us extract the appropriate rates that should be used to discount cash flows at different maturities. There are various relationship between yield and their maturity. Most common patterns:
    - Upward sloping
    - Downward sloping (inverted)
    - Flat
    - Hump-shaped (rising and then falling)

# The Term Structure of Interest Rates

- **Interest rate uncertainty and forward rates:** Suppose you buy a 2-year zero coupon bond and hold it till maturity. Alternatively another investor buys a 1 year zero coupon bond and roll it over at the end of the first year for another year (i.e. buy another 1 year zero). If interest rates are known with certainty then these two strategies should yield the same return.

$$(1+y_{0\to 2})^2 = (1+r_{0\to 1})(1+r_{1\to 2})$$

- In case we deal with uncertain interest rates and investors are risk-neutral (care only about expected returns), then the previous relationship can be written as

$$(1+y_{0\to 2})^2 = (1+r_{0\to 1})[1+E_0(r_{1\to 2})]$$

- However, investors are risk averse, so to get rid of uncertainty and establish a link between current and future rates, we use forward rates.

## The Term Structure of Interest Rates

- Now suppose you buy a 2-year zero coupon bond and hold it till maturity. Alternatively another investor buys a 1 year zero coupon bond and agrees today the rate at which he will roll over his investment at the end of the first year for another year (i.e. the forward rate). These two strategies should yield the same return because by locking the rate today (the forward rate), there is no uncertainty.

$$(1+y_{0\to 2})^2 = (1+r_{0\to 1})(1+f_{1\to 2})$$

- In case we deal with uncertain interest rates and investors are risk-neutral (care only about expected returns), then the previous relationship can be written as

$$(1+y_{0\to 2})^2 = (1+r_{0\to 1})[1+E_0(r_{1\to 2})]$$

- However, the interest rate that will actually prevail in the future will not necessarily be equal to the corresponding forward rate extracted today.

# The Term Structure of Interest Rates

- **Pure Expectations Theory**: This theory explains that spot rates are determined by expectations of what rates will be in the future.
- Suppose for an upward sloping yield curve with rates increasing for longer maturities, the 2 year rate is greater than the 1 year rate. This is so because the market believes that the 1 year rate will most likely go up next year (because most people believe inflation will rise and thus to maintain the same real rate of interest, the nominal rate must increase). This majority belief that the interest rate will rise translates into a market expectation. An expectation is only an average guess.
- According to this theory, the forward rate equals the today's market consensus expectation of future spot rate.

$$(1+y_{0\to 2})^2=(1+r_{0\to 1})[1+E_0(r_{1\to 2})]=(1+r_{0\to 1})(1+f_{1\to 2})$$

$$E_0(r_{1\to 2})=f_{1\to 2}$$

# The Term Structure of Interest Rates

- An upward sloping yield curve is an indication that the interest rates are expected to increase with maturity.
- A flat yield curve is an indication that the interest rates are expected to remain same.
- A downward sloping yield curve is an indication that the interest rates are expected to fall with maturity.

# The Term Structure of Interest Rates

▪**Liquidity Preference Theory**: This theory assumes that investors require a higher expected return (premium) to induce them to hold bonds with maturities different from their investment horizons.

$$f_n = E_0(r_n) + \text{ liquidity premium}$$

▪If the investor has a short term horizon, then he needs premium to hold a long term bond (upward sloping curve, ceteris paribus), because he would be exposed to interest rate risk.

▪If the investor has a long term horizon, then he needs a premium to hold a short term bond –downward sloping ( because he runs the risk of re-investing his capital at an uncertain rate)

▪Advocates of the liquidity preference theory of the term structure believe that short term investors dominate the market so that the forward rate will generally exceed the expected future spot rate or short rate.

# The Term Structure of Interest Rates

- **Segmented Markets Theory**: According to this theory, investors and issuers of debt seem to have a strong preference for debt of certain maturity and may be indifferent to yield differentials among maturities.
- Therefore, if investors are sufficiently risk averse, they will operate only in their desired maturity spectrum.
- What determines the yield at each maturity is solely the supply and demand for funds at this specific maturity.
- As a result, examining flows of funds into these market segments can help predict changes in the yield curve.

# Interest Rate Sensitivity

- Bond prices and yields are inversely related; bond prices decrease when yields rise, and that the price curve is convex, meaning that decreases in yields have bigger impacts on price than increases in yields of equal magnitude.
- Prices of long term bonds tend to be more sensitive to interest rate changes than prices of short term bonds. If rates increase, for example, the bond is less valuable as its cash flows are discounted at a now-higher rate. The impact of the higher discount rate will be greater as that rate is applied to more distant cash flows.
- The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increase.
- Interest rate risk is inversely related to the bond's coupon rate. Prices of low coupon bonds are more sensitive to changes in interest rates than prices of high coupon bonds.
- Bonds with higher yield to maturity is less sensitive to changes in interest rates and vice versa, i.e. bond price sensitivity falls with yield to maturity.

# Duration

- Bond price sensitivity falls with yield to maturity. A higher yield reduces the present value of all of the bond's payments, but more so for more distant payments. Therefore, at a higher yield, a higher fraction of the bond's value is derived from its earlier payments, which have lower effective maturity and interest rate sensitivity.
- To deal with the ambiguity of the "maturity" of a bond making many payments, we need a measure of the average maturity of the bond's promised cash flows to serve as a useful summary statistic of the effective maturity of the bond.
- Frederick Macaulay termed the effective maturity concept the duration of the bond. Macaulay's duration equals the weighted average of the times to each coupon or principal payment made by the bond. The weight associated with each payment time clearly should be related to the "importance" of that payment to the value of the bond. In fact, the weight applied to each payment time is the present value of the payment divided by the bond price.

$$w_t = \frac{\mathrm{CF}_t / (1+y)^t}{\text{Bond Price}}$$

# Duration

- Macaulay's Duration is given by, $D = \sum_{t=1}^{T} t \ x \ w_t$
- Duration is a very useful measure of bond portfolio's interest rate sensitivity. It is a useful tool for immunization against interest rate risk.
- It can be shown that when interest rate changes, the proportional change in bond' price is related to the change in its YTM (y).

$$\frac{\Delta P}{P} = -D \text{ x } [\frac{\Delta(1+y)}{(1+y)}]$$

- Instead of Macaulay's duration, professionals commonly use modified duration, defined as, $D^* = \frac{D}{(1+y)}$ and $\Delta(1+y) = \%\Delta y$ $\quad i.e. \frac{\Delta P}{P} = -D^* \text{ x } \Delta y$ or, $D^* = -\frac{1}{P}\frac{\Delta P}{\Delta y}$
- The percentage change in bond price is just the product of modified duration and the change in the bond's yield to maturity. Because the percentage change in the bond price is proportional to modified duration- it is a natural measure of the bond's exposure to interest rate changes.

# Rules for Duration

1) The duration of a zero-coupon bond equals its time to maturity (since there are no interim cash flows)
2) Holding maturity constant, a bond's duration is lower when the coupon rate is higher. The higher the coupon payments, the higher the weights on the early payments, and the lower is the weighted average maturity (duration) of the payments.
3) Holding the coupon rate constant, a bond's duration generally increases with its time to maturity (because of introducing more payments in the distant future). However, duration need not always increase with time to maturity. It turns out that for some deep discount coupon bonds, duration may fall with increases in maturity.
4) Ceteris paribus, the duration of a coupon bond is higher, when the bond's yield to maturity is lower. Because at lower yields the more distant payments made by the bond have relatively greater present values and accounts for greater weights of the bond's value.

# Limitation of Duration

- The equation below states that the percentage change in the value of a bond approximately equals the product of modified duration times the change in the bond's yield. $$\frac{\Delta P}{P} = -D^{*} \text{ x } \Delta y$$
- This equation asserts that the percentage change in price is directly proportional to the change in the bond's yield. If this were exactly so, there should be a straight line relationship between the percentage change in bond's price and change in its yield. However, the true price-yield relationship is said to be convex and the curvature of the price-yield curve is called the convexity of the bond.
- Duration underestimates the increase in bond price when the yield falls, and it overestimates the decline in price when the yield rises.
- The duration rule is a good approximation for small changes in the bond yield or interest rates, but it is less accurate for larger changes. To improve this approximation, we need a correction term, known as convexity.

# Convexity

- The convexity of a bond equals the second derivative of the price-yield curve divided by bond price.

$$\text{Convexity, C} = \frac{1}{P}\frac{d^2 p}{dy^2} = \frac{1}{P(1+y)^2}\sum_{1}^{T}[\frac{CF_t}{(1+y)^t}(t^2+t)]$$

- The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -D^* \text{ x } \Delta y + 1/2 \text{ x C x } (\Delta y)^2$$

# Immunization

- Immunization programs aim at protecting bond portfolios against interest rates shifts. They attempt to eliminate the portfolio's sensitivity to shifts in the yield curve by matching the duration of the assets to the duration of the liabilities (duration matching).
- Since duration is a measure of sensitivity for bond returns, two bonds (or portfolios of bonds) with the same duration will have their values changed by the same amount due to a common interest rate shock.
- If duration matching is successful, then any rise/ fall in the present value of the portfolios assets due to a shift in the yield curve will be offset by a fall/ rise of equal magnitude in the present value of the portfolios liabilities. Therefore, the net position will be immunized against these shifts. Note: The duration of a bond portfolio is equal to the weighted average of the constituent bonds'' durations (weighted according to their relative value).

# Immunization Strategies

- Focused strategy: Find a portfolio of bonds with each bond having a duration close to the duration of the liability.
- Barbell strategy: Use bonds with very different durations and mix them accordingly to construct the desired duration.
- A barbell strategy provides more flexibility (necessary as time passes) relative to the focused strategy, but it also incorporates a greater degree of potential inaccuracy

# Risks or Problems with Immunization

- Successful immunization depends on the use of a correct measure of duration, which depends on the assumed shape of the yield curve. For an incorrect choice, we won't have a perfectly immunized portfolio.
- Even if a portfolio is perfectly immunized, with the passage of time or small changes in the yields, the portfolio ceases to be immunized. Hence, portfolio rebalancing is necessary to immunize it according to the new conditions (i.e. it is an active strategy!). Only exact cash flow matching implies an always immunized portfolio (this is obviously more difficult and costly to construct).
- Duration provides a rough approximation. Incorporating convexity considerably enhances the approximation. Therefore, most bond portfolio managers engage in immunization by matching both convexity and duration. Convexity matching leads to a very successful immunization, but it also implies a higher cost of constructing the portfolio since fewer sets of bonds can achieve both targets

# Example 1:

▪Consider a semi-annual 8% coupon bond with 2 year maturity i.e. n = 4, C = $40, M = $1,000 and yield to maturity, y = 10%. Calculate the bond`s current price (P), modified duration and convexity. If the bond`s yield to maturity increases by 0.02 percentage points, calculate the change in the bond price and therefore, the new price.

| **Time** | **Cash Flow** | **Present Value** | **Weight** | **Time*Weight** |
|---|---|---|---|---|
| **1** | 40 | 38.095 | 0.0395 | 0.0395 |
| **2** | 40 | 36.281 | 0.0376 | 0.0752 |
| **3** | 40 | 34.554 | 0.0358 | 0.1074 |
| **4** | 1040 | 855.61<br>Price, Σ = 964.54 | 0.887 | 3.548<br>Duration, D Σ= 3.77 |

Here modified duration, $D^* = D/(1+y) = 3.77/1.05 = 3.59$(semi-annual)

$$i.e.\frac{\Delta P}{P} = -D^* \text{ x } \Delta y \ = -3.59x0.0001 = -0.0359\%$$

$$i.e.\Delta P = P(-0.0359\%) = -\$0.346$$

# Example 1:

| Time | Cash Flow | Present Value | Pv($t^2$+t) |
|---|---|---|---|
| **1** | 40 | 38.095 | 76.19 |
| **2** | 40 | 36.281 | 217.686 |
| **3** | 40 | 34.554 | 414.648 |
| **4** | 1040 | 855.61<br>Price, Σ = 964.54 | 17112.2<br>Σ=17820.724 |

$$\text{Convexity, C} = \frac{1}{P(1+y)^2}\sum_{1}^{T}[\frac{CF_t}{(1+y)^t}(t^2+t)] = \frac{1}{964.54(1.05)^2}x17820.724 = 16.758$$

- The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -D^* \text{ x } \Delta y + 1/2 \text{ x C x } (\Delta y)^2$$

$$= -3.59x0.0001 + 0.5x16.758x(0.0001)^2 = -0.0003589 = -0.0359\%$$

$$i.e.\Delta P = P(-0.0359\%) = -\$0.346$$

# Example 2:

- Consider a coupon paying bond with n = 3, C = \$200, M = \$1,000 and r = 8% continuously compounded. Calculate the bond`s current price (P), modified duration and convexity. If the bond`s yield to maturity increases by 0.5 percentage points (i.e. 50 basis points), calculate the change in the bond price and therefore, the new price.
- The bond price is:

$$P_0 = [c_1 e^{-rt_1} + c_2 e^{-rt_2} + .... + (c_n + M) e^{-rt_n}]$$

$$= [200e^{-0.08*1} + 200e^{-0.08*2} + 1200e^{-0.08*3}] = 1299$$

- The first derivative:

$$\frac{dP_0}{dr} = -t_1 c_1 e^{-rt_1} - t_2 c_2 e^{-rt_2} - ... - t_n (c_n + M) e^{-rt_n}$$

$$= -200e^{-0.08*1} - 2x200e^{-0.08*2} - 3x1200e^{-0.08*3} = -3357.34$$

# Example 2:

$$\text{Modified Duration} = -(\frac{1}{P_0})[\frac{dP_0}{dr}] = -[\frac{1}{1299}][-3357.34] = 2.58$$

- The second derivative:

$$\frac{d^2P_0}{dr^2} = t_1^2 c_1 e^{-rt_1} + t_2^2 c_2 e^{-rt_2} + t_3^2 (c_3 + M) e^{-rt_3}$$

$$= 200e^{-0.08*1} + 2^2 x 200e^{-0.08*2} + 3^2 x 1200e^{-0.08*3} = 9361.9192$$

$$\text{Convexity, C} = \frac{1}{P_0}\frac{d^2P_0}{dr^2} = \frac{9361.9192}{1299} = 7.2$$

- The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -D^* \text{ x } \Delta y + 1/2 \text{ x C x } (\Delta y)^2$$

$$= -2.58x0.005 + 0.5x7.2x(0.005)^2 = -0.0129 = -1.29\%$$

# Example 2:

- An increase in interest rate by 0.5% to 8.5%, leads to a drop in the bond price by 1.29% or approximately $16.75.
- The bond price after the change in the interest rate is $1282.24
- **Simple Approach:**

| Time | Cash Flow | Present Value | Weight | Time*Weight |
|---|---|---|---|---|
| **1** | 200 | 184.62 | 0.142 | 0.142 |
| **2** | 200 | 170.43 | 0.131 | 0.262 |
| **3** | 1200 | 943.95<br><br>Price, Σ = 1299 | 0.727 | 2.181<br><br>Duration, D Σ= 2.58 |

$$i.e.\frac{\Delta P}{P} = -D^{*} \text{ x } \Delta y \ = -2.58x0.005 = -1.29\%$$

$$i.e.\Delta P = P(-1.29\%) = -\$16.75$$

# Example 2:

- **Simple Approach:**

| Time | Cash Flow | Present Value | PV*(t^2) | [PV*(t^2)]/p |
|---|---|---|---|---|
| **1** | 200 | 184.62 | 184.62 | 0.142 |
| **2** | 200 | 170.43 | 681.71 | 0.524 |
| **3** | 1200 | 943.95<br>Price = 1299 | 8495.58 | 6.54<br>Σ=7.2 |

- The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -D^{*} \text{ x } \Delta y + 1/2 \text{ x C x } (\Delta y)^{2}$$

$$= -2.58x0.005 + 0.5x7.2x(0.005)^{2} = -0.0128 = -1.28\%$$

$$i.e.\Delta P = P(-1.28\%) = -\$16.64$$

# Example 3 (Constructing an Immunized Portfolio):

- An insurance company must make a payment of \$19,487 in 7 years. The market interest rate is 10%, so the present value of the obligation is \$10,000. The company's portfolio manager wishes to fund the obligation using 3-year zero coupon bonds and perpetuities paying annual coupons. How can the manager immunized the obligation?
- Answer: Immunization requires that the duration of the portfolio of assets equal the duration of the liability. We can proceed in four steps:

1) Calculate the duration of the liability: It is a single payment obligation with duration of 7 years.
2) Calculate the duration of the asset portfolio: The portfolio duration is the weighted average duration of each component asset, with weights proportional to the funds placed in each asset. The duration of the zero coupon bond is simple its maturity, 3 years. The duration of the perpetuity is $(1+y)/y = 1.10/0.10 = 11$ years.

$$\text{Asset duration} = w \times 3 \text{ years} + (1-w) \times 11 \text{ years}$$

# Example 3 (Constructing an Immunized Portfolio):

Asset duration = Liability duration

Asset duration = w x 3 years + (1-w) x 11 years =7 years

i.e. w = 0.5

3. The manager should invest half of the portfolio in the zero coupon bonds and the rest half in perpetuities. This will result in an asset duration of 7 years.

4. Because the obligation has a present value of $10,000, the manager must purchase $5,000 of the zero coupon bonds and $5,000 of the perpetuities. (note that the face value of the zero will be $5000 x $(1.10)^3$ = $6,655