Financial Management



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<u>Financial Management</u> <u>An Overview of Financial Management and Financial Environment</u>

Corporate Life Cycle:

- Starting up as a Proprietorship: Many companies begin as a proprietorship, which is an unincorporated business owned by one individual. Starting a business as a proprietor is easy—one merely begins business operations after obtaining any required city or state business licenses.
- The proprietorship has three important advantages: (1) it is easily and inexpensively formed, (2) it is subject to few government regulations, and (3) its income is not subject to corporate taxation but is taxed as part of the proprietor's personal income.
- However, the proprietorship also has two important limitations: (1) it may be difficult for a proprietorship to obtain the capital needed for growth; (2) the proprietor has unlimited personal liability for the business's debts, which can result in losses that exceed the money invested in the company (creditors may even be able to seize a proprietor's house or other personal property)
- **Partnership:** Some companies start with more than one owner, and some proprietors decide to add a partner as the business grows. A partnership exists whenever two or more persons or entities associate to conduct a non-corporate business for profit.
- Partnerships may operate under different degrees of formality, ranging from informal, oral understandings to formal agreements. Partnership agreements define the ways any profits and losses are shared between partners.
- Regarding liability, the partners can potentially lose all of their personal assets, even assets not invested in the business, because under partnership law, each partner is liable for the business's debts. Therefore, in the event the partnership goes bankrupt, if any partner is unable to meet his or her propata liability then the remaining partners must make good on the unsatisfied claims, drawing on their personal assets to the extent necessary.
- However, it is possible to limit the liabilities of some of the partners by establishing a limited partnership, wherein certain partners are designated general partners and others limited partners. In a limited partnership, the limited partners can lose only the amount of their investment in the partnership, while the general partners have unlimited liability. The limited partners typically have no control—it rests solely with the general partners—and their returns are likewise limited.
- In both regular and limited partnerships, at least one partner is liable for the debts of the patinership. However, in a limited liability partnership (LLP), sometimes called a limited liability company (LLC), all partners enjoy limited liability with regard to the business's liabilities, and their potential losses are limited to their investment in the LLP.
- Many Owners: A Corporation: A corporation is a legal entity created under state laws, and it is separate and distinct from its owners and managers. This separation gives the corporation three major advantages: (1) unlimited life—a corporation can continue after its original owners and managers are deceased; (2) easy transferability of ownership interest—ownership interests are divided into shares of stock, which can be transferred far more easily than can proprietorship or partnership interests; and (3) limited liability—losses are limited to the actual funds invested.

- The corporate form offers significant advantages over proprietorships and partnerships, but it also has two disadvantages: (1) Corporate earnings may be subject to double taxation—the earnings of the corporation are taxed at the corporate level, and then earnings paid out as dividends are taxed again as income to the stockholders. (2) Setting up a corporation involves preparing a lot of documentation, which is more complex and time consuming than creating a proprietorship or a partnership. 3)For proprietorships, partnerships, and small corporations, the firms owners are also its managers. This is usually not true for a large corporation, which means that large firm's stockholders, who are it's owners, face a serious problem- agency problem, because managers are hired as agents to act on behave of the owners.
- Agency problems can be addressed by a company's corporate governance, which is the set of rules that controls the company's behavior towards its directors, managere employees, shareholders, creditors, customers, competitors, and community.
- Shareholders are the owners of a corporation, and they purchase stocks because they want to earn a good return on their investment without undue risk exposure. In most cases, shareholders elect directors, who then hire managers to run the corporation on a day-to-day basis. Because managers are supposed to be working on behalt of shareholders, they should pursue policies that enhance shareholder value. Consequently, management's primary objective is stockholder wealth maximization.
- When we say management's objective should be to reaximize stockholder wealth, we really
 mean it is to maximize the fundamental price or invinsic value of the firm's common stock,
 not just the current market price.

Efficient Capital Markets:

- An efficient capital market is one in which stock markets fully reflect available information.
- Suppose Eastern Housing, being a real estate company, got a contract from government to build 10,000 apartments for Dhaka city people. Eastern Housing found a positive NPV for this project which means if the project goes ahead it will certainly increase shareholder' wealth. If this information is released in the media on Wednesday morning, the price of the shares of Eastern Housing will immediately adjust to this new information.
- The efficient market hypothesis predicts that the price of shares of Eastern Housing on Wednesday afternoon will already reflect the information contained in the Wednesday morning media release.
- Efficient market hypothesis states that financial market process all relevant information about securities quickly and efficiently, that is, the security price usually reflects all the information available to investors concerning the value of the security.
- According to this hypothesis, as new information about a security becomes available, the
 price of the security quickly adjusts so that at any time, the security price equals the market
 consensus estimate of the value of the security. If this were so, there would be neither
 underpriced nor overpriced securities.

Implications of EMH for Investors and Firms:

- Because information is reflected in prices immediately, investors should only expect to obtain a normal rate of return. Awareness of information when it is released does an investor no good. The price adjusts before the investor has time to trade on it.
- Firms should expect to receive fair value for securities that they sell. Fair means that the price they receive from issuing securities is the present value. Thus, valuable financing opportunities that arise from fooling investors are unavailable in efficient capital markets.
- From the figure in the following slide we can see, the solid line represents the path taken by the stock in an efficient market. In this case, the price adjusts immediately to the new information with no further price changes. The dotted line represents a slow reaction. Here it takes the market 30 days to fully absorb the information. Finally, the broken line illustrates an over-reaction and subsequent correction back to the true price.
- The broken line and the dotted line show the paths that the stock price might take in an inefficient market. If the price of the stock takes several days to adjust trading profits would be available to investors who suitably timed their purchases and sales.



- Andrie Shleifer argues that there are three conditions any one of which will lead to efficiency:
- Rationality: If all investors act rationally, with the release of new information in the marketplace, all investors will adjust their estimates of stock prices in a rational way. In my previous example of Eastern Housing, investors will use the information in the media release in conjunction with existing information about the firm, to determine the NPV of

the new project. If the information in the release implies that the NPV of the new project is 200 million taka and there are 10 million shares, investors will calculate that the NPV is 20 taka per share. The price of shares would rise immediately by 20 taka per share.

- Independent Deviations from Rationality: If the new information is unclear, some investors may act irrationally optimistic and while some may act irrationally pessimistic. If we same number of irrationally pessimistic and optimistic investors in the market, prices would likely rise in a manner consistent with market efficiency, even though most investors would be classified as less than fully rational. Thus, market efficiency does not require rational investors- only countervailing irrationalities.
- Arbitrage: Suppose in a market we have irrational amateurs and the rational professionals. The amateurs get caught up in their emotions, at times believing irrationally that a stock is undervalued and at other times believing the opposite. If the passions of the different amateurs do not cancel each other out, they would tend to carry stocks either above or below the efficient prices.
- However, professionals evaluate information objectively and estimate stock prices coldly and clearly and act accordingly. If a stock is underpriced, they would buy it and if it is overpriced they would sell it. Professionals try to arbitrage by simultaneous purchase and sale of different, but substitute securities. If the arbitrage of professionals dominates the speculation of amateurs, markets would still be efficient.

Types of Market Efficiency:

Weak form of Efficiency: A capital market is said to be weakly efficient, if it fully incorporates the information in past stock prices. Weak form of affinition mathematically represented as

 $Pt = P_{t-1} + Expected return + Random error_t$

- The above equation states that the price today is equal to the sum of the past observed price plus the expected return on the stock plus a random component occurring over the interval.
- Weak form efficiency is about the weakest type of efficiency that we would expect a financial market to display because historical price information is the easiest kind of information about a stock to acquire. If it were possible to make extraordinary profits simply by finding patterns in stock price movements, everyone would do it, and any profits would disappear in the scramble.
- Semi-strong and Strong form Efficiency: A market is semi-strong form efficient if prices reflect (incorporate) all publicly available information, including information such as published accounting statements for the firm, as well as historical price information.
- A market is strong form efficient if prices reflect all information, public or private.

Financial Management Time Value of Money

Simple Interest and the Future Value:

- Simple Interest = P x r x tHere, P = Present Value or Principal r = Rate of interestt= Time period
- Future Value, F = Principal + Interest = P + Prt = P(1+rt)
- Example: Compute interest amount and the future value of \$600 invested at 9.59 rate for 10 months. Sol: Here, Principal, P = \$600, Interest rate, r = 7.5% = 7.5/100 = 0.075%Time periods, t =10 months = 10/12 year =0.83 years Interest = Prt = $$600 \times 0.075 \times 0.83 =$ \$37.5Future value = \$600+\$37.5 =\$637.5

Simple Interest and Bank Discount:

- In many loans, the interest charge is computed with on the amount the borrower receives, but on the amount that is repaid later. The interest charge for a loan computed in this manner is called the bank discount, the amount the borrower receives is called the proceeds or present value of a loan. The future amount to be paid back is F, now called the maturity value of the loan.
- Example: If \$1,000 is borrowed at 12% for 6 months, the borrower receives P and pay back F = \$1,000 after 6 months. Time periods, t = 6 months =0.5 years Future value, F = P P F rt $F - Frt = $1000 - $1000 \times 0.12 \times 0.5 = 940 Present value,

Compound Interest and the Future Value:

- pose, \$5,000 is invested at 10% interest compounded each year. The amount at the end of the first year would be,
- $F1 = 5,000 + 5,000 \ge 0.10 \ge 1 = $5,500$
- This \$5,500 becomes the principal at the beginning of the second year, and the amount at the end of the second year is,
- $F2 = 5,500 + 5,500 \ge 0.10 \ge 1 = 6,050$
- Thus in the second year, interest is earned on not only the \$5,000 invested, but also on the \$500 of interest earned in the first year. This common practice of computing interest on interest is called compounding interest.

Derivation of Compound Interest Formula:

Future value at the end of one year,

 $F_1 = P(1 + r x 1) = P(1 + r) = P(1 + r)^{1}$

Future value at the end of two years,

$$F_{2} = P(1+r)(1+r \ x \ 1) = P(1+r)(1+r) = P(1+r)^{2}$$

Future value at the end of three years,

$$F_{3} = P(1+r)^{2}(1+r) = P(1+r)^{3}$$

Future value at the end of n years,

 $F_{\perp} = P(1+r)^{t}$

Compound Interest and the Future Value:

- interest and the Future Value: Example: find the future value of \$1,000 at 7% per year for 10 years. Sol: Here, Present value, P = \$1,000, t = 10 years, r = 7% = 0.07%Future value, $F_r = P(1 + r)^r = 1000(1 + 0.07)^{10} = $1967^{-1/5}$ Practice question: If \$500 'uture value?'
- Practice question: If \$500 is invested at 6% compounded annually, what will be the future value 30 years later?
- Example: Find the future value of \$500 at % compounded quarterly for 10 years. **Sol:** Here, present value, P = \$500, time periods, t = 10 years x 4 quarters/year =40 quarters Yearly interest rate = 8% = 0.08Quarterly interest rate, r = 0.08 / 4 = 0.02 $= \$00(1+0.02)^{40} = \$1104.019 \approx \$1104.02$ Future value, $F_t = P(1 + r)^{\frac{1}{2}}$
- Example: If \$800 is nvested at 6% compounded semiannually(every 6 months), what will be the amount in 5 years? **Sol:** Here, present value, P = \$800, time periods f = 5 years x 2 semi-annual period/year =10 periods Yearly interest rate = 6% = 0.06Semi-annual interest rate, r = 0.06/2 = 0.03Future value, $F_t = P(1+r)^t = 800(1+0.03)^{10} = 1075.13
- Example: Compute the future value of \$5,000 at 9% compounded monthly for 10 years.

Sol:Here, present value, P = \$5,000, t = 10x12 = 120 periods Monthly interest rate, r = 0.09/12 = 0.0075

Future value, $F_t = P(1+r)^t = 5000(1+0.0075)^{120} = $12,256.78$

Example: A bank pays 7.25% compounded daily on 90 day notice accounts. If \$500 is deposited in such an account, what will be the amount in 90 days? (use 365 days per year).

Sol: Here, present value, P = \$500, time periods, t = 90 days Yearly interest rate = 7.25% = 0.0725Daily interest rate, r = 0.0725/365

Future value,
$$F_t = P(1+r)^t = 500(1+\frac{0.0725}{365})^{90} = $509.0178 \approx $509.02$$

- Practice question: Find the future value of \$2,500 invested at
- *Example:* At 8% compounded annually, how many years will it take for \$2,000 to grow to \$3,000? Sol: Here, present value, P = \$2,000, Future value, F = \$3,000 the end of the e

i.e. $t \ge ln(1.08) = ln(1.5)$

i.e.
$$t \ge ln(1.08) = ln(1.5)$$

i.e. $t \ge ln(1.5)$
i.e. $t = \frac{ln(1.5)}{ln(1.08)} = 5.268 \approx 5.27$ years

- Practice question: How many years will it take for \$1,000 to grow to \$2,000 at 9% compounded annually
- Example: At what interest rate compounded annually will a sum of money double in 10 years? 2

Sol: Let, present value, P = \$1, and Future value, F = \$2, time periods, t = 10 years, Yearly interest rate = ?

Further value,
$$F_r = P(1 + r)$$

i.e. $2 = 1(1 + r)^{10}$

i.e.
$$(1+r)^{10} = 2$$

i.e. $ln(1+r)^{10} = ln(2)$; [Taking ln in both sides] *i.e.* 10 ln(1 + r) = ln(2)*i.e.* $ln(1+r) = \frac{ln(2)}{10}$ $i.e.(1+r) = e^{\frac{ln(2)}{10}}$ $i.e.r = e^{\frac{ln(2)}{10}} - 1 = 0.07177 = 7.17\%$

Alternate sol: Future value, $F_t = P(1+r)^t$ or, $2 = 1(1+r)^{10}$ or, $(1+r)^{10} = 2$ $or, (1+r) = 2^{\frac{1}{10}}$ $or, r = 2^{\frac{1}{10}} - 1 = 0.07177 = 7.177\%$

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Compound Interest and the Present Value:

Example: What is the present value of \$2,500 payable 4 years from now at 8% compounded quarterly?

Sol: Here, present value, P = ?, and Future value, F = \$2,500time periods, t = 4x4=16Yearly interest rate = 8% = 0.08Quarterly interest rate, r = 0.08/4 = 0.02Future value, $F_t = P(1+r)^t$ *i.e.* $2500 = P(1+0.02)^{16}$ *i.e.* $P = \frac{2500}{1.02^{16}} = \1821.11

- Practice question: What is the present value of \$4,000 payable in 20 years at 8% compounded semi-annually?
- Example: How much must be deposited now in an account paying 7.3% compounded daily in order to have just enough in the account 3 years from how to make \$10,000 available for investment in a business enterprise?

Sol: Here, present value, P = ?, and Future value, F = \$10,000efin, unohammad time periods, t = 3x365 = 1095Yearly interest rate = 7.3% = 0.073daily interest rate, r =0.073/365 Future value, $F_{t} = P(1+r)^{t}$ *i.e.* $10000 = P(1 + \frac{0.073}{365})^{1095}$

i.e.
$$P = \frac{10000}{\left(1 + \frac{0.073}{365}\right)^{1095}} = \$8033.39$$

- Practice question: How much must be deposited now in an account paying 8% compounded monthly in order to have just enough in the account 5 years from now to make a \$10,000 doxy payment on a home?
- Example: Find the effective rate(or equivalent annual rate) of 12% compounded monthly. **Sol:** Here, time periods, t = 12 months Yearly interest rate = 12% = 0.12Monthly interest rate, r = 0.12/12 = 0.01Effective Rate, r_e = $(1 + r)^{t} - 1 = (1 + 0.01)^{12} - 1 = 0.1268 = 12.68\%$
- Practice question: Find the effective rate of 16% compounded quarterly.
- Example: Find the effective rate(or equivalent annual rate) of 15% compounded daily.

Sol: Here, time periods, t = 365 daysYearly interest rate = 15% = 0.15Daily interest rate, r = 0.15/365

Effective Rate, $r_e = (1+r)^t - 1 = (1+\frac{0.15}{365})^{365} - 1 = 0.161798 \approx 16.18\%$

Derivation of Continuous Compounding Formula:

• Future value of \$1 at 100% compounded daily for one year is,

Future value, $F_t = P(1+r)^t = 1(1+\frac{1}{365})^{365} = 2.7145674 Future value of \$1 at 100% compounded hourly for one year (365daysx24hours=8760hrs)is, Future value, $F_t = P(1+r)^t = 1(1+\frac{1}{8760})^{8760} = 2.7181266 W e know, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} = 2.718281828$ Future value of \$1 at j % rate compounded m times of Future value, $F_t = P(1 + r)^t$ year Future value of \$1 at j % rate compounded m times a year for t years is, Future value, $F_t = P(1+r)^t = 1(1+\frac{j}{m})^{mt} = (1+\frac{j}{m})^{mt}$ let, $P = \frac{m}{j}$; so $\frac{j}{m} = \frac{1}{P}$; and m = Pj $F_t = (1+\frac{j}{m})^{mt} = (1+\frac{1}{P})^{Pjt} = [(1+\frac{1}{P})^{Pjt}]^{p}$ *m P* Now as m becomes larger and larger, P() also becomes larger and larger so, $(1 + \frac{1}{m})^m \approx (1 + \frac{1}{p})^p \approx$ $F_t = Pe^{rt}$;[where r is the continuous compounding rate] Example: Find the future value of \$500 at 8% compounded continuously for 9 years and 3 months Sol: Here, Present value, P = \$500, interest rate, r = 8% = 0.08Time periods, t = 9 years and 3 months = 9.25 years Future value, $F_t = Pe^{rt} = \$500 xe^{0.08 x9.25} = \1047.97

- Practice question: Find the future value of \$2,000 at 10% compounded continuously for 9 months.
- Example: How much must be deposited now in an account earning 7.5% compounded continuously if the amount in the account 8 years from now is to be \$10,000?
 Sol: Here, Present value, P =?, Future value, F = \$10,000
 Interest rate, r = 7.5% =0.075, time periods, t = 8years

Future value, $10000 = Pe^{0.075x8}$; $P = \frac{10,000}{e^{0.075x8}} = $5,488.11$

- Example: Find the effective rate of 10 percent compounded continuously. Effective rate, $r_e = e^r - 1 = e^{0.1} - 1 = 0.10517 = 10.517\%$
- Example: A bank states that the effective interest on savings accounts that earn continuous interest is 7%. Find the nominal rate.

Here, Effective rate, r = 7% = 0.07, find the nominal rate, r

 $r_{1} = 0.07 = e^{r} - 1;$ i.e. $e^{r} = 1.07$

an effective

 e. (ine=1)
 or, r = 6.765%
 Practice question: What nominal rate compounded continuously gives an effective rate of 8 percent?
 Perpetuity:
 Perpetuity is a constant stream of cash flows for an infinite period.
 Perpetuity is a constant stream of cash flows for an infinite period.
 Perpetuity is a constant stream of cash flows for an infinite period. Perpetuity is a constant stream of cash flows for an infinite period of time. Example: The British bonds called 'consols'. An investor purchasing a consol is entitled to receive yearly interest from the British Government forever.

Present value of Perpetuity or Consol,
$$P = \frac{c}{1 + c} \frac{c}{(1+r)^2} + \frac{c}{(1+r)^3} + \dots = \frac{c}{1 + c}$$

Growing Perpetuity:

- P = $\frac{c}{1+r} + \frac{c(1+g)}{(1+r)^2} + \frac{c(1+g)^2}{(1+r)^3} + \frac{c(1+g)^2}{(1+r)^3} + \frac{c}{r-g}$ Where, C = constant stream of cash flow muities:
- Where, C = constant stream of cash flows, g = constant growth in cash flows

Annuities:

- An annuity is series of periodic payments, usually made in equal amounts.
- Annuity extain: An annuity that begins and ends on designated dates is called an annuity certain e. loan transactions and rent payments.
- Simple Annuity: An annuity whose payment intervals coincide with the interest intervals is Called a simple annuity, e.g. a transaction whose payments is made monthly and interest Sis also monthly compounding. When the two interval do not coincide, the annuity is called complex.
- Ordinary annuity: An annuity whose payment is made at the end of each payment interval is called an ordinary annuity.
- Annuity due: An annuity whose payment is made at the beginning of each payment interval is called an annuity due.

• Present value of Ordinary Annuity,
$$P = \frac{c}{1+r} + \frac{c}{(1+r)^2} + \frac{c}{(1+r)^3} + \dots + \frac{c}{(1+r)^T}$$

Ordinary Annuity:

Periods	Now (0)	1	2	3	•••••	Т	T+1	T+2	T+3	∞
Consol 1		С	С	С		С	С	С	С	∞
Collsof 1	C/r									
Consol 2							С	С	С	∞
						C/r				
Annuity		С	С	С		С				n)
Present value of Ordinary Annuity, $P = \frac{c}{1+r} + \frac{c}{(1+r)^2} + \frac{c}{(1+r)^3} + \dots + \frac{c}{(1+r)^T}$										
Present value of consol 1 = $\frac{c}{r}$; Value of consol 2 at time T = $\frac{c}{\sqrt{r}}$										
Present valu	e of co	nsol	$2 = \frac{1}{2}$	$\frac{x}{x}$	$(+ r)^{-T}$			refilt		
Present valu	e of Or	dina	ry A	nnuit	y, P = P	V of con	sol 1 – P	V of cons	ol 2	
$=\frac{c}{r}-\frac{c}{r}x(1+$	$(r)^{-T} =$	$\frac{c}{r}$ [1 - r	- (1 +	$r)^{-T}$]	310	mae			
Future value	e of O	rdina	ıry A	nnu	ity, $F = $	$c \oplus c(1 +$	r) + c(1 + c)	$(r+r)^{2}+$	c(1 + r)	·) ^{<i>T</i>-1}
Present valu	ie of O	rdin	ary A	۹nnu ش	ity, $P \equiv$	$\frac{c}{r} - \frac{c}{r}x(r)$	$(1+r)^{-T}$		0	
Future value, $F = P(1+r)^{T}$ $r = \frac{c}{r}x(1+r)^{-T}](1+r)^{T} = \frac{c}{r}(1+r)^{T} - \frac{c}{r}$										
Future value, $F = -[(1+r)^T - 1]$										
Example: If	\$100 i	š dep	osite	d in	an accou	nt at the	end of e	very qua	rter for t	the next \sharp

5 years, how much will be in the account at the time of the final deposit if interest is 8% compounded quarterly. What will be the present value? Sol: Here, no. of payments, T = 5x4=20 payments; r = 0.08/4=0.02

~	Periods	Now(0)	1	2	3	•••••	T=20				
Annu	Annuity		\$100	\$100	\$100	•••	\$100				
	Annuity						FV =?				
FV	$FV = \frac{c}{r}[(1+r)^{T} - 1] = \frac{100}{0.02}[(1+0.02)^{20} - 1] = \2429.74										

using annuity table: FV = $100 x FV IFA_{20,2\%} = 100 x 24.297 = 2429.7$

$$PV = \frac{c}{r} \left[1 - \frac{1}{(1+r)^{T}}\right] = \frac{100}{0.02} \left[1 - \frac{1}{(1+0.02)^{20}}\right] = \$1635.14$$

using annuity table: PV = \$100 xPV IFA 20.2% = \$100 x16.351 = \$1635.1

Example: If \$100 is deposited in an account each month for 10years and the account earns 7% compounded monthly, how much will be in the account after the last deposit is made.

Sol: Here, no. of payments, T = 10x12=120 payments; r = 0.07/12

Periods	Now(0)	1	2	3	••••	T=120			
A		\$100	\$100	\$100	•••••	\$1000			
Annuity									
$FV = \frac{c}{r} [(1+r)^{T} - 1] = \frac{100}{0.07 / 12} [(1+0.07 / 12)^{20} - 1] = \17308448									

Practice question: Sums of \$500 are deposited in an account at the end of each 6-month period for 10 years. Find the amount in the account after the last deposit is made if interest is computed at 6% compounded semi-annually. (Use both formula and annuity 20 table)

Example: How much should be deposited in a sinking fund at the end of each quarter for 5 years to accumulate \$10,000 if the fund earns 8% compounded quarterly.

Sol: No. of payments, $T = 5x4=20$ payments; r	: =8,4=	=2% =0.02;	FV = \$10,000
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	Periods	Now(0)	1	2	3	•••••	T=20		
A			C	\$C	С	••••	С		
	Annuity						FV =\$10,000		
FV =	$FV = 10,0000 = \frac{c}{r} [(1+r)^{T} + \frac{c}{10000} = \frac{c}{100000000000000000000000000000000000$								
<i>or</i> ,10	r = 0000 x 0.02 = C	1(.02) - 1)	.02						
or,C	$pr, C = \frac{200}{(1.02^{20} \text{ A})} = \411.56								
using	annuit table:								
FV =	10000 = CxFV	$V \text{IFA}_{20,2\%} = C$	x 2 4 . 2 9	97					
i.e.	$\sum_{n=1}^{\infty} \frac{10,000}{24.297} = \4	11.57							

Practice Question: A company wants to accumulate \$100,000 to purchase replacement machinery 8 years from now. To accomplish this, equal semi-annual payments are made to a fund that earns 7% compounded semi-annually. Find the amount of each payment.

Example: What sum deposited now in an account earning 8% interest compounded quarterly will provide quarterly payments of \$1,000 for 10 years, the first payment to be made 3 months from now?

Periods	Now(0)	1	2	3	•••••	T=40
Annuity	PV =?	1,000	1,000	1,000	••••	1,000

Sol: Here, No. of periods, T=10x4 =40; C = \$1,000; r =8/4 =2% =0.02; PV=?

$$PV = \frac{c}{r} \left[1 - \frac{1}{\left(1 + r\right)^{T}}\right] = \frac{1000}{0.02} \left[1 - \frac{1}{\left(1 + 0.02\right)^{40}}\right] = \$27, 355.48$$

using annuity table: $PV = $1000 x PV IFA_{40,2\%} = $1000 x 27.355 = 27355

Total interest earned = 40 x \$1,000 - \$27,355.48 = \$12,644.52

Example: Sam borrowed \$5,000 to buy a car. He will amortize the loan by monthly payments over 3 years. a) find the monthly payment amount if interest is 12% compounded monthly b) find the total amount sam will pay.

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Sol: Here, No. of periods, T=3x12 =36; C = ?; r =12/12 =1% =0.01; PV=\$5,000

	Periods	Now(0)	1	2	1233	•••••	T=36			
	Annuity	PV =\$5,000	С	offai	С		С			
<i>PV</i> = \$5 <i>or</i> ,50 = using an	$PV = \$5,000 = \frac{c}{r} [1 - \frac{1}{(1 + r)^{T}}] = \frac{C}{0.01} [1 - \frac{1}{(1 + 0.01)^{36}}]$ $pr,50 = C (1 - 1.01^{-36}); \qquad or, C = \frac{1}{(1 + 0.01)^{-36}} = \166.07 using annuity table: PV = \\$CXVIFA $_{36.1\%} = \$Cx30.108$									
$or, C = \frac{1}{2}$ Total Pa	$\frac{PV}{30.108} = \frac{\$5}{30}$ syment= 36	5,000 0.100 5166.07 = \$5,9	978.47							

Amortization Schedules:

• When a mortgage payment is made or debt is amortized, the interest due is subtracted first and the remainder is then applied to the outstanding balance.

Example: A \$70,000 condominium is to be purchased by paying \$10,000 in cash and a \$60,000 mortgage for 30 years at 9.75% compounded monthly. a) find the monthly payment on the mortgage b) what will be the total amount of interest paid? c) show amortization schedule for the first two payments.

Periods	Now(0)	1	2	3	•••••	T=360
Annuity	PV =\$60,000	С	С	С		С

Sol: PV=\$60,000; No. of periods, T=30x12=360; r = 0.0975/12

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 $PV = \$60,000 = \frac{c}{r} \left[1 - \frac{1}{(1+r)^{T}}\right] = \frac{C}{0.0975/12} \left[1 - \frac{1}{(1+0.0975/12)^{360}}\right]$ $or, 487.5 = C(1 - 1.008125^{-360});$ $or, C = \frac{487.5}{(1 - 1.008125^{-360})} = \515.49 Total interest paid= 360 x \$515.49 - \$60,000 = \$125,577.3

At the time the first payment is due, one month has passed and the interest on the \$60,000 =\$60,000 x 0.0975/12 =\$487.5

Period	Monthly PaymentInterest chargedBala Red		Balance Reduced	New Balance
1	\$515.49	\$60,000x0.0975/12 =\$487.5	\$27.99	60,000-27.99 =\$\$9,972.01
2	\$515.49	59,972.01x0.0975/12 =\$487.27	\$28.22	\$9,972.01–28.22 =\$59,943.8

Multi-step Problems:

Example: Sam wants to determine how much he should deposit in a retirement account now at 8% compounded quarterly so that the amount in the account 10 years from now will provide an income of \$5,000 every 6 months for 12 years, with the first \$5,000 to be received in 10.5 years from now. Sam estimate that 10 years from now he should be able to earn 6% compounded semi-annually on the account. How much should Sam deposit now?

Now	Year-10	shi?	2	3	• • • • •	T=24
\$38,350	Value=\$84,680	\$3.000	\$5,000	\$5,000		\$5,000
8% c	om. quarterly	Inter	est rate = 6°	% compounded	l semi-ann	ually

using annuity table: Value (year $10 = $5,000 \text{ xPV IFA}_{24,3\%} = $5,000 \text{ x16.936} = $84,680$ Mr. Sam will need \$84,680 in his account after 10 years to receive \$5,000 thereafter in every 6 months for 12 years.

$$PV = \$84, 6\$9x(1+0.02)^{-40} = \$38, 350.76$$

Mr. Sam should deposit \$38,350.76 today to receive \$84,680 after 10 years.

Example: James wishes to provide himself with an income of \$5,000 every six months, starting 15.5 years from now and continuing for 20 years. He deposits \$25,000 in the account now, and he has a guaranteed inheritance of \$10,000, which he will receive 10 years from now and add to the account. He knows these sums will not provide the income he wants, so he plans to make periodic deposits to the account at the end of every 6 months for 15 years to make up the difference. How much should the periodic deposits be if all interest is compounded at 6% semi-annually?

Now	Year-10	Year-15	Year	-15.5 (1)	2	3	•••••	T=40
			\$5	5,000	\$5,000	\$5,000		\$5,000
\$25,000 ·								
	\$10,000 -	>						
Sho	rtage							2

Year-0.5(T1)	Year-1 (T2)	Т3	••••	Year-15(T=30)
				\$.211C
C?	C?	C?		(DZ)

@ year 15, Sam need = $$5,000 \times PV$ IFA $_{40,3\%}$ = $$5,000 \times 23.115 = 115575 Future value of \$25,000 @ year-15 = $$25,000 \times (1+0.03)^{30} = $60,681.56$ Future value of \$10,000 @ year-15 = $$10,000 \times (1+0.03)^{10} = $13,439.16$

Now	Year-10	Year-15	Year-15.5	2	3	••••	T=40
			\$3,000	\$5,000	\$5,000		\$5,000
		Need=\$115,575	27				
\$25,000 -	>	\$60,68 2.56					
	\$10,000 -	> \$ \$,439.16					
Shortage \$41,454.27							

Year-0.5FT1)	Year-1 (T2)	Т3	•••••	Year-15(T=30)
No.				\$41,454.27
NC?	C?	C?		C?

Future Value, $$41,454.27 = CxFVIFA_{30,3\%} = Cx47.575$

i.e.
$$C = \frac{\$41, 454.27}{47.575} = \$871.35$$

Mr. Sam should deposit \$871.35 every 6-month for 15 years to cover the shortage

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<u>Financial Management</u> Time Value of Money Problems

Question 1-2: In problems 1 through 2, find the future value at the stated nominal interest rate compounded annually.

- 1) \$200; 20 years; 5 percent
- 2) \$300; 10 years; 6 percent

Question 3-4: In problems 3 through 4, find the future value using the appropriate interest and number of periods.

3) \$150; 8 years; 8 percent compounded quarterly

4) \$600; 20 years; 8 percent compounded semi-annually

Question 5: How many years will it take for \$5,000 to amount to \$20,000 at 7% compounded annually?

Question 6: How many years will it take for a sum of money to double at 10% compounded semi-annually?

Question 7: Find the rate of interest compounded annually at which a sum of money will double in 20 years.

Question 8: Find the rate of interest compounded semi-andually at which \$5,000 will grow to \$12,000 in 8 years.

Question 9: A bank pays 5.25% compounded daily on savings accounts running for 6 years. Using 365 days per year, compute the future value of a deposit of \$5,000 for 6 years.

Question 10: How many years will it take for \$5,000 to grow to \$10,000 at 9% compounded annually?

Question 11: At what rate of interest compounded annually will \$1,000 grow to \$5,000 in 10 years?

Question 12-14: In problems 12 through 14, compute the present value:

12) \$1,000; due in 20 years; 8 percent compounded semi-annually

13) \$5,000; due in 5years; 10 percent compounded quarterly

14) \$1,000; due in 25ears; 12 percent compounded daily (use 365 days/year)

Question 15: What sum of money deposited now at 8% compounded quarterly will provide just enough money to pay a \$1 million debt due 7 years from now?

Question 16. What sum of money invested now at 12% compounded monthly will provide just enough to pay a debt of \$50,000 due in 15 years?

Question 17: If output per labor-hour increases by 5% compounded annually and is currently 100 units per labor-hour, what was output per labor-hour 5 years ago?

Question 18: An account bearing interest at 6% compounded semi-annually was established 10 years ago. The account balance now is \$9,030.55. What was the initial amount when the account was established?

Question 19-20: Compute the future value:

19) \$4,000; 8 percent compounded continuously; 5 years 8months

20) \$5,000;12 percent compounded continuously; 6months

Question 21-22: Compute the present value:

21) \$2,000; 7 percent compounded continuously; 3 years 2months

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22) \$250;12 percent compounded continuously; 1year 6months

Question 23-24: The rate in problems 5 through 6 are nominal rates, find the effective rate when the interest rate is compounded continuously:

23) 12%, Sol: 24) 8%, Sol:

Question 25-26: The rate in problems 25 through 26 are effective interest rates, find the nominal rate when the interest rate is compounded continuously:

25) Effective rate = 5%26) Effective rate = 10%

Question 27: How much will a deposit of \$5000 grow to in 20 years at 7.2% interest compounded continuously?

Question 28: How much should be deposited now at 8.4% compounded continuously if the amount in the account 10 years from now is to be \$8000?

Question 29: Sam invests \$15,000 in a bank account paying \$4% compounded continuously for 15 years. How much will be in the account at the end of this time?

Question 30: History tells us that Peter Minuit purchased Manhattan Island in New York from the Indians for \$24 about 370 years ago. If the \$24 had been invested at 5% compounded continuously, what would be its amount after 370 years.

Question 31: A company issues \$1 million of bonds and sets up a sinking fund at 8% compounded quarterly to accumulate \$1 million by 10 years to redeem the bonds. Find the quarterly payment to the sinking fund.

Question 32: In order to accumulate \$15,000 for a down payment on a home 8 years from now, the Jonses are going to deposit a sum of money at the end of each 6-month period in an account earning 8% compounded semi-annually. What should be the amount of each deposit?

Question 33: When Kathy was born, her parents decided to deposit \$500 every 6 months thereafter for 15 years in an account earning 6% compounded semi-annually. How much will be in the account after the last deposit is made?

Question 34: A sum of money invested now at 10% compounded semi-annually is to provide payments of \$5,500 every 6 months for 8 years, with the first payment due 6 months from now. How much should be invested? How much interest will the investment earn?

Question 35: The directors of a company have voted to establish a fund that will pay a retiring accountant or his estate \$1,000 per month for the next 3 years, with the first payment to be made a month from now. How much should be placed in the fund if it earns interest at 12% compounded monthly? How much interest will the fund earn during its existence?

Question 36: A company has borrowed \$50,000 at 8% compounded quarterly. The debt is to be amortized by equal payments each quarter over 10 years. a) find the quarterly payment b) how much interest will be paid?

Question 37: A real estate developer borrows \$100,000 at 12% compounded monthly. The debt is to be discharged by monthly payments over the next 3 years. a) find the monthly payment b) how much interest will be paid?

Question 38: A company borrows \$100,000 at 12% compounded semi-annually. The debt is amortized by making equal payments at the end of each 6 months for 7 years.

- a) Find the amount of each payment
- b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
- c) How much of the second payment is for interest, and by how much does it reduce the balance owed?

Question 39: Mr. Smith have taken out a \$35,000, 30 year mortgage on their home, with interest at 8.75% compounded monthly.

- a) Find the amount of each payment
- b) How much of the first payment is for interest, and by how much does it reduce the balance owed?

Question 40: Mr. Fran borrowed \$2,000 from Silverbank and signed a note promising to discharge the debt with interest at 12% compounded monthly at a maturity date? years from now. Six months later, Silverbank needed more cash and sold France note to Goldbank. Goldbank computed the maturity amount of Fran's note and gave Silverbank the present value of this amount, computed at 8% compounded quarterly How much did Silverbank receive?

Question 41: Sue borrowed \$7,000 at 12% compounded monthly for 3 years to buy a car. How much will she have to pay at the end of each month to discharge the debt?

Question 42: How much should be deposited now at 7% compounded annually to provide an income of \$20,000 at the end of each year for the next 22 years?

Question 43: Mr. James will make 20 equal semi-annual deposits to an account earning 8% compounded semi-annually. Then, after the last deposit, she will use the amount in the account to establish an ordinary annuity carning 6% compounded annually which will provide her with \$10,000 at the end of each year for 5 years. How much should his semi-annual deposit be?

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1. Selim has taken a loan of Taka 56,000 at 12.5 percent annual rate for a period of 6 years. How much does he have to pay at the end of six years to fully pay off the loan?

2. Sadek has a fixed deposit that will mature at Taka 16,00,000 three years from now. He needs money urgently and he wants to mortgage the fixed deposit. The bank agreed to accept the mortgage at an annual rate of 11.5 percent and lend him the maximum. How much will be receive as loan?

3. Shahid has a fixed deposit that will mature at Taka 24,00,000 four years from now. He need money urgently and he wants to mortgage it. The bank agreed to accept the mortgage at an annual rate of 15.25 percent and lend him the present value of the fixed deposit but not to exceed 75 percent of the maturity value of the fixed deposit. How much will he receive as loan?

4. Shofiq made an investment of Taka 68,000. If the investment grows at an annual rate of 18.5 percent for 7 years, what will be the value of the investment at the end of the seventh year?

5. Siddique had made an investment of Taka 1,25,000 five years ago. He was promised a return of 17.5 percent per year. He wants to withdraw the investment now. How much should he get?

6. Sajed just purchased two bighas of land for a total price of Taka 38,00,000. He expects the land value to appreciate by 9.5 percent per year. At what price should the land sell at the end of six years?

7. Solaiman purchased three bighas of land for a total price of Taka 48,00,000 six years ago. The land value appreciated at 8 percent during the first 4 years and at 12 percent in the last two years. What is the price of the land today?

8. Shaker made an investment twelve years ago. The amount was Taka 48,000. The investment grew at 9 percent in the first three years, at 11 percent the following three years, at 13.5 percent in the next three years, and at 15 percent in the final three years. What is the value of the investment today?

9. Karim plans to save Taka 20,000 per year and deposit the savings in a bank account at the end of each year. The first deposit will be made at the end of this year. The bank account will pay 11 percent interest per year. How much would be the total accumulated savings if he does this for 12 years?

10. Kamrul needs a loan of Taka 58,000. He approaches a bank and the bank agrees to lend him the required amount at 12.5% annual rate if he pays off the ban in five years in five equal annual installments, the first payment to be made at the end of year 1. How much does he have to pay per year? Build an amortization schedule.

11. Kofil wants to set up a plan so that he will have at least Taka 50,00,000 by the end of his twenty fifth year in the job. His plan is to set aside a certain amount every year and deposit it in an account that will pay him 11 percent annually. How much does he need to set aside per year?

12. Kabir has set up a bank account that pays 12 percent annual interest. Kabir's goal is to have Taka 10,000 at the end of 6 years. He will deposit Taka 1,280 per year at the end of each year until his target is achieved. How much does he need to deposit at the end of the sixth year if his account balance is to exactly equal Taka 10,000?

13. A Taka 80,000 loan was set op to be paid off in six equal annual installments. The annual rate is 15 percent. (a) What is the size of required installment? (b) How much is the remaining balance after four years?

14. Khaleque wants to set up a bank account for his granddaughter now to cover her for four years of college expenses. The annual college cost next year is expected to be Taka 10,00,000. This is expected to go up by 8 percent per year. How much does he need to place in the account today if the account is credited with 11 percent interest annually?

15. Without doing any math or using a calculator, at 7 percent annual growth rate, how long do you think it will take an initial investment of Taka 10,00,000 to become Taka 20,00,000?

16. Kaisar has a piece of land and he planted mahogany tree saplings at a cost of Taka 1,00,000. He expects to sell the trees for Taka 2,00,000 at the end of seven years. What rate of return will he get? (Answer without using calculator or any tables)

17. Kashem approached Hashem for a loan of Taka 12,00,000. Kashem told Hashem that he will pay him double the amount in five years. What rate of return is Kashem promising? (Exact Rate)

18. Kashfia had borrowed Taka 65,000 from Habib four years ago. She paid him Taka 1,00,000 today as a full pay-off. What interest rate did Kashfia pay?

19. Keka wants to accumulate Taka 12,00,000 by depositing Taka 1,00,000 in an account every year at the end of the year, the first deposit to be made at the end of this year. The

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account will credit her with an annual interest of 9 percent. (a) How long will she need to accumulate the desired amount? (b) What deposit will she need to make to make the terminal amount exactly Taka 12,00,000?

20. Build an amortization table for a loan of Taka 36,000 to be paid off in four equal annual installments and the interest rate is 12.5 percent per year.

21. Kamaruzzaman has taken a loan of Taka 80,000 for a period of 5 years. He is paying off the loan in five equal annual installment of Taka 24,432.75 per year. What interest rate is he paying?

22. What is the value of a land that will generate Taka 1,00,000 per year for ever? The expected return per year is 12 percent.

23. Helal needs to borrow Taka 72,000 for a period of 5 years. following options:

Determine the amount needed to pay off the loan at the end of the 5th year under each option. Which option is the best?

What are the EAR for option (d) and (e)?

.or ever? The .s 5 years. He looked into the .s 5 years. He looked into the .s 14.6% interest with quarterly compounding 14.6% interest with quarterly compounding 14.5% interest with daily compounding e. 14.35% interest with daily compounding the amount needed to pay off the loan at the and re-hich option is the best? the EAR for option (d) and (e)? of land is used to prod-xcted to be Tr' percer 24. A piece of land is used to produce vegetables. The net market value of the output this year is expected to be Taka 125,000. The Segetables generally experience an annual inflation of 7 percent. If an investor expects to earn 18 percent annual return, what is the fair value of the land?

25. If you deposit the following amounts at the end of the indicated years and the account earns 12 percent interest, how much will be in the account at the end of the five years? Year 1 Taka 12,000, year 2 Taka 16,000, year 3 Taka 18,000, year 4 Taka 20,000, and year 5 Taka 21,000.

26. If a contract is expected to generate the following net cash flows and the expected return is 16 percent, what is the present value of the contract? Year 1 Taka 22,000, year 2 Taka 26,000, year 3 Taka 28,000, year 4 Taka 30,000, and year 5 Taka 35,000.

27. A proposed investment will cost Taka 1, 22,000. It will generate the following cash flows: Year Y Taka 32,000, year 2 Taka 29,000, year 3 Taka 38,000, year 4 Taka 42,000, and year Taka 41,000. What rate of return is obtained from the investment?

28. If a business generates Taka 12,00,000 per year and an investor wants 20 percent annual return, what is the value of the business?

29. What is more valuable - (a) Taka 1,50,000 per year for ever or (b) Taka 3,00,000 for 4 years? Annual rate is 20 percent.

30. A loan of Taka 12,00,000 requires 5 equal annual installment at 16 percent with an extra provision that in the 6th year, a lump sum payment of Taka 2,00,000 will be made. What is the size of annual payments?

31. What is the future value of 7 equal annual deposits of Taka 40,000 at 9 percent if the first deposit is made today?

32. What is the present value of 6 equal annual deposits of Taka 45,000 at 11 percent if the first deposit is made today?

33. A bank says that it will credit depositors with 10 percent annual interest on a continuous basis. If you make a deposit of Taka 16,000 in the account for 6 years, how much will be the accumulated value in the account?

34. A bank says that it will credit depositors with 12 percent annual interest on a continuous basis. If you deposit Taka 12,000 every year in the account for 4 years, how much will be the accumulated value in the account?

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Financial Management Bonds and Stocks Valuation

Bonds Characteristics:

- Bonds represent long term debt securities that are issued by government agencies or corporations. The issuer of bond is obligated to pay interest (coupon) payments periodically such as (annually or semi-annually) and the par value (principal or face value) at maturity. The coupon rate, maturity date, and par value of the bond are part of the bond indenture, which is the contract between the issuer and the bondholder.
- Bonds are often classified according to the type of issuer. Treasury bond are issued by the treasury, federal agency bonds are issued by federal agencies, municipal bonds are issued by state and local governments, and corporate bonds are issued by corporation.
- Most bonds have maturities between 10 to 30 years. Bonds can also be dassified by the ownership structure as either bearer bonds or registered bonds. Bearer bonds require the owner to clip coupons attached to the bonds and send them to the issuer to receive coupon payments. Registered bonds require the issuer to maintain records of who owns the bond and automatically send coupon payments to the owners.

Different types of Bonds:

- ferent types of Bonds: Treasury Bonds and Notes: The federal govt Treasury commonly issues treasury notes or treasury bonds to finance federal govt expenditures. The minimum denomination for treasury notes or bonds is \$1,000. The key difference between a note an a bond is that note maturities are usually less than 10 years, whereas bond maturities are 10 years or more. An active over the counter secondary market allows investors to sell treasury notes or bonds prior to maturity. Investors receive semi-annual interest payments from the treasury. Although the interest is taxed by the federal govt as ordinary income, it is exempt from state and local taxes.
- Striped Treasury Bopds: The cash flows of bonds are commonly transformed (striped) by securities firms so that one security represents the principal payment only while a second security represents the interest payments. For example, consider a 10 year treasury bond with a par value of \$100,000 that has a 12% semi-annual coupon rate. This bond could be striped into principal only (PO) security that will provide \$100,000 upon maturity and an interest only (IO) security that will provide 20 semi-annual payments of \$6,000 each.
- Investors who desire a lump-sum payment in the distant future can choose the PO part, and in Sestors desiring periodic cash inflows can select the IO part.
- Inflation-Indexed Treasury Bonds: Inflation-indexed bonds provide returns tied to the inflation rate. These bonds commonly referred to as TIPS (Treasury inflation protected securities) are intended for investors who wish to ensure that the returns on their investments keep up with the increase in prices over time. The coupon rate offered on TIPS is lower than the rate on typical Treasury bonds, but the principal value is increased by the amount of the inflation rate every six months.
- For example, consider a 10 year inflation indexed bond that has a par value of \$10,000 and a coupon rate of 4%. Assume that during the first 6 months since the bond was issued, the inflation rate was 1%. The principal of the bond is increased by \$100 (1% of \$10,000). Thus the coupon payment after 6 month will be 2% of the new par value (\$10,100) = \$202.

- Savings Bonds: Savings bonds are issued by the treasury, but can be purchased from many financial institutions. They are attractive to small investors because they can be purchased with as little as \$25. Large denominations are available as well. Savings bonds have a 30 year maturity and do not have a secondary market.
- Federal Agency Bonds: Federal agency bonds are issued by federal agencies. The bonds are backed both by the mortgages that are purchased with the proceeds and by the federal government.
- Municipal Bonds: State and local govt often issue municipal bonds to finance their budget deficit. Payments on general obligation bonds are supported by the municipal govt's ability to tax, whereas payments on revenue bonds must be generated by revenues of the projects (toll way, toll bridge, etc.). Municipal bonds typically promise semi-annual payments. Common purchasers of these bonds include financial and non-financial institutions as well as individuals. The minimum denomination of municipal bonds is typically \$5,000. A secondary market exists for them, although it is less active than the one for treasury bonds. Most municipal bonds contain a call provision, which allows the issuer to repurchase the bonds at a specified price before the bonds mature. A municipality may exercise the call option to repurchase the bonds it interest rates decline substantially because it can reissue bonds at the lower interest rate and reduce its cost of financing.
- Variable Rate Municipal Bonds: Variable rate municipal bonds have a floating interest rate based on a benchmark interest rate. The coupon payment adjusts to movements in the benchmark interest rate (LIBOR). Some variable rate bonds are convertible to a fixed rate bond until maturity under specified conditions. In general, these bonds are desirable to investors who expect that interest rate will rise.
- Corporate Bonds: When corporations need borrow for longer term periods, they issue corporate bonds which usually promise the owner interest on a semi-annual basis. The minimum denomination is \$1,000. Larger bond offerings are normally achieved through public offerings, which must first be registered with the SEC. The bonds issued by smaller corporations tend to be less liquid because their trading volume is relatively low.
- Although most corporate bonds have maturities between 10 to 30 years. Corporations such as Boeing, Ford and Chevron have recently issued 50-year bond. These bonds can be attractive to insurance companies that are attempting to match their long term policy obligations.

Bond Indenture:

- The bood indenture is a legal document specifying the rights and obligations of both issuing firm and the bondholders. Federal law requires that for each bond issue of significant size a trustee be appointed to represent the bondholders in all matters concerning the bond issue. The trustee's duties include monitoring the issuing firm's activities to ensure compliance with the terms of the indenture. If the terms are violated, the trustee initiates legal action against the issuing firm and represents the bondholders in that action.
- Sinking fund provision: Bond indentures frequently include a sinking fund provision or a requirement that the firm retire a certain amount of the bond issue each year. This provision is considered to be an advantage to the remaining bondholders because it reduces the payments necessary at maturity. For example, a bond with 20 years maturity could have a provision to retire 5% of the bond issue each year.
- **Protective covenants**: Bond indentures normally place restrictions on the issuing firm that are designed to protect the bondholders from being exposed to increasing risk during the

investment period. These so called protective covenants frequently limit the amount of dividends and corporate officer's salaries the firm can pay and also restrict the amount of additional debt the firm can issue.

Call provision: Some corporate bonds are issued with call provisions allowing the issuer to repurchase the bond at a specified call price before the maturity date. For example, if a company issues a bond with higher coupon rate when market interest rates are high, and interest rates later fall, the firm might like to retire the high coupon bond and issue new bonds with lower coupon rate to reduce coupon payments. This is called refunding. Callable bonds typically comes with a period of call protection, an initial time during which the bonds are not callable. Such bonds are referred to as deferred callable bonds. A call provision normally requires the firm to pay a price above par value when it calls its bonds. The difference between the bond's call price and par value is the call premium. Bondholders normally view a call provision as a disadvantage because it can disrupt their investment plans and reduce their investment returns. As a result, firms must pay slightly higher rates of interest on bonds that are callable, other things being equal \Im FIN (D)

Bond Collateral:

- Bonds can be classified according to whether they are secured by collateral and by the nature of that collateral. Usually the collateral is a more gage on real property (land and buildings).
 - First Mortgage Bond: A first mortgage bond has first claim on the specified assets.
 - Chattel Mortgage Bond: A chattel mortgage bond is secured by personal property.
 - Debentures: Bonds unsecured by specific property are called debentures (backed only by the general credit of the issuing fine). These bonds are normally issued by large, financially sound firms whose ability to service the debt is not in question.
 - Subordinated Debentures: Subordinated debentures have claims against the firms assets that are junior to the daims of both mortgage bonds and regular debentures. Owners of subordinated debentures receive nothing until the claims of mortgage bondholders, regular term owners, and secured short term creditors have been satisfied. The main our burchasers of subordinated debt are pension funds and insurance companies.

Other types of **Bonds**:

- Convertible Bond: Convertible bond allows investors to exchange the bond for a stated number of shares of the firm's common stock. This conversion feature offers investors the potential for high returns if the price of the firm's common stock rises. Investors are therefore willing to accept a lower rate of interest on these bonds, which allows the firm to obtain financing at a lower cost.
- **Puttable bonds**: While the callable bond gives the issuer the option to extend or retire the bond at the call date, the extendable or put bond gives this option to the bondholder. If the bond's coupon rate exceeds current market interest rate, the bondholder may choose to extend the bond's life. If the bond's coupon rate is too low, it will be optimal not to extend; the bondholder instead claims principal which can be invested at current yields.
- **Inverse Floaters**: These bonds are opposite to floating rate bonds; the coupon rate on these bonds falls when the general level of interest rates rises.

• Junk Bonds: Credit rating agencies assign credit ratings to corporate bonds based on their perceived degree of credit risk. Those bonds that are perceived to have high risk are referred to as speculative grade or junk bond. Junk bonds are also known as high-yield bonds.

On the contrary, bonds with good credit rating are called investment-grade bond.

Preferred Stock:

- Although preferred stock is considered to be equity, it is often included in the fixed income securities like bond. This is because, like bonds, preferred stock promises to pay a fixed stream of dividends. However, unlike bonds, the failure to pay the promised dividend does not result in corporate bankruptcy. Instead, the dividends owed simply accumulates, and the common stockholder may not receive any dividends until the preferred stockholders have been paid in full.
- In the event of bankruptcy, preferred stockholders' claim to the firm's easiets have lower priority than those of bondholders, but higher priority than those of compon stockholders.

Bond Price and Impact of Change in Interest Rate on it.

• Yield to Maturity (YTM): Yield to Maturity is defined as the interest rate that makes the present value of future cash flows from the bond equal to its market price. In other words it is the internal rate of return to the bond.

Bond Price,
$$P = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^6} + \frac{C_3}{(1+y)^6} + \frac{(C_n + M)}{(1+y)^n}$$

- **Yield Curve**: Yield curve is a plot of **yield** to maturity as a function of time to maturity. The yield curve is also called the term structure of interest rates and it helps us extract the appropriate rates that should be used to discount cash flows at different maturities. There are various relationship between yield and their maturity. Most common patterns:
- Upward sloping
- Downward sloping (inverted)
- Flat
- Hump-shaped (rising and then falling)

Rules of Interest Rate Sensitivity of a Bond:

- Bondprices and yields are inversely related; bond prices decrease when yields rise, and that the price curve is convex, meaning that decreases in yields have bigger impacts on price than increases in yields of equal magnitude.
- Prices of long term bonds tend to be more sensitive to interest rate changes than prices of short term bonds. If rates increase, for example, the bond is less valuable as its cash flows are discounted at a now-higher rate. The impact of the higher discount rate will be greater as that rate is applied to more distant cash flows.
- The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increase.
- Interest rate risk is inversely related to the bond's coupon rate. Prices of low coupon bonds are more sensitive to changes in interest rates than prices of high coupon bonds.

 Bonds with higher yield to maturity is less sensitive to changes in interest rates and vice versa, i.e. bond price sensitivity falls with yield to maturity.

Effective Maturity Concept (Duration):

- Bond price sensitivity falls with yield to maturity. A higher yield reduces the present value of all of the bond's payments, but more so for more distant payments. Therefore, at a higher yield, a higher fraction of the bond's value is derived from its earlier payments, which have lower effective maturity and interest rate sensitivity.
- To deal with the ambiguity of the "maturity" of a bond making many payments, we need a measure of the average maturity of the bond's promised cash flows to serve as a useful summary statistic of the effective maturity of the bond.
- Frederick Macaulay termed the effective maturity concept the duration of the bond. Macaulay's duration equals the weighted average of the times to each couper or principal payment made by the bond. The weight associated with each payment time clearly should be related to the "importance" of that payment to the value of the bond. In fact, the weight applied to each payment time is the present value of the payment divided by the bond price.

$$w_{t} = \frac{C F_{t} / (1 + y)^{t}}{B \text{ ond Price}}$$

- Macaulay's Duration is given by, $D = \sum_{t=1}^{n} t x w_{t}$
- Duration is a very useful measure of bond portfolio's interest rate sensitivity. It is a useful tool for immunization against interest rate risk
- It can be shown that when interest rate changes, the proportional change in bond' price is related to the change in its YTM (y).

Bond price consitivity,
$$\frac{\Delta P}{P} = -\frac{D}{(1+y)} \ge \Delta y \%$$

Rules for Duration:

- 1) The duration of a zero-coupon bond equals its time to maturity (since there are no interim cash flows)
- 2) Holding maturity constant, a bond's duration is lower when the coupon rate is higher. The higher the coupon payments, the higher the weights on the early payments, and the lower is the weighted average maturity (duration) of the payments.
- 3) Hadding the coupon rate constant, a bond's duration generally increases with its time to the advertee of introducing more payments in the distant future). However, duration heed not always increase with time to maturity. It turns out that for some deep discount coupon bonds, duration may fall with increases in maturity.
- 4) Ceteris paribus, the duration of a coupon bond is higher, when the bond's yield to maturity is lower. Because at lower yields the more distant payments made by the bond have relatively greater present values and accounts for greater weights of the bond's value.

Limitation of Duration:

- Bond price sensitivity equation asserts that the percentage change in bond price is directly proportional to the change in the bond's yield. If this were exactly so, there should be a straight line relationship between the percentage change in bond's price and change in its yield. However, the true price-yield relationship is said to be convex and the curvature of the price-yield curve is called the convexity of the bond.
- Duration underestimates the increase in bond price when the yield falls, and it overestimates the decline in price when the yield rises.
- The duration rule is a good approximation for small changes in the bond yield or interest rates, but it is less accurate for larger changes. To improve this approximation, we need a correction term, known as convexity.

Convexity:

rrection term, known as convexity. **Exity:** The convexity of a bond equals the second derivative of the price yield curve divided by bond price by bond price.

Convexity, C =
$$\frac{1}{P} \frac{d^2 p}{dy^2} = \frac{1}{P(1+y)^2} \sum_{1}^{T} \left[\frac{C R}{(R + y)^2} (t^2 + t) \right]$$

The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -\frac{D}{1+y} \times \Delta y + 1/2 \sum_{x=0}^{\infty} C \times (\Delta y)^{2}$$

Immunization:

- Immunization programs aim at protecting bond portfolios against interest rates shifts. They attempt to eliminate the portfolio's sepsitivity to shifts in the yield curve by matching the duration of the assets to the duration of the liabilities (duration matching).
- Since duration is a measure of sensitivity for bond returns, two bonds (or portfolios of bonds) with the same duration will have their values changed by the same amount due to a common interest rate shock.
- If duration matching is successful, then any rise/ fall in the present value of the portfolios assets due to a shift in the yield curve will be offset by a fall/ rise of equal magnitude in the present value of the portfolios liabilities. Therefore, the net position will be immunized against these hifts. Note: The duration of a bond portfolio is equal to the weighted average of the constituent bonds" durations (weighted according to their relative value).

de de Immunization Strategies:

- Focused strategy: Find a portfolio of bonds with each bond having a duration close to the duration of the liability.
- Barbell strategy: Use bonds with very different durations and mix them accordingly to construct the desired duration.
- A barbell strategy provides more *flexibility* (necessary as time passes) relative to the focused strategy, but it also incorporates a greater degree of potential inaccuracy.

Risks or Problems with Immunization:

- Successful immunization depends on the use of a correct measure of duration, which depends on the assumed shape of the yield curve. For an incorrect choice, we won't have a perfectly immunized portfolio.
- Even if a portfolio is perfectly immunized, with the passage of time or small changes in the yields, the portfolio ceases to be immunized. Hence, portfolio rebalancing is necessary to immunize it according to the new conditions (i.e. it is an active strategy!). Only exact cash flow matching implies an always immunized portfolio (this is obviously more difficult and costly to construct).
- Duration provides a rough approximation. Incorporating convexity considerably enhances the approximation. Therefore, most bond portfolio managers engage in immunization by matching both convexity and duration. Convexity matching leads to a very successful immunization, but it also implies a higher cost of constructing the portfolio since fewer sets of bonds can achieve both targets

Example: Consider a semi-annual 8% coupon bond with 2 year maturity i.e. n = 4, C =\$40, M =\$1,000 and yield to maturity, y = 10%. Calculate the bond s current price (P), modified duration and convexity. If the bond's yield to maturity increases by 0.02 percentage points, calculate the change in the bond price and therefore, the new price.

Time	Cash Flow	Present Value	Weight	Time*Weight
1	40	38.095	0,0395	0.0395
2	40	36.281	0 .0376	0.0752
3	40	34.554	0.0358	0.1074
4	1040	855.61 Price, Σ - 264 .54	0.887	3.548 Duration, D Σ = 3.77

$$i.e.\frac{\Delta P}{P} = -\frac{D}{(1+y)} \ge \Delta y = -\frac{3.77}{10.5} \ge 0.01\% = -0.0359\%$$

 $P \quad (1 + y)$ B ond price will decrease by 0.0359% *i.e.* $\Delta P = P(-0.0359\%) = -\$964.54 \times 0.0359\% = -\0.346 B ond price will decrease by \$0.346 New Price = \$664.54 - \$0.346 = \$964.19

6	J Dime	Cash Flow	Present Value	Pv(t + t)		
2	1	40	38.095	76.19		
/	2	40	36.281	217.686		
	3	40	34.554	414.648		
	4	1040	855.61	17112.2		
			Price, $\Sigma = 964.54$	Σ=17820.724		
Convexity, C = $\frac{1}{P(1+v)^2} \sum_{i=1}^{T} \left[\frac{CF_i}{(1+v)^i} (t^2+t) \right] = \frac{1}{964.54(1.05)^2} x17820.724 = 16.758$						

The percentage change in the value of a bond after accounting convexity equals,

$$\frac{\Delta P}{P} = -D^* \ge \Delta y + 1/2 \ge C \ge (\Delta y)^2$$

= -3.59 \exp(0.0001 + 0.5 \exp(16.758 \exp(0.0001)^2) = -0.0003589 = -0.0359%
i.e. \Delta P = P(-0.0359%) = -\\$0.346

Example: An insurance company must make a payment of \$19,487 in 7 years. The market interest rate is 10%, so the present value of the obligation is \$10,000. The company's portfolio manager wishes to fund the obligation using 3-year zero coupon bonds and perpetuities paying annual coupons. How can the manager immunized the obligation?

Sol: Immunization requires that the duration of the portfolio of assets equal the duration of the liability. We can proceed in four steps:

1. Calculate the duration of the liability: It is a single payment obligation with duration of 7 years.

2. Calculate the duration of the asset portfolio: The portfolio duration is the weighted average duration of each component asset, with weights proportional to the bunds placed in each asset. The duration of the zero coupon bond is simple its maturity, by years. The duration of the perpetuity is (1+y)/y = 1.10/0.10 = 11 years.

Asset duration = $w \ge 3$ years + (1-w) ≥ 11 years Asset duration = Liability duration Asset duration = $w \ge 3$ years + (1-w) ≥ 11 years =7 years

i.e. w = 0.5

3. The manager should invest half of the portfolio in the zero coupon bonds and the rest half in perpetuities. This will result in an asset duration of 7 years.

4. Because the obligation has a present value of \$10,000, the manager must purchase \$5,000 of the zero coupon bonds and \$5,000 of the perpetuities. (note that the face value of the zero will be $5000 \times (1.10)^3 = 56055$



Bond Valuation Problems

Question 1: How much should you pay today to purchase a 10% semi-annual coupon bond of 3 years maturity where the bond comes with face value of \$1,000 and yield to maturity 12%?

Question 2: Ford issued a 30 years maturity bond 28 years before where it promises to make 12% semi-annual coupon payment and a payment of \$1,000 face value at maturity. How much should you pay today to purchase this bond if yield to maturity for 2 years Treasury note is 15%?

Question 3: Sam purchased a 50 years maturity bond with 2 years remaining at a price of \$867 where the bond promises to make 10% semi-annual coupon payment and a single payment of \$1,000 at maturity. Find the rate of yield to maturity (YTM) Sam will receive from his investment in bond.

Question 4: Mr. Karim paid \$750 to purchase a 30 years maturity bond with 2.5 years remaining, where the bond promises to make 8% semi-annual coupon payment and a single payment of \$1,000 at maturity. Find the rate of yield to maturity (YTM) Mr. Karim will receive from his investment in bond.

Question 5: Mr. Imdad is considering a 12% semi-anneal coupon bond with 3 years maturity where face value is \$5,000 and yield to maturity, y = 8%. How much Mr. Imdad should pay to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity increases by 0.6 percentage points, calculate the percentage change in the bond price and therefore, the new price.

Question 6: Mr. Azad is working in Lanka Bangla Finance Company in Bangladesh. Lanka Bangla invests a significant amount of their capital in Treasury Securities issued by Bangladesh Bank. Bangladesh Bank is going to increase their lending rate from 5% to 5.5%. Find out the impact of this change in interest rate on Lanka Bangla' Treasury security portfolio of 100,000,000 if the semi-annual duration of the portfolio is 2.67.

Question 7: Mr. Saikat is planning to invest in a 7% semi-annual coupon bond with 2.5 years maturity where face value is \$10,000 and yield to maturity, y = 10%. How much Mr. Saikat should may to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity decreases by 2.2 percentage points, calculate change in the bond price and therefore, the new price.

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Equity or Stock Valuation Models:



Constant Growth DDM or Gordon Model:

 $or, P_0 = \frac{D_1}{r_0 - r_0}$

- The constant growth DDM is valid only when "g" is less than "r". If dividends were expected to grow forever at a rate equal or faster than the discount rate, the value of the stock would be infinite or negative. If an analyst derives an estimate of g that is greater than discount rate (required return), that growth rate must be unsustainable in the long run.
- The appropriate valuation model to use in this case is a multistage DDM.
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$$P_0 = \frac{D_1}{r-g}$$
 and $P_1 = \frac{D_2}{r-g} = \frac{D_1(1+g)}{r-g} = \frac{D_1}{r-g}(1+g) = P_0(1+g)$

Therefore, the DDM implies that in the case of constant growth of dividends, the rate of price appreciation in any year will equal that constant growth rate, g.

Stock Prices and Investment Opportunities:

- A low reinvestment-rate plan allows the firm to pay higher initial dividends, but results in a lower dividend growth rate. Eventually, a high reinvestment-rate plan will provide higher dividends. If the dividend growth generated by the reinvested earnings is high enough, the stock will be worth more under the high reinvestment strategy.
- Current year's earnings =Total equity x ROE = 100 millionEarnings per Share (3 million shares outstanding) =15/3 = 5 morel•
- Earnings per Share (3 million shares outstanding) =15/3 =\$5 per share
- If the company pay out all of these earnings as dividends (cash cov), maintaining a perpetual dividend flow of \$5 per share, then the value per share will $(b_{0}) \neq 5/r$
- i.e if the required return is 12.5 %, then the value per share will be \$7.125=\$40 per share.
- However, if 60% of the earnings (\$15 million) is reinvested, then the value of the firm's assets will increase by 0.60 x 15 million = 9 million or by 9%
- Now endowed with 9% more assets, the company earns % more income, and pays out 9% higher dividends, i.e. the growth rate of the dividend will be 9%.
- Growth rate of the dividend,
- $g = ROE \times Earnings Retention Ratio = 0.15 \times 0.09 = 9\%$
- Value of the company per share will be, $P_0 = \frac{1}{(r-g)} = \frac{2}{(.125-0.09)} = \frac{57.14}{(r-g)} =$
- When growth prospects decided to reduce current dividends and reinvest some of its earnings in new investments, its stock price increased. The increase in the stock price reflects the fact that the planned investments provide an expected rate of return greater than the required rate.
- In other words the investment opportunities should have positive net present value. The value of the firm rises by the NPV of these investment opportunities. This net present value is called Net Present Value of Growth Opportunities (NPVGO).
- Value or Price per share, $P_0 =$ No-growth value per share + NPVGO
- 57.14 = 5/.125 NPVGO or, NPVGO = 17.14 per share.

Example: ABC company is run by entrenched management that insists on reinvesting 60% of its earnings in projects that provide an ROE of 10%, despite the fact that the firm's required rate of return or capitalization rate, r = 15%. The firm's year end dividend will be \$2 per share, paid out of earnings of \$5 per share. At what price will the stock sell? What is the net present value of growth opportunities? Why would such a firm be a takeover target for another firm?

Sol: Given current management's investment policy, the dividend growth rate will be, g = ROEx Retention Ratio = $0.10 \times 0.60 = 0.06 = 6\%$

- The stock price should be = 2 / (0.15 0.06) = 22.22
- NPVGO = \$22.22 (\$5/0.15) = \$22.22 \$33.33 = -\$11.11
- NPVGO is negative because the net present value of the firm's projects is negative. The rate of return from these projects is less than the required return or opportunity cost of capital.

Such a firm would be subject to takeover, because another firm could buy the firm for the market price of \$22.22 per share and increase the value of the firm by changing its investment policy (pay out all earnings; value = $\frac{5}{0.15} = 33.33$).

Present Value of Operating Cash Flows:

In this model, we are deriving the value of the total firm because we are discounting the total operating cash flows prior to the payment of interest to the debt holders. Therefore, once we estimate the value of the total firm, we must subtract the value of the debt to arrive

$$V_{0} = \sum_{t=1}^{\infty} \frac{O C F_{t}}{(1 + WA C C)^{t}}$$

$$V_0 = \frac{OCF_t}{WACC - g_{OCE}}$$

Free Cash Flow Valuation Method:

- $V_{0} = \sum_{r=1}^{\infty} \frac{OCF_{r}}{(1 + WACC)^{r}}$ If we are dealing with a mature firm whereby its operating cash flows have reached a stage of stable growth, we can adapt the constant growth DDM as follows: $V_{0} = \frac{OCF_{r}}{WACC g_{OCF}}$ **e Cash Flow Valuation Method:** An alternative approach to the dividend discount model values the firm of it. This approach is particularly useful for firms that pay no dividends, for which the dividend discount model would be difficult to implement, But free cash flow model may be applied to any firm and can provide useful insights about firm value beyond the DDM.
- One approach is to discount the free cash for the firm (FCFF) at the weighted average cost of capital to obtain the value of the firm, and subtract the then-existing value of the debt to find the value of equity.
- Another approach is to focus from the start on the free cash flow to equity holders (FCFE), discounting those directly at the cost of equity to obtain the market value of equity.
- The free cash flow to the firm is the after tax cash flow that accrues from the firm's operations, net of investments in capital, and net working capital. It include cash flows available to both debt and equity holders.
- FCFF = EBIT(1 + Depreciation-Capital Expenditures-Increase in NWC
- FCFE = FCFF Interest Expense(1-t_c)+Increase in net debt

Where
$$V_{t} = \sum_{t=1}^{T} \frac{FCFF_{t}}{(1 + WACC)^{t}} + \frac{V_{t}}{(1 + WACC)^{t}}$$
; Where $V_{t} = \frac{FCFF_{t+1}}{WACC - g}$

and equity value, we subtract the existing market value of debt from the derived value the firm. Alternatively, we can discount free cash flow to equity (FCFE) at the cost of equity.

Value of Equity =
$$\sum_{t=1}^{T} \frac{FCFE_{t}}{(1+r_{e})^{t}} + \frac{V_{t}}{(1+r_{e})^{t}}$$
; Where $V_{t} = \frac{FCFE_{t+1}}{r_{e}-g}$

As in the dividend discount model, free cash flow models use a terminal value to avoid adding the present values of an infinite sum of cash flows. The terminal value may simply be the present value of a constant growth perpetuity.

Earnings Multiplier or P/E Ratio Model:

- Many investors prefer to estimate the value of common stock using an earnings multiplier model.
- We know, Value or Price per share, P0 = No-growth value per share + NPVGO

$$P_{0} = \frac{EPS_{1}}{r} + NPVGO$$

$$P/E \text{ ratio} = \frac{P}{EPS} = \frac{1}{EPS} \left[\frac{EPS_{1}}{r} + NPVGO \right] = \frac{1}{r} + \frac{NPVGO}{EPS}$$

$$= \frac{1}{r} \left[1 + \frac{NPVGO}{EPS/r} \right] = \frac{1}{r} \left[1 + \frac{NPVGO}{No \text{ growth value of the firm}} \right]$$
if NPVGO=0 then P/E ratio = $\frac{1}{r}$

$$P_{0} / E_{1} = \frac{D_{1}}{r-g} / E_{1} = \frac{D_{1} / E_{1}}{r-g} = \frac{\text{Dividend Payout Ratio}}{r-g} = \frac{1-b}{r-ROE \times b}$$
Here, b = earnings retention ratio

Example: If we assume a stock has an expected dividend payout of 50%, a required rate of return of 12%, and an expected growth rate for dividends of 9%, then find the P/E ratio and the value of the stock per share for a current ERS of \$2.

$$P_0 / E_1 = \frac{\text{Dividend Payout Ratio}}{r - g} = \frac{0.50}{0.12 - 0.09} = 16.7$$

Here, $E_0 = \$2$ and $E_1 = \$2x1.09 = \2.18
The value or Price per share will be, $P_0 = 16\sqrt{7x2}.18 = \36.41

Price-Book Value Ratio Method: The ratio has been widely used by analysts in the banking industry as a measure of relative value. The book values of banks are considered good indicators of value because most bank assets are liquid assets such as bonds and commercial anni loans.

$$P / B V = \frac{P_t}{B V_{t+1}}$$

Price-Sales Ratio Method: Advocates of this method believe that strong and consistent sales growth is a recorrement for a growing company. Although, they note the importance of an above average profit margin, the growth process must begin with sales. In addition, given all the data the balance sheet and income statement, sales information is subject to less manipulation than any other data item.

$$P / S = \frac{P_t}{S_{t+1}}$$
Equity Valuation Problems

Question 1: A company promises to pay a fixed dividend of \$10 per preferred share. If your required return from investment is 15% then how much should you offer to purchase this share?

Question 2: ABC Textiles re-invests 50% of its earning on growth projects that yield an ROE of 20%. If company's year-end projected earnings per share (EPS) is \$10 and your required return from investment is 15% then how much should you offer per share to purchase ABC Textiles' share? How much would you offer if ABC act like a cash cow? Calculate Net Present Value of Growth Opportunity (NPVGO). Explain why ABC should act like a cash cow when its growth projects generate returns below the required return from investment.

Question 3: Beximco Pharmaceuticals follows a dividend payout policy of 40% on its earnings. The company has a historical record of achieving an average ROE of 15%. How much should you offer per share if its projected year-end dividend is 200 taka per share and your required return from investment is 12%. Calculate Net Present Value of Growth Opportunity (NPVGO).

Find NPVGO if required return increases to 16%.

Aohannnad

Question 4: Olympic Industries follows a dividend payout policy of 25% on its earnings. The company has a historical record of achieving an average ROE of 20%. How much should you offer per share if its projected year-end dividend is 100 taka per share and your required return from investment is 22%. Calculate Net Present Value of Growth Opportunity (NPVGO).

Question 5: XYZ Pharmaceuticals follows a dividend payout policy of 30% on its earnings. The company has a historical record of achieving an average ROE of 10%. Explain why XYZ can be a potential target for hostile takeover by other companies if investors' required return from investment is above 10%.

Question 6: If we assume a stock has an expected dividend payout of 50%, a required rate of return of 12%, and an expected growth rate for dividends of 9%, then find the P/E ratio and the value of the stock per share for a current EPS of \$2.

Financial Management Capital Budgeting: Basic Investment Appraisal Techniques

Investment Appraisal Techniques:

- ROCE (Return on Capital Employed) or Accounting Rate of Return (ARR) Method
- Payback Period Method
- Net Present Value (NPV) Method
- Internal Rate of Return (IRR) Method

ROCE or ARR method:

- Disadvantages:

 Simplicity
 Links with other accounting measures
 Disadvantages:
 It ignores time value of money i.e. it fails to take account of either the project life or the timing of cash flows.
 It varies depending on accounting policies
 It may ignore working capital
 mple: A project involves an immediate purchased
 Moo. It would generate annual cash for machine purchased 3. It may ignore working capital **Example: A project involves an immediate purchase of an item of machine costing** \$110,000. It would generate annual cash flows of \$24,400 for five years, starting in year 1. The machine purchased would have a scrap value of \$10,000 in five years, when the project terminates. Depreciation is on a straight line basis.

Depreciation (straight line) = (110,000, 10,000)/5 = \$20,000

$$ROCE = \frac{\text{Average annual PATE x100\%}}{\text{Initial capital costs or Average capital Investment}} = \frac{(24,400 - 20,000)x100}{\text{Initial capital costs} = 110,000}\% = 4\%$$

$$\text{Average capital investment} = \frac{110,000 + 10,000}{2} = 60,000$$

$$ROCE = \frac{(24,400 - 20,000)x100}{\text{Average apital investment} = 60,000}\% = 7.33\%$$

Decision role: If the expected ROCE for the investment is greater than the target or required r hurdle rate of return then the project should be accepted.

Accounting Profits Vs Cash Flows

An capital investment appraisal it is more appropriate to evaluate future cash flows than accounting profits, because:

- Profits can not be spent
- Profits are subjective
- Cash is required to pay dividends

Cash Flows and Relevant Costs:

Capital budgeting decisions must be based on cash flows, not accounting income. The only cash flows that should be taken into consideration in capital investment appraisal are:

- 1) Cash flows that will happen in the future
- 2) Cash flows that will arise only if the capital project goes ahead
- 3) Cash flows are direct revenues from the project and relevant costs are future costs that will be incurred or saved as a direct consequence of undertaking the investment.
- 4) The relevant cash flow for a project is the additional free cash flow that the company can expect if it implements the project.
- 5) FCFF = EBIT(1 $-t_c$)+Depreciation-net increase in capital expenditures-increase in NWC

We should ignore the following costs:

- 1) Sunk costs: Costs that have already been incurred are not relevant investment appraisal decision. For example, a company makes a non-refundable deposit as a down payment for an equipment and then reconsiders whether it want the equipment after all. The money that has already been spent can not be recovered and so is not relevant to the current decision about obtaining the equipment.
- 2) Committed costs: Costs that will be incurred anyway, whether or not a capital project goes ahead, can not be relevant to a decision about investing in the project. Fixed cost expenditures are an example of committed costs For the purpose of investment appraisal, a project should not be charged with a mount for a share of fixed costs that will be incurred anyway.
- 3) Non-cash expense, such as depreciation, conviewer be relevant to investment appraisal.

Payback period method:

- The payback period is the time a project will take to pay back the money spent on it. It is based on expected cash flows and provides a measure of liquidity.
- Payback period = Initial investment / Annual cash flow
- Decision rule: Only select projects which pay back within the specified time period or choose between option on the basis of the fastest payback.

Example: Find the pay	back period	of the following project.
mine	Year	Cash Flow (\$,000)
nair	0	(3,100)
Nor	1	1,000
2	2	900
	3	800
	4	500
	5	500

Year	Cash Flow (\$,000)	Cumulative cash flows
0	(3,100)	
1	1,000	(2,100)
2	900	(1,200)
3	800	(400)
4	500	100
5	500	600

- Payback is between the end of year 3 and end of year 4. If we assume a constant rate of cash flow through the year, we could estimate payback period will be three years plus (400/500) of 4th year, which is 3.8 years or 3 years 10 months.
- Same way discounted payback period can also be calculated.

Advantages:

- It is easily understood and easily calculated It is useful for equipment with rapidly changing technology; Thew plant is likely to be scrapped in a short period because of obsolescence, a quick payback is essential.
- It favors quick return because rapid project payback to rapid company growth, however, such policy may overlook many profitable investment opportunities because of slow payback period.
- Rapid payback also minimizes risk. It is likely that earlier cash flows can be estimated with greater certainty.
- Rapid payback maximizes liquidity.
- It uses cash flows, not accounting profit and so is likely to produce an optimistic figure.

Disadvantages:

- It ignores cash flows after the payback period
- It ignores the timings of the cash flows (time value of money)
- It is subjective -no definitive investment signal: there is no objective measure as to what length of time should be set as the minimum payback period. Investment decisions are therefore subjective

Net Present Value (NPV) method:

- NPV represents the extra cash flow earned (after deducting initial investment) from the project above the required rate of return.
- NEV involves adding present values of all cash flows associated with the project. In NPV alculation outflows and inflows are treated as negative and positive respectively.
- If NPV is positive, it indicates that the project will earn higher return than the required return and the project is financially viable.
- If NPV is zero, it indicates that the project will break-even at the required rate of return from the project and the project is financially viable.
- If NPV is negative, it indicates that the project will fail to earn required rate of return from the project and it is not financially viable.
- If the company has two or more mutually exclusive projects under consideration then it should choose the one with the highest NPV.

Sol:

Example: Find the NPV of the project to assess whether it should be undertaken. (cost of capital is 6%)

Year	0	1	2	3	4	Total
Cash	(25,000)	6,000	10,000	8,000	7,000	
PVs	(25,000)	5,660.37	8,899.96	6,716.95	5,544.65	1,821.95

The project gives excess cash flows of \$1,822 above the required return. As the project has positive NPV, the project is financially viable and can be accepted.

Advantages of NPV:

- 1) It considers time value of money: discounting cash flows to PV takes account of the impact of intervention of the impact of intervention. impact of interest, inflation, and risk over time.
- It is an absolute measure of return: the NPV of an investment represents the actual surplus raised by the project arefindy
- 3) This method is based on cash flows not profits
- 4) It considers the whole life of the project

Disadvantages of NPV:

- 1) This method is difficult to explain to managers. To yoterstand the meaning of the NPV calculated requires an understanding of discounting. The method is not as intuitive as techniques such as payback period method.
- 2) It requires knowledge of the cost of capital
- 3) It is relatively complex.

Internal Rate of Return (IRR) method

- The IRR is another project appraised method that use discounted cash flow techniques
- The IRR represents the discount rate at which the NPV of an investment is zero. As such it represents a break-even cost of capital.
- Decision rule: Investment type of project (i.e. projects with initial outflow and followed by inflows from the project): Projects with IRR greater than the cost of capital should be accepted.
- Financing type of projects (i.e. cash inflows precede cash outflows. This type of projects generate revenue first- customers pay in advance.): Accept the project when the IRR is less than the discount rate.
- Calculate two NPVs for the project at two different costs of capital for which the NPV is positive and negative respectively but close to zero.

Find the IRR using the formula: $IRR = L + \left[\frac{NPV_L}{NPV_L - NPV_H}x(H - L)\right]$

Where, L = lower rate of interest; H = higher rate of interest

Example: Find the IRR of a potential project with NPV of \$50,000 at a discount rate of 10% and -\$10,000 at a rate of 15%.

$$IRR = 0.10 + \left[\frac{50,000}{50,000 - (-10,000)}x(0.15 - 0.10)\right] = 0.10 + \left[\frac{50,000}{60,000}x0.05\right] = 14.167\%$$

Advantages of IRR:

- 1) It considers the time value of money.
- 2) IRR gives a percentage and therefore easily understood
- 3) This method uses cash flows not profits
- 4) It considers the whole life of the project

Disadvantages of IRR:

- 1) It is not a measure of absolute profitability.
- 2) Interpolation only provides an estimate and an accurate estimate requires the use of a spreadsheet program.
- 3) It is fairly complicated to calculate
- 4) Non-conventional cash flows may give rise to multiple IRRs which means the yahoo. interpolation method can't be used.

Modified IRR (MIRR) method:

- Suppose the cash flows from a project are (-\$100, \$230, -\$132), Because the project has a negative cash flow, a positive cash flow, and another negative cash flow, we say that the project's cash flows exhibit two changes of sign, or flip-flops, this type of project will have multiple IRR, such as this project have two IRR: 10% and 20%.
- In a case like this the IRR does not make any sense; which one of the two IRR should we use? There is no good reason to use one over the other, IRR simply can not be used here.
- In theory, a project with K changes in sign in the cash flows can have up to K number of IRR.
- This method combine cash flows until on the change in sign remains.

		1	2
	\$100	\$230	-\$132
Invest. rate = Borrowing rate =14%	al Are	-\$132/1.14 = -\$115.789	
		\$114.21	
V-ar	\$114.21/1.14=\$100.184		
NPV @14%	\$0.184		
and the	1		
Periods	Now(0)	1	2
Not	-\$100	\$230	-\$132
\searrow Invest. rate = 14%		-\$132/1.14 = -\$115.789	
Borrowing rate =15%		\$114.21	
	\$114.21/1.15=\$99.313		
NPV @15%	-\$0.686		

$$IRR = 0.14 + \left[\frac{0.184}{0.184 - (-0.686)}x(0.15 - 0.14)\right] = 14.21\%$$

Excel formula: MIRR(values, finance rate, reinvest rate)=MIRR(A1:A3,15%,14%)=14.55% Mohammad Kamrul Arefin, MSc. in Quantitative Finance, University of Glasgow Page | 40

	А
1	(\$100)
2	\$230
3	(\$132)
4	14.55%

This method violates the spirit of the IRR approach. The basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. This number is internal, intrinsic; it does not depend on discount rate. By contrast, MIRR is clearly a function of the discount rate.

Capital Rationing using Profitability Index (PI):

count rate. **pital Rationing using Profitability Index (PI):** When we have multiple financially viable projects (i.e. projects with positive NPV) but can not account all due to conital abortion not accept all due to capital shortage, we can rank projects based of profitability index (PI) and allocate capital accordingly.

 Profitability Index (PI) - 	N P V	0
- Trontaointy index (TT) -	Initial Investm	ent 2
	<u>Capital Bu</u>	dgeting Problems
Question 1:	[O ^O
	Year V	(240,000)
		80,000
	$\sqrt{\frac{2}{3}}$	70,000
2		40,000 20,000
xL211	6	15,000
a) Find out the navbac	7 k pariod and d	12,000 iscounted navback n

a) Find out the payback period and discounted payback period of the above project b) Find NeW of the project at 15% hurdle rate. Question 2 Year Cash Flow

Questior	12
~	2
	٢
\mathbf{y}^{*}	

Year	Cash Flow
0	(25,000)
1	6,000
2	10,000
3	8,000
4	7,000

a) Find out the payback period and discounted payback period of the above project

b) Find NPV of the project at 22% hurdle rate.

Question 3:

Year	Cash Flow
0	(50,000)
1	18,000
2	25.000

3	20,000
4	10,000

- a) Find out the payback period and discounted payback period of the above project
- b) Find NPV of the project when required rate of return is 12%.

Ouestion 4: Find IRR of the following project

01 0	
Discount Rate	NPV
18%	18849.1602
20%	8144.71879
22%	-1513.96197
24%	-10236.373

Question 5: Find IRR of the following project

Discount Rate	NPV
20%	5159.0364
22%	2747.0105
24%	563.18611
26%	-1415.7951
28%	-3210.5872
30%	-5141.7951

Stin ayahoo.com Question 6: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are divisible. Determine which projects should be chosen to maximize the return to the business.

Projects	Initial Investment	NPV
	(\$000)	(\$000)
Α	×40.×	20
В	0100	35
С	50	24
D	60	18
Ε	50	(10)

Question 7: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are indivisible. Determine the optimal project selection.

	Projects	Initial Investment	NPV
	alle	(\$000)	(\$000)
1	X	40	20
~ ````````````````````````````````````	B	100	35
J.	С	50	24
, *	D	60	18

Question 8: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are divisible and project A and C are mutually exclusive. Determine the optimal project selection.

Projects	Initial Investment	NPV
	(\$000)	(\$000)
Α	40	20
В	100	35
С	50	24
D	60	18

Financial Management Risk & Return

- Holding Period Return (HPR): $R_t = \frac{P_t P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} 1$
- Holding Period Return (HPR) including dividend: $R_t = \frac{P_t P_{t-1} + D_t}{P_{t-1}} = \frac{P_t + D_t}{P_{t-1}} 1$
- **Continuously compounded return :** $R_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$
- Expected Return: This is the return that an investor expects a security to carn over the next period. Off course this is only an expectation, the actual return may be either higher or lower. An investor's expectation may simply be the average return perperiod a security has earned over the past.

W ithout probability:
$$E(r) = \overline{r}$$

- Here, r = return and p = probability corresponding return
- With probability: $E(r) = \overline{r} = \sum_{n=1}^{\infty} r_{n}$ Here, r = return and p = probability and the function of the functio **Risk**: The uncertainty about the future return from an investment is called the risk or volatility and can be measured using variance or standard deviation of the return.

Variance,
$$\sigma^2$$
 (without probability) = $\frac{\sum (r_i - \overline{r})^2}{n-1} = \frac{1}{n-1} [\sum r_i^2 - \frac{(\sum r_i)^2}{n}]$
Variance, σ^2 (with probability) = $\sum (r - \overline{r})^2 p = \sum r^2 p - [E(r)]^2$
ple:

Example:

	Security	1	В		С		D	
	Rate of return	Frob	Rate of	Prob	Rate of return	Prob	Rate of return	Prob.
		1	5%	1	-10%	1⁄4	- 20%	1⁄4
.0					0	1⁄4	10	1⁄2
Ly.					20	1/2	40	1⁄4

- Expected return:
- Security A: 6%
- Security B: 5%
- Security C: $-(0.25 \times 10\%) + (0.25 \times 0) + (0.5 \times 20\%) = 7.5\%$
- Security D: -(0.25x20%)+(0.5x10%)+(0.25x40%)=10%

Example:

Purchase price=\$100						
State of the Economy Prob Year-end price Cash Dividends Holding Period Return (HPF						
Boom	0.3	129.5	4.5	?		
Normal	0.5	110.0	4.0	?		
Recession 0.2 80.5 3.5 ?						
Expected Return = ?						

Sol:

Purchase price=\$100						
State of the Economy Prob Year-end price Cash Dividends Holding Period Return (HP						
Boom	0.3	129.5	4.5	(129.5+4.5-100)/100=0.34		
Normal	0.5	110.0	4.0	(110+4-100)/100-0.14		
Recession	0.2	80.5	3.5	(80.5+3.5-100) 00=-0.16		
Expost	Even a stad D strum = $(0.24 \times 0.2) + (0.14 \times 0.5) + (-0.14 \times 0.2) = 0.14$					

Expected Keturn = (0.34x0.3)+(0.14x0.5)+(0

$$= [(0.34^{2}x0.3) + (0.14^{2}x0.5) + (-0.16^{2}x0.2)] - 0.14^{2} = 0.03$$

Standard Deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{0.03} = 0.1732$

Portfolio Returns and Risk:

- Triance, $\sigma^2 = \sum r^2 p [E(r)]^2$ (0.34² x0.3) + (0.14² x0.5) + (-0.16² x0.2)] 0.14² = 0.03 and ard Deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{0.03} = 0.1732$ The second return is the weighted average of the expected returns of all asset held in your portfolio.
- w_i are weights, i.e. the investment proportions, and $E(R_i)$ expected return on individual
- asset. Expected return on a portfolio: $E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$
- Expected return on a portfolio (2 assets): $E(R_p) = w_A x R_A + w_B x R_B$
- Expected return on a portfolio (3 assets): $E(R_p) = w_A x R_A + w_B x R_B + w_C x R_C$
- Variance of a portfolio (2 assets): $\sigma_{p}^{2} = w_{a}^{2} \sigma_{A}^{2} + w_{B}^{2} \sigma_{B}^{2} + 2w_{A}w_{B}\sigma_{AB} = w_{A}^{2} \sigma_{A}^{2} + w_{B}^{2} \sigma_{B}^{2} + 2w_{A}w_{B}\sigma_{A}\sigma_{B}\rho_{A,B}$

Here σ_p^2 = Portfolio Variance; σ_A^2 = Variance of Security A

 \mathbf{N}_{B}^{\prime} = Variance of Security B; w_{A} = Proportion of investment on A

 w_B = Proportion of investment on B; $\rho_{A,B}$ = Correlation coefficient between A & B

Variance of a portfolio (3 assets):

$$\sigma_{p}^{2} = w_{A}^{2}\sigma_{A}^{2} + w_{B}^{2}\sigma_{B}^{2} + w_{C}^{2}\sigma_{C}^{2} + 2w_{A}w_{B}\sigma_{AB} + 2w_{A}w_{C}\sigma_{AC} + 2w_{B}w_{C}\sigma_{BC}$$
$$= w_{A}^{2}\sigma_{A}^{2} + w_{B}^{2}\sigma_{B}^{2} + w_{C}^{2}\sigma_{C}^{2} + 2w_{A}w_{B}\sigma_{A}\sigma_{B}\rho_{A,B} + 2w_{A}w_{C}\sigma_{A}\sigma_{C}\rho_{A,C} + 2w_{B}w_{C}\sigma_{B}\sigma_{C}\rho_{B,C}$$

Year	Prob.	GM	BP	Portfolio	
2	1/4	0.10	0.15	0.1333	
	1⁄4	-0.05	0.10	0.05	
	1/4	0.08	0	0.0267	
	1/4	0.15	-0.01	0.0433	$\overline{\mathbf{a}}$
Expected	Rate of return	0.07	0.06	0.0633	cott

Example: Calculate the expected return on a portfolio that has one third of your wealth invested in GM and two thirds in BP.

Covariance and Correlations: Returns on individual securities are related to one another. Covariance is a statistic measuring the inter-relationship between two securities. Alternatively, this relationship can be restated in terms of the correlation between the two

Securities. A sample covariance is $Cov(x, y) = \sigma_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{1}{n-1} [\sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}]$ Correlation Coefficient $(x, y) = r_{xy} = \frac{\sigma_{xy}}{\sigma_{xy}} = \frac{\sigma_{xy}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}}$ Example: Out of \$100. if an investor

Example: Out of \$100, if an investor invests \$60 in Security A and remaining \$40 in security B, then find the expected return and variance of the portfolio.

	.1 0	7					
	State of the Economy	Rate of Return from Security A	Rate of Return from Security B				
	Bgeth	-20%	5%				
	Normal 10%		20%				
	Recession	30%	-12%				
V	Depression	50%	9%				
Σ	$x^{2} = 0.39; \qquad \sum y^{2}$	$= 0.065; \sum x = 0.7; \sum y =$	0.22; $\sum xy = 0.019;$				
\overline{x}	$= 0.175; \overline{y} = 0.055$; $\sigma_x = 0.2586$; $\sigma_y = 0.115$					
C	Correlation Coefficient $(x, y) = r_{in} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{n}$						
	$\sqrt{\{\sum x_i^2 - \frac{(\sum x_i)^2}{n}\} \{\sum y_i^2 - \frac{(\sum y_i)^2}{n}\}}$						

$$= \frac{0.019 - \frac{0.7 \times 0.22}{4}}{\sqrt{(0.39 - \frac{0.7^2}{4})(0.065 - \frac{0.22^2}{4})}} = -0.1639$$

Expected Return, $E(R_i) = w_A x R_A + w_B x R_B = 0.6 x 0.175 + 0.4 x 0.055 = 0.127$ $\sigma_{p}^{2} = w_{A}^{2}\sigma_{A}^{2} + w_{B}^{2}\sigma_{B}^{2} + 2w_{A}w_{B}\sigma_{A}\sigma_{B}\rho_{A,B}$ $= 0.6^{2} x 0.2586^{2} + 0.4^{2} x 0.115^{2} + 2 x 0.6 x 0.4 x 0.2586 x 0.115 x (-0.1639) = 0.02385$ $\sigma_{p} = \sqrt{0.02385} = 0.1544 = 15.44\%$ Systematic and Non-systematic Risk:

- When an investor diversifies across assets that are not perfectly correlated, the portfolio's risk is less than the weighted average of the risks of the fodividual securities in the portfolio. The risk that is eliminated by diversification is ealed unsystematic risk (also called unique, diversifiable, or firm specific risk). Since the market portfolio contains all the risky assets, it must be well a diversified portfolio the risk that remains and can not be diversified away, is called the systematic risk (also called non-diversifiable or market risk).
- The concept of systematic risk applies to individual securities as well as to portfolios. Some securities' returns are highly correlated with overall market returns. Examples of firms that are highly correlated with market returns are luxury goods manufacturers such as Ferrari Automobiles and Harley Davidson Motorcycles. These firms have high systematic risk (i.e they are very responsive to marke
- Other firms, such as utility companies, respond very little to changes in the systematic risk factors. These firms have very little systematic risk. Hence total risk (measured by standard deviation) can be broken down into its component parts: unsystematic risk and systematic risk.



Figure 4: Risk vs. Number of Portfolio Assets

Relationship between Risk and Expected Return (CAPM: Capital Asset Pricing Model):

 Capital Asset Pricing Model (CAPM) implies that the expected return on a security is linearly related to its beta. Because the average return on the market has been higher than the average risk free rate over long periods of time, market premium is presumably positive. Thus the formula implies that the expected return on a security is positively related to its beta.

Capital Asset Pricing Model (CAPM):

$$E(R_{i}) = R_{f} + \beta_{i} x [E(R_{m}) - R_{f}]$$
Risk-free rate + Beta of the security x Market Risk Premium

- If $\beta=0$, that is, the expected return on the security is equal to the risk free rate. Because a security with zero beta has no relevant risk, its expected return should equal the risk-free rate.
- If $\beta=1$, that is, the expected return on the security is equal to the expected return on the market. This makes sense because the beta of the market portfolio is also 1.
- Security Market Line (SML) is the graphical depiction of the capital asset pricing model (CAPM).



 The CML uses total risk σ on the X-axis. Hence only efficient portfolios will plot on the CML. On the other hand, the SML uses beta (systematic risk) on the X-axis. So in a CAPM world, all properly priced securities and portfolios of securities will plot on the SML.

Calculate and Interpret Beta:

- The sensitivity of an asset's return to the return on the market index in the context of the market model is referred to as its beta. Beta measures the responsiveness of a security to movements in the market portfolio.
- The contribution of a security to the variance of a diversified portfolio is best measured by beta. Therefore, beta is the proper measure of the risk of an individual security for a diversified investor.
- Beta measures the systematic risk of a security. Thus, diversified investors pay attention to the systematic risk of each security. However, they ignore the unsystematic risk of individual securities, since unsystematic risks are diversified away in a large portfolio.

$$\beta_{i} = \frac{Cov(i,m)}{\sigma_{m}^{2}} = \frac{\rho(i,m)\sigma_{i}\sigma_{m}}{\sigma_{m}^{2}} = \rho(i,m)\frac{\sigma_{i}}{\sigma_{m}}$$

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• One useful property is that the average beta across all securities, when weighted by the proportion of each security's market value to that of the market portfolio, is 1. That is, the beta of the market portfolio is 1. For aggressive securities beta>1; for defensive securities beta<1 and for neutral securities beta=1

Example: The expected return on the market is 15%, the risk free rate is 8%, and the beta for the stock A is 1.2. Compute the rate of return that would be expected (required) on this stock.

 $E(R_{A}) = 0.08 + 1.2(0.15 - 0.08) = 0.164 = 16.4\%$; Here, $\beta_{A} > 1$; $E(R_{A}) > E(R_{M})$

Example: The expected return on the market is 15%, the risk free rate is 8%, and the beta for the stock B is 0.8. Compute the rate of return that would be expected (required) on this stock.

 $E(R_{B}) = 0.08 + 0.8(0.15 - 0.08) = 0.136 = 13.6\%$; Here, $\beta_{B} < 1$; $E(R_{B}) < E(R_{M})$

Example: Acme, Inc., has a capital structure that is 40% debt and 60% equity. The expected return on the market is 12%, and the risk free rate is 4%. What discount rate should an analyst use to calculate the NPV of a project with an equity beta of 0.9 if the firm's after tax cost of debt is 5%

- Sol: The required return on equity for this project is, 0.04+0.9(0.12-0.04)=0.112=11.2%
- The appropriate discount rate is a weighted average of the costs of debt and equity for this project, 0.40x0.05+0.6x0.112=0.0872=8.72%

Example: Suppose risk free rate and market return acc. 7% and 15% respectively. Compute the expected and required return on each stock, determine whether each stock is undervalued, overvalued or properly valued and outline an appropriate trading strategy.

Stock	Price Today	E(Price) in 1 Year	E(Dividend) in 1 Year	Beta
Α	\$25	\$27	\$1.00	1.0
В	40	45	2.00	0.8
С	15	17	0.50	1.2

n
7) = 15.0%
7) = 13.4%
7) = 16.6%
Souther South International Street Southers

Stock A is *overvalued*. It is expected to earn 12%, but based on its systematic risk it should earn 15%. It plots *below* the SML.

Stock B is *undervalued*. It is expected to earn 17.5%, but based on its systematic risk it should earn 13.4%. It plots *above* the SML.

Stock C is *properly valued*. It is expected to earn 16.6%, and based on its systematic risk it should earn 16.6%. It plots *on* the SML.



- Short sell Stock A.
 - Buy Stock B.
 - Buy, sell, or ignore Stock C.

Assumptions of CAPM:

- Investors make their investment decisions according to mean-variance rule
- Investors can borrow or lend at the risk-free rate.
- No transactions cost for diversification
- Investors are price takers and have homogeneous expectations or beliefs regarding future expected returns, variances and covariance.
- All assets are marketable and perfectly divisible.
- There are no market imperfections such as taxes, regulations, or destrictions on short selling.

Arbitrage Pricing Theory (APT) or Multi-factor model vs Single Factor or Single Index Model or CAPM:

- APT has three major assumptions: 1) Capital markets are perfectly competitive 2) Investors always prefer more wealth to less wealth with certainty 3) The stochastic process generating asset returns can be represented as a K factors model.
- Multi-factor models to predict expected returns commonly use macro economic factors such as GDP growth, inflation, consumer confidence, along with fundamental factors such as earnings, earnings growth, firm size and research expenditures. The general form of a multi factor model with k factors is as follows:
- Expected Return, $E(r_i) = R_1 + \beta_1 E(factor_1) + \beta_2 E(factor_2) + \dots + \beta_k E(factor_k)$
- Single index model in contrast is a single factor or market model. The only factor is the expected excess return on the market portfolio (market index). The form of the single index model is as follows:
- Expected Return, $E(\mathbf{r}_i) = R_f + \beta_i [E(\mathbf{r}_m) R_f] \ or, E(\mathbf{r}_i) R_f = \beta_i [E(\mathbf{r}_m) R_f]$
- In this case, the beta for asset i is a measure of how sensitive the excess return on asset i is to the excess return on the overall market portfolio.

Risk & Return Problems

Security	Amount Invested (\$)	Beta				
Stock A	5,000	0.75				
Stock B	10,000	1.10				
Stock C	8,000	1.36				
Stock D	7,000	1.88				

Question 1: Suppose you have invested \$30,000 in the following four stocks:

The risk-free rate is 4% and the expected return on the market portfolio is 15%. Based on the CAPM, what is the expected return on the above portfolio?

Question 2: You have been provided the following data on the securities of three tirms and the market:

Security	\overline{R}	σ_{i}	ρ _{i,m}	man.
Stock A	0.13	0.12	(i)	J 0.90
Stock B	0.16	(ii)	0.4	0) 1.10
Stock C	0.25	0.24	0.75	(iii)
Market	0.15	0.1	(iv) (iv)	(v)
Risk-free Asset (r_f)	0.05	(vi)	(viii)	(viii)

Assume the CAPM holds true:

- a) Fill in the missing values in the table.
- b) Provide an evaluation of the investment performance of the three firms
- c) What is your investment recommendation Why?

Question 3:

a) What is the expected return and variance on an equally weighted portfolio of the following three stocks?

State of	Probability of State	Expected Retrun		
Economy	of Economy	Stock A	Stock B	Stock C
Boom	0.8	0.07	0.15	0.33
Bust		0.13	0.03	-0.06

b) What is the expected return and variance of a portfolio invested 20% each in A and B, and the remaining in C?

Question 4: Your portfolio is invested 30% each in stock A and C and the remaining in B? What is the expected return, variance and standard deviation of the portfolio?

~ D	State of	Prob.	Expected Return		
St.	Economy		Α	В	С
~	Boom	0.20	0.30	0.45	0.33
	Good	0.35	0.12	0.10	0.15
	Poor		0.01	-0.15	-0.05
	Bust	0.15	-0.06	-0.30	-0.09

Question 5: You own a stock portfolio invested 25% in stock Q, 20% in stock R, 15% in stock S, and the remaining in stock T? The betas for these four stocks are 0.75, 1.90, 1.38 and 1.16 respectively. What is the portfolio beta?

Question 6: You own a portfolio equally invested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.85 and the total portfolio is equally as risky as the market, what must the beta be for the other stock in your portfolio? Question 7: A stock has a beta of 1.25, the expected return on the market is 12% and the risk free rate is 5%. According to CAPM, what must the expected return on this stock be?

Question 8: A stock has an expected return of 14.2%, the risk free rate is 4% and the market risk premium is 7%. According to CAPM, what must the beta of this stock be?

Question 9: A stock has an expected return of 10.5%, its beta is 0.73, and the risk free rate is 5.5%. According to CAPM, what must the expected return on market be?

Question 10: A stock has an expected return of 16.2%, a beta of 1.75, and the expected return on the market is 11%. According to CAPM, what must the risk free rate be

Question 11: A stock has a beta of 0.92, and an expected return of 10.3%. A risk free asset currently earns 5%.

- a) Based on CAPM, what is the expected return on a portfolio that is equally invested in the two assets?
- b) If a portfolio of the two assets has a beta of 0.50, what are the portfolio weights?
- c) If a portfolio of the two assets has an expected return of 9%, what is its beta?
- d) If a portfolio of the two assets has a beta of 1.84, what are the portfolio weights? How do you interpret the weights for the two assets in this case? Explain.

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Question 12: Stock Y has a beta of 1.35 and an expected return of 14%. Stock Z has a beta of 0.85 and an expected return of 11.5%. The market risk premium is 6.8%.

- a) If the risk free rate is 5.5%, are these stocks correctly priced?
- b) What would the risk free rate have to be for the two stocks to be correctly priced?

Question 13:

a) If your portfolio is invested **40%** each in A and B and the remaining in C, what is the portfolio expected return, variance and the standard deviation?

State of	Prob.	Expected Return			
Economy		Α	В	С	
Boom	0.35	0.20	0.35	0.60	
Normal	0.40	0.15	0.12	0.05	
Bust		0.01	-0.25	-0.50	

- b) If the expected T-bill rate is 3.80%, what is the expected risk premium on the portfolio?
- c) If the expected inflation rate is 3.50%, what are the approximate and exact expected real returns on the portfolio? What are the approximate and exact expected real risk premiums on the portfolio?

Question 14: You want to create a portfolio equally as risky as the market, and you have \$1,000,000 to invest. Given this information, fill in the rest of the following table:

Asset	Investment (\$)	Beta
Stock A	180,000	0.75
Stock B	290,000	1.30
Stock C		1.45
Risk Free Asset		

Question 15: You have \$100,000 to invest in a portfolio containing stock X, stock Y, and a risk-free asset. You must invest all of your money. Your goal is to create a portfolio that has an expected return of 10.7% and that has only 80% of the risk of the overall market.

If X has an expected return of 17.2% and a beta of 1.8, Y has an expected return of 8.75% and a beta of 0.5, and the risk free rate is 7%, how much money will you invest in stock X?

Question 16: You have invested 40% of your capital in stock A and the remaining in stock B. Historical return over last 5 years shows the following pattern:

Voor	Stock A	Stock B
1 cal	(x)	(y)
2014	25%	15%
2013	32%	10%
2012	15%	30%
2011	20%	-5%

Find the expected portfolio return and variance.

Question 17: Security F has an expected return of 10% and a standard deviation of 26% per year. Security G has an expected return of 17% and a standard deviation of 58% per year.

- a) What is the expected return on a portfolio composed of 30% of security F and 70% of security G?
- b) If the correlation between the returns of security F and Security G is 0.25, what is the standard deviation of the portfolio described in part (a)?

Question 18: Suppose the expected returns of stocks A and B are 13% and 19% respectively whereas standard deviations are 38% and 62% respectively. Calculate expected return and standard deviation of a portfolio that is composed of 45% of A and 55% of B when the correlation between the returns on A and B is 0.5.

Question 19: You have been provided the following data on the securities of three firms and the market:

Security		σ,	$\rho_{i,m}$	β,
Stock A	Q.19	0.27	(i)	0.85
Stock B	0.14	(ii)	0.5	1.50
Stock C	0.17	0.7	0.35	(iii)
Market 🧃 🔗	0.12	0.2	(iv)	(v)
Risk-free Asset (r_f)	0.05	(vi)	(vii)	(viii)

Assume the **CAPM** holds true:

a) Fill in the missing values in the table.

b) Are these stocks A, B and C correctly priced?

c) What is your investment recommendation? Why?

Question 20: The market portfolio has an expected return of 12% and a standard deviation of 19%. The risk free rate is 5%.

- a) What is the expected return on a well diversified portfolio with a standard deviation of 7%?
- b) What is the standard deviation of a well diversified portfolio with an expected return of 20%?

Question 21: A portfolio that combines the risk free asset and the market portfolio has an expected return of 9% and a standard deviation of 13%. The risk free rate is 5%, and the expected return on the market portfolio is 12%. Assume the CAPM holds. What expected

rate of return would a security earn if it had a 0.45 correlation with the market portfolio and a standard deviation of 40%?

Question 22: Suppose the risk free rate is 4.8% and the market portfolio has an expected return of 11.4%. The market portfolio has a variance of 0.0429. Portfolio Z has a correlation coefficient with the market of 0.39 and a variance of 0.1783. According to the CAPM, what is the expected return on portfolio Z?

Question 23: Suppose market risk premium is 7.5% and the risk free rate is 4%. Which of the following stock has the most systematic risk? Which one has the most unsystematic risk? Which stock is riskier? Explain.

State of Economy	Probability	Return from Stock A	Return from Stock R
Recession	0.15	0.09	-0.30 C ^O
Normal	0.55	0.42	0.12 0.
Good	0.30	0.26	0,440

Ouestion 24: Suppose you observe the following situation:

-	0.00	0120		
ippose y	ou observe th	e following sit	tuation:	ayar
Se	curity	Beta	Expected	l Return
	Α	1.4	0.1	50
	В	0.9	0.1	ÉÉ
				7

Mohammad Kammu Arefins unohammad Assume these securities are correctly priced. Based on the CAPM, what is the expected

Financial Management Cost of Capital

Cost of Capital:

- If a firm's capital is financed with equity, then the cost of capital would be the required rate of return on equity. However, most firms use different types of capital, and due to differences in risk, these different securities have different required rates of return.
- The cost of capital used to analyze capital budgeting decisions should be a weighted average cost of the various capital sources (WACC: Weighted Average Cost of Capital)

$$WACC = w_{d}r_{d}(1-T) + w_{p}r_{p} + w_{e}r_{e}$$

- **Cost of debt**= $r_d (1 T)$
- **Cost of Preferred stock**, \mathbf{r}_{p} = Preferred Dividend / Price
- Cost of Common Equity, \mathbf{r}_e : Cost of equity can be estimated using the following methods: Capital Asset Pricing Model (CAPM): $\mathbf{E}(\mathbf{R}_i) = \mathbf{R}_f + \beta_i [\mathbf{E}(\mathbf{R}_m) \mathbf{R}_f]$

Discounted cash flow method:

$$P_0 = \frac{D_1}{r_e - g}$$
; i.e. $r_e = \frac{D_1}{P_0} + g$ = Div yield + Growth or Capital gain

Cost of Capital Problems and Solut

Question 1: The Dybvig Corporation's common stock has a beta of 1.15. If the risk free rate is 4.5% and the expected return on the market is 11%, what is Dybbig's cost of nâ equity capital?

Question 2: The Devon Co. just issued a dividend of \$2.4 per share on its common stock. The company is expected to maintain a constant 5.5% growth rate in its dividends indefinitely. If the stock sells for \$52 a share, what is the company's cost of equity?

Question 3: Stock in Country Road Industries has a beta of 0.85. The market risk premium is 8%, and T-bills are currently yielding 5%. The company's most recent dividend was \$1.6 per share, and dividends are expected to grow at a 6% annual rate indefinitely. If the stock sells for \$37 per share, what is your best estimate of the company's cost of equity?

Question 4: Advance Inc., is trying to determine its cost of debt. The firm has a debt issue outstanding with 12 years to maturity that is quoted at 95% of face value. The issue makes semiannual payments and has a coupon rate of 8% annually. What is Advance's pretax cust of debt? If the tax rate is 35%, what is after tax cost of debt?

Question 5: Shanken Corp. issued a 30 years, 7% semiannual bond 7 years ago. The bond currently sells for 108 percent of its face value. The company's tax rate is 35%.

- a) What is the pretax cost of debt?
- b) What is the after tax cost of debt?
- c) Which is more relevant, the pretax or the after tax cost of debt? Why?
- d) Suppose the book value of the debt issue is \$60 million. In addition, the company has a second debt issue on the market, a zero coupon bond with seven years left to maturity; the book value of this issue is \$80 million and the bonds sell for 73% of par. What is the company's total book value of debt? The total market value? What is your best estimate of the after tax cost of debt now?

Question 6: Fama's LIamas has a weighted average cost of capital of 9.8%. The company's cost of equity is 15%, and its cost of debt is 7.5%. The tax rate is 35%. What is Fama's debt-equity ratio?

Question 7: Filer manufacturing has 7.5 million shares of common stock outstanding. The current share price is \$49, and the book value per share is \$4. Filer Manufacturing also has two bond issues outstanding. The first bond issue has a face value of \$60 million, pays 7% semi-annual coupon and sells for 93% of par. The second issue has a face value of \$50 million, pays 6.5% semiannual coupon, and sells for 96.5% of par. The first issue matures in 10 years, the second in 6 years.

- a) What are Filer's capital structure weights on a book value basis?
- b) What are Filer's capital structure weights on a market value basis?
- c) Which are more relevant, the book value or market value weights? Why? ,
- d) Suppose the company's stock has a beta of 1.2, the risk free rate is 5.2%, and the market risk premium is 7%. Assume that the overall cost of debt is the weighted average of the two outstanding debt issues. The tax rate is 35%. What is the company's WACC?

Question 8: Kose Inc., has a target debt equity ratio of 0.65. Its WACC is 11.2%, and the tax rate is 35%.

- a) If Kose's cost of equity is 15%, what is its pretax cost of debt?
- b) If instead you know that the after tax cost of debt is 6.4%, what is the cost of equity?

Question 9: Given the following information for Huntington Power Co. find the WACC. Assume the company's tax rate is 35%, market tak premium 7% and risk free rate 6%.

Debt	5,000 8% semiannual coupon bonds of \$1,000 par value, 20 years maturity
	remaining, selling for 103% of par
Common stock	160,000 shares outstanding, selling for \$57 per share; the beta is 1.10.

Question 10: Titan Mining Corporation has 8.5 million shares of common stock outstanding and 200,000 7.5% semiannual bonds outstanding with par value of \$1,000 each. The common stock currently sells for \$34 per share and has a beta of 1.2 and the bonds have 15 years to maturity and sell for 93% of par. The market risk premium is 7%, T-bills are yielding 5%, and company's tax rate is 35%.

- a) What is the firm's market value capital structure?
- b) If Titan Mining is evaluating a new investment project that has the same risk as the first's typical project, what rate should the firm use to discount the project's cash flows?

Question 11: Suppose your company needs \$20 million to build a new assembly line. Your target debt-equity ratio is 0.75. The floatation cost for issuing new equity is 8%, but the floatation cost for debt is only 5%. Your boss has decided to fund the project by borrowing money because the floatation costs are lower and the needed funds are relatively small.

- a) What do you think about the rationale behind borrowing the entire amount?
- b) What is your company's weighted average floatation cost, assuming all equity is raised externally?
- c) What is the true cost of building the new assembly line after taking floatation costs into account?

Question 12: Southern Alliance Company needs to raise \$45 million to start a new project and will raise the money by selling new bonds. The company will generate no internal equity for the foreseeable future. The company has a target capital structure of 65% common stock, 5% preferred stock, and 30% debt. Floatation costs for issuing common stock are 9%, for new preferred stock 6%, and for new debt 3%. What is the true initial cost figure Southern should use when evaluating its project?

Question 13: Och, Inc. is considering a project that will result in initial after tax cash savings of \$3.5 million at the end of the first year, and these savings will grow at a rate of 5% per year indefinitely. The firm has a target debt-equity ratio of 0.65, a cost of equity of 15%, and an after tax cost of debt of 5.5%. The cost saving proposal is somewhat riskier than the usual projects the firm undertakes; management uses the subjective approach and applies an adjustment factor of +2% to the cost of capital for such risky projects. Under what circumstances should Och take on the project?

Question 14: Goodbye Inc. recently issued new securities to finance a new TV show. The project cost \$15 million, and the company paid \$850,000 in floatation costs. In addition, the equity issued had a floatation cost of 7% of the amount raised, whereas the debt issued had a floatation cost of 3% of the amount raised. If Goodbye issued new securities in the same proportion as its target capital structure, what is the company's target debt-equity ratio?

Question 15: Photochronograph Corporation (PC), manufactures time series photographic equipment. It is currently at its target debt-equity ratio of 0.7. It's considering building a new \$45 million manufacturing facility. This new plant is expected to generate after tax cash flows of \$6.2 million a year in perpetuity. The company raises all equity from outside financing. There are three financing options:

- 1) A new issue of common stock: The floatation costs of the new common stock would be 8% of the amount raised. The required return on the company's new equity is 14%.
- A new issue of 20 year bonds: The floatation costs of the new bonds would be 4% of the proceeds. If the company issues these new bonds at an annual coupon rate of 8%, they will sell at par.
- 3) Increased use of accounts payable financing: Because this financing is part of the company's ongoing daily business, it has no floatation costs, and the company assigns it a cost that is the same as the overall firm WACC. Management has a target ratio of accounts payable to long term debt of 0.20 (assume there is no difference between the pretax and after tax accounts payable cost.)

What is the NPV of the new plant? Assume that PC has a 35% tax rate.

Question 16: Trower Corp. has a debt equity ratio of 1.2. The company is considering a new plant that will cost \$145 million to build. When the company issues new equity, it incurs a floatation cost of 8%. The floatation cost on new debt is 3.5%.

- a) What is the initial cost of the plant if the company raises all equity externally?
- b) What if it typically uses 60% retained earnings?
- c) What if all equity investments are financed through retained earnings?

Question 17: Suppose a firm has both a current and a target debt-equity ratio of 0.6, a cost of debt of 5.15%, and a cost of equity of 10%. The corporate tax rate is 34%. The firm is considering taking on a warehouse renovation costing \$60 million that is expected to yield cost savings of \$12 million a year for six years. Should the firm take on the warehouse renovation?

Question 18: Consider a firm whose debt has a market value of \$40 million and whose stock has a market value of \$60 million (3million outstanding shares of stock, each selling for \$20 per share). The firm pays a 5% rate of interest on its new debt and has a beta of 1.41. The corporate tax rate is 34%. The risk premium on the market is 9.5% and that the current Treasury bill rate is 1%. What cost of capital the firm should use to discount its future cash flows?

Question 19: The Weinstein Corporation has a target capital structure of 80% equity and 20% debt. The floatation costs for equity issues are 20% of the amount raised; the floatation costs for debt issues are 6%. If Weinstein needs \$65 million for a new manufacturing facility, how much it will have to raise?

Question 20: Tripleday Printing Company is currently at its target debt-equity ratio of 100%. It is considering building a new \$500,000 printing plant in Kansas. This-new plant is expected to generate after tax cash flows of \$73,150 per year forever. The tax rate is 34%. To finance the project, the firm is considering all debt or all equily option. The issuance costs of the new common stock and new debt would be about 10% and 2% respectively. The required return on the company's new equity is 10% and cost of new debt is 10%. Should the firm accept the project? 34%. To finance the project, the firm is considering all debt or all equity option. The

<u>Financial Management</u> <u>Sources of Finance</u>



racions	issue to Consider N
Cost	Debt usually cheaper than equity
Duration	Long-term finance wally more expensive but secure Firms should match duration of asset purchased with duration of liability.
Term structure of interest rates	Usually short term loan is cheaper but not always.
Gearing	Using more debt is cheaper but high gearing is risky
Accessibility	Not all sources are available to all firms

Short Terre Sources of Finance:

1) Bank overdrafts 2) Bank loans 3) Better management of working capital

3) Leasing 4) Sale and leaseback

Leasing:

- A lease is a contract between a lessor and a lessee for the hire of a particular asset.
- Lessor retains ownership of the asset
- Lessor conveys the right of the use of the asset to the lessee for an agreed period and in return lessor receives specified rentals

- **Operating lease**: The lease period is less than the useful life of the asset. The lessor relies on subsequent leasing or eventual sale of the asset to cover his capital outlay and show a profit. In this type of lease, the lessor is normally responsible for repairs and maintenance.
- Finance Lease- Long Term Source of Finance: The key difference between an operating lease (short term) and a finance lease (medium to long term) is that the former equates to renting an asset whereas the latter equates to borrowing money in order to purchase the asset. Long term lease agreements are likely to be a finance lease.
- One lease exists for the whole useful life of the asset
- The lessor does not retain the risks or rewards of ownership. Lessee is responsible-for repairs and maintenance. The lease agreement can not be cancelled; the lessee the 'a yahoo.co liability for all payments.

Sale and Leaseback: Short Term Source of Finance:

- A company that owns its own premises can obtain finance by selling the property for cash and rent it back from the buyer under sale and leaseback agreement. Although this can provide an immediate source of cash, often more than that Would be obtained from a mortgage, there are a number of disadvantages including:
- The company loses ownership of the property and will therefore miss out on any ohanni appreciation in the property's future value.

Raising Equity Finance:

- There are three main sources of equity finance:
 - 1. Internally generated funds- retained earnings
 - 2. Private Placement
 - 3. Public Issue: a) Initial Public Offering (IPO) or unseasoned issue b) Seasoned issue or right issues
- Internally generated funds: Internally generated funds are earnings retained in the business, i.e. undistributed profits attributable to ordinary shareholders.
- Such finance is the ap and quick to raise, require no transaction costs, professional assistance or time delay.

Private Placement of Equity:

- Anges: For most startups, the first round of external financing comes through a private pacement of equity to one or two individual investors, called angels. In return for a typical investment in the range of \$50,000 to \$400,000, the angels receive stock and perhaps also a seat on the board of directors. Because angels can influence the strategic decision of the company, it is best that they bring experience and industry contacts to the table, not just cash.
- The Securities Act restrict the number and type of investors who may participate in private placements that are not registered with the SEC. Accredited investors include the officers and directors of the company, high-wealth individuals, and institutional investors.
- In a non-registered private placement, the company may issue securities to an unlimited number of accredited investors, but only to 35 non-accredited investors. In addition, none of the investors can sell their securities in the secondary market to the general public.

- Venture Capital Fund: As the company grows, its financing requirements may exceed the resources of individual investors, in which case it is likely turn to a venture capital fund. A venture capital fund is a private limited partnership, which raises fund from a relatively small group of primarily institutional investors, including pension funds, college endowments, and corporations. The managers of a venture capital fund, called venture capitalists or VCs, are usually very knowledgeable and experience in a particular industry.
- The venture capital fund buys shares of the portfolio of companies, and the VCs sit on the companies' board of directors. The venture capital fund usually has a pre-specified life of seven to ten years, after which it is dissolved, either by selling the portfolio companies stock and distributing the proceeds to the funds' investors or by directly distributing-the yahoo.com stock to the investors.

The Public Issue of Equity:

- The Basic Procedure for a IPO or unseasoned issue:
 - 1. Management's first step in any issue of securities to the public to obtain approval from the board of directors.
 - 2. Next, the firm must prepare and file a registration statement with the SEC. This statement contains a great deal of financial information, including a financial history, details of the existing business, proposed financing, and plans for the future.
 - 3. The SEC studies the registration statement during a waiting period. During this time, the firm may distribute prospectus contains much of the information put into the registration statement, and is given to potential investors by the firm. The company can not sell the securities during the waiting period; however, oral offers can be made. A registration statement will become effective on the 20th day after its filing unless the SEC sends a letter of comment suggesting changes.
 - 4. The registration statement doe not initially contain the price of the new issue. On the effective date of the registration statement, a price is determined and a full-fledged selling effort gets under way.

Investment Bank in the process of IPO or unseasoned issue:

- Investment banks are involved in issuing IPO securities. In addition to aiding in the sale of securities, the may facilitate mergers, and other corporate restructuring, act as brokers to both individual and institutional clients.
- Investment bankers perform following services in IPO issue:
 - 1. Formulating the method used to issue the securities
 - Pricing the new securities
 - Selling the new securities

There are three basic methods of issuing securities for cash:

Firm commitment: Under this method, the investment bank (or a group of investment banks-syndicate) buys the securities for less than the offering price and accepts the risk of not being able to sell them. Because this function involves risk, we say that the investment banker underwrites the securities in a firm commitment. To minimize the risk, investment bankers combine to form an underwriter group (syndicate) to share the risk and to help sell the issue.

- The difference between the underwriter's buying price and the offering price is called the spread or discount. It is the basic compensation received by the underwriter. The issuer receives the full amount of the proceeds less the spread, and all the risk is transferred to the underwriter.
- Best Efforts: The underwriter bears risk with a firm commitment method by buying the entire issue. Conversely, the syndicate avoids this risk under a best efforts offering method because it does not purchase the shares, instead it merely acts as an agent, receiving commission for each share sold. The syndicate is legally bound to use its best efforts to sell the securities at the agreed upon offering price. If the issue can not be sold at the offering price, it is usually withdrawn. This form of underwriting has become relatively rare.
- Dutch Auction Underwriting: In this method, the underwriter does not set a fixed price for the shares to be sold. Instead, the underwriter conducts an auction in which investors bid for shares. The offer price is determined based on the submitted bids. This method is ernayano comparatively new in IPO market while more common to bond market.

Green Shoe Provision:

Many underwriting contracts contain a Green Shoe Provision, which gives the members of the underwriting syndicate the option to purchase additional shares at the offering price. This provision usually last for about 30 days and involve no more than 15% of the newly issued shares. This option is a benefit to the underwriting syndicate and a cost to the issuer. If the market price of the new issue goes above the offering price within 30 days, the underwriters can buy shares from the issuer and immediately reall the shares to the public.

Right or Seasoned Issue:

- A right issue is an offer to existing shareholders to subscribe for new shares, at a discount to the current market price, in proportion to their existing holdings. Shareholders no wishing to take up their rights an sell them on the stock market
- Advantages of right issue; 1) it is cheaper that a public share issue 2) No need to place shares with the help of investment banker
- Theoretical Ex-rights price (TERP): The new share price after the issue is known as the theoretical ex-risers price and is calculated by finding the weighted average of the old price and the rights price, weighted by the number of shares.

TERP Market value of shares already in issue + Proceeds from new share issue Number of shares in issue after the rights issue

Example: X company, which has an issued capital of 2 million shares, having a current market price of \$2.70 each, makes a rights issue of one new share for every two existing shares at a price of \$2.10. Find theoretical ex right price.

Market value of shares already in issue + Proceeds from new share issue T E R P =Number of shares in issue after the rights issue $\frac{\$2.70 \text{ x } 2 \text{ million} + \$2.10 \text{ x } 1 \text{ million}}{2 \text{ million} + 1 \text{ million}} = \2.5 per share

Hybrid Financing:

- There are three types of long term hybrid financing: Preferred stock, warrants, and convertibles.
- Preferred Stock: Although preferred stock is considered to be equity, it is often included in the fixed income securities like bond. This is because, like bonds, preferred stock promises to pay a fixed stream of dividends. However, unlike bonds, the failure to pay the promised dividend does not result in corporate bankruptcy. Instead, the dividends owed simply accumulate, and the common stockholder may not receive any dividends until the preferred stockholders have been paid in full.
- In the event of bankruptcy, preferred stockholders' claim to the firm's assets has lower priority than those of bondholders, but higher priority than those of common stockholders.
- Warrants: A warrant is a derivative certificate issued by a company that gives the holder the right to buy a stated number of shares of the company' stock at a specified price for some specified length of time. Generally, warrants are issued along with debt, and they are used to induce investors to buy long term debt with a lower componerate that would otherwise be required.
- Warrants are long term call options that have value because holders can buy the firm's common stock at the exercise price regardless of how high the market price climbs. This option offers the low interest rate on the bonds and makes the package of low yield bonds plus warrants attractive to investors.
- Warrants generally are used by small, rapidly growing firms as sweeteners when they sell debt or preferred stock
 - Warrant Example: ABC company wants to raise \$50 million of debt finance by issuing 30 years bonds in the market. Company's CFO and investment banker found that the bonds would be difficult to sell and that a coupon rate of 10% would be required. However, as an alternative'they also suggested that investors might be willing to purchase the bonds at 8% coupon rate if the company would offer 20 warrants with \$1000 bond, each warrant entitling the holder to purchase one share of common stock at an exercise price of \$22 per share. The stock was selling for \$20 per share and the warrant would expire after 10 years.
- Convertibles: Convertible securities are bonds or preferred stocks that under specified terms and conditions, can be exchanged for (that is converted into) common stock at the option of the holder. Unlike the exercise of warrants, which brings in additional finds to the firm, conversion does not provide new capital, debt or preferred stock is simply replaced by common stock in the balance sheet. Exercise of this option will reduce the debt or preferred stock and will improve the firm's financial strength and make it easier to raise additional capital.
 - Conversion Ratio and Conversion Price: One of the most important provisions of a convertible security is the conversion ratio, CR, defined as the number of shares of stock a bondholder will receive upon conversion. Related to the conversion ration is the conversion price, P_c, which is the effective price investors pay for the common stock when conversion occurs. Once CR is set, the value of P_c is established.

Conversion Price, $Pc = \frac{Par \text{ value of bond given up}}{Shares received or CR}$

Financial Management Dividend Payout Policy

Distribution to Shareholders:

- A profitable company regularly faces three important questions:
 - 1. How much of its free cash flow should it pass on to shareholders?
 - 2. Should it provide this cash to shareholder by raising the dividend or by repurchasing stock?
 - 3. Should it maintain a stable, consistent payment policy, or should it let the payments vary as conditions change?
 There are only six good uses of free cash flow:

 Re-invest in the business by issuing stock dividends
 Pay interest expenses
 Pay debt principal
 Pay dividends
 Repurchase stock
- - 6. Buy non-operating assets such as treasury bills or other marketable securities.

The Level of Distributions and Firm Value:

- Can a company increase its value through its choice of distribution policy, defined as the level of distributions, the form of distributions (cash dividends vs stock repurchases) and the stability of distributions?
- The answer to above question depends on investor' preferences for returns as dividend versus capital gains. A high distribution ratio and a high payout ratio mean that a company pays large dividends and has small or zero stock repurchases. In this situation, dividend yield is relatively high and the expected capital gain is low. Conversely, if a company has large distribution ratio but small payout ratio, then it pays low dividends but regularly repurchases stock, resulting in a low dividend yield but a relatively high expected capital gain.

Investor' Preferences for Dividend Yield versus Capital Gains:

- Dividend Irrelevance Theory: Merton Miller and Franco Modigliani (MM) theory states that dividend policy has no effect on either the price of a firm's stock or its cost of capital. They argued that the firm's value is determined only by its basic earning power and its business risk. In other words, MM argued that the value of the firm depends only on the income produced by its assets, not on how this income is split between dividends and retained earnings. The principal conclusion of MM theory is that dividend policy does not affect the required return on equity. MM proposition is based on the assumption of no tax and brokerage costs. However, in real world tax and brokerage cost do exist, so the MM irrelevance theory may not be true.
- Bird-in-the-Hand Theory: Dividends Are Preferred: Myron Gordon and John Linter opposed the MM theory and state that required return on equity decreases as the dividend payout is increased because investors are less certain of receiving the capital gains than

they are of receiving dividend payments. Gordon and Linter said, investors value a dollar of expected dividends more than a dollar of expected capital gains because the dividend yield component is less risky than the expected capital gain.

Tax Preference Theory: Capital Gains are Preferred: Due to time value effects, a dollar of taxes paid in the future has a lower effective cost than a dollar paid today. So even if dividends and gains are taxed equally, capital gains are taxed only when the investor sell shares. If an investor holds his shares for long term, then he will be concerned only on taxes on dividend income and thereby prefer to have companies minimize dividends. If so, investors would be willing to pay more for low payout companies than for otherwise 100.com similar high payout companies.

Stock Dividends and Repurchase of Stock:

- Stock Dividend: Stock dividend is paid out in shares of stock. It is not true dividend because no cash leaves the firm. Rather, a stock dividend increases the number of shares outstanding, thereby reducing the value of each share. A stock windend is commonly expressed as a ratio; for instance, with a 5% stock dividend a marcholder receives 1 new share for every 20 currently owned.
- Stock Repurchase: Instead of paying dividends, a firm may use cash to repurchase shares of its own stock. Share repurchases are typically accomplished in one of three ways:
 - 1. Companies may simply purchase their own stock just as anyone would buy shares of a particular stock. In these open market purchases, the firm does not reveal itself as the buyer. Thus, the seller does not know whether the shares were sold back to the firm or to just another investor.
 - 2. Companies could institute a tender offer. Here, the firm announces to all of its stockholders that it is willing to buy a fixed number of shares at a specific price. The firm can also purchase shares from an auction.
- 3. Finally, firms may repurchase shares from specific individual stockholders, a procedure called a targeted repurchase. Companies engage in targeted repurchases for a variety of reasons. A single large stockholder can be bought out at a price lower than that in tender offer. The legal fees in a targeted repurchase may also be lower than those in more typical buyback Mohammad



Financial Management Working Capital Management

The Elements of Working Capital:

- Working capital is the capital available for conducting the day-to-day operations of an organization; normally the excess of current assets over current liabilities.
- Working capital management is the management of all aspects of both current assets and current liabilities, to minimize the risk of insolvency while maximising the return on assets.



- 1. The cost of funding it
- 2. The opportunity cost of lost investment opportunities because cash is tied up and unavailable for other uses
- Working capital is an investment which affects cash flows in the following ways:
 - 1. When inventory is purchased, cash is paid to acquire it.
 - 2. Receivables represent the cost of selling goods or services to customers, including the costs of the materials and the labor incurred.
 - 3. The cash the up in working capital is reduced to the extent that inventory is financed by trade payables. If suppliers supply goods to the firm on credit, the firm's cash flows are improved and working capital is reduced.

Objectives of Working Capital Management:

- The main objective of working capital management is to get the balance of current assets and current liabilities right.
 - 1. Ensuring current assets are sufficiently liquid to minimize the risk of insolvency
 - 2. Investing in less liquid assets in order to maximize return.

Trade-off between Liquidity versus Profitability:

- Liquidity versus Profitability: The decision regarding the level of overall management in working capital involves a trade-off between liquidity versus profitability. Liquidity is as important as profitability. Unprofitable companies can survive if they have liquidity whereas profitable companies can fail if they run out of cash to pay their liabilities (wages, amounts due to suppliers, overdraft interest etc). Liquidity in the context of working capital management means having enough cash or ready access to cash to meet all payment obligations when these fall due. The main sources of liquidity are: cash in the bank, short term investments that can be cashed in easily and quickly, cash inflows from normal trading operations, an overdraft facility or other ready source of extra borrowing.
- Some examples of transactions that have this trade-off effect on liquidity and on profits are as follows:
- Purchase of non-current assets for cash. The cash will be paid in full to the supplier when the asset is delivered; however profits will be charged gradually over the the of the asset in the form of depreciation.
- Receiving a bulk purchase discount for buying more inventory that is currently required improves profitability but reduces liquidity.
- Sale of goods on credit attracts more customers so improves profitability but reduces liquidity. Profits will be credited in full once the sale has been confirmed; however the cash may not be received for some considerable periods afterwards.
- With some payments such as tax there may be a significant timing difference between the impact on reported profit and the cash flow.
- Offering an early settlement discount to customers will improve liquidity at the cost of profitability.

Policies Regarding Working Capital Management

- Aggressive vs Conservative Approach: A firm choosing to have a lower level of working capital than rivals is said to have an aggressive approach, whereas a firm with a higher level of working capital has a conservative approach.
- An aggressive approach will result in higher profitability with higher risk, while a conservative approach will result in lower profitability and lower risk.
- Over-capitalization in working capital: If there are excessive inventories, accounts receivable and cash, and very few accounts payable, there will be an over-investment by the company in current assets. Working capital will be excessive and the company will be over-capitalized.
- **Overtrading**: Cash flow is the lifeblood of the thriving business. Effective and efficient management of the working capital investment in essential to maintain a control of business cash flow. Management must have full awareness of the profitability versus liquidity trade-off.
- For example, healthy trading growth typically produces: increased profitability, the need to
 increase investment in non-current assets and working capital. In contrast to overcapitalization, if the business does not have access to sufficient capital to fund the increase,
 it is said to be "overtrading." this can cause serious trouble for the business as it is unable to
 pay its business creditors.

Working Capital Ratios-Liquidity:

- Two key measures, the current ratio and the quick ratio, are used to assess short-term liquidity. Generally a higher ratio indicates better liquidity.
- **Current Ratio**: Current ratio measures how much of the total current assets are financed by current liabilities.

$$C urrent R atio = \frac{C urrent A ssets}{C urrent Liability}$$

- A current ration of 2:1 means that current liabilities can be paid twice over out of existing current assets.
- Quick (Acid Test) Ratio: The quick or acid test ratio measures how well current liaburities are covered by liquid assets. This is particularly useful where inventory holding periods are long.

Quick Ratio =
$$\frac{C \text{ urrent A ssets - Inventory}}{C \text{ urrent I iability}}$$

- A quick ratio of 1:1 means that the company is able to meet existing rabilities if they all fall due at once.
- These liquidity ratios are a guide to the risk of cash flow problems and insolvency. If a company suddenly finds that it is unable to renew its short term liabilities (for instance if the bank suspends its overdraft facilities) there will be a danger of insolvency unless the company is able to turn enough of its current assets into cash quickly.
- In general, high current and quick ratios are considered 'good' because it indicates that the firm has the resources to meet its commitments as they fall due. However, it may also indicate that working capital is not being used efficiently, or the company have too much idle cash tied up in the business that could be used to earn profit.
- Ideal current and quick ratio varies depending on the nature of the business. For example, a supermarket business operating a MP system will have little inventory and since most of its sales are for cash they will have few receivables. In addition, the ability to negotiate long credit periods with suppliers can result in a large payables figure. This can result in net current liabilities and a current ratio below 1 but does not mean the business has a liquidity problem.

Cash Operating Cycle

- The cash operating cycle is the length of time between the company's outlay on raw materials wages and other expenditures and the inflow of cash from the sale of goods.
- The faster a firm can push items around the cycle the lower its investment in working capital.

Purchases	Sales		Receipts from receivables
Inventory Days		Receivable Days	
Payable Days		Cash operating cycle	
Pay Pay	yables		

Calculation of the Cash Operating Cycle:



Example: Evaluate working capital management of the following company:

	20	006	20	007
Current Assets	\$000	\$000	\$000	\$000
Inventory	37		42	
Receivables	23		29	
Bank	5	_	3	_
	65	-	74	-
Current				2
Liabilities				
Trade Payables	55		36	CO'
Tax Payables	10	-	10	0."
	65	-	46	- 210
Revenue		196	con O	209
Opening			.ext	
Inventory	29	2	37	
Purchases	159	8/	162	
less: Closing Inventory	(37)		(42)	
Cost of Sales		151		157
Gross Profit		0145		52

- a) Calculate liquidity ratios in 2006 and 2007
 b) Calculate the length of operating cycle in 2006 and 2007
 c) Comment on your results and appraise how effectively the working capital is being managed.

Sol:			
Liquidity Ratios		2006	2007
Current Ratio		$\frac{65}{} = 1$	$\frac{74}{} = 1.6$
Current Liability		65	46
$Ouick Berlin = \frac{Current Assets - Inventory}{Current Assets - Inventory}$		$\frac{65-37}{2} = 0.43$	$\frac{74-42}{2} = 0.7$
Current Liability		65	46
		2006	2007
Inventory Holding Period = $\frac{\text{Inventory}}{\text{Cost of sales}} x365$	$\frac{(29+37)/2}{151}x365 = 80 \mathrm{days}$		$\frac{(37+42)/2}{157}x365 = 92 \mathrm{days}$
Trade Receivable Days = $\frac{\text{Receivables}}{\text{Credit Sales}} x365$	$\frac{23}{196}x365 = 43 days$		$\frac{29}{209}x365 = 51days$
Trade Payable Days = $\frac{\text{Trade Payables}}{\text{Credit Purchases}} x365$	$\frac{55}{159}x365 = 126.3$ days		$\frac{36}{162}x365 = 81.1$ days
Cash Operating Cycle	80+43-126.3=(3.3days)		92+51-81.1=61.9days

Comments:

- Liquidity Ratio: The company improved in its liquidity condition in 2007 from 2006. Liabilities payables are 70% covered by cash and receivables (liquid assets, close to cash) in 2007.
- The inventory holding period has lengthened. In general, the shorter the stock holding period the better. It is expensive to hold stock for longer period; the company should focus on its sells effort by providing training to sales staff and effective promotion strategy.
- The receivables collection period scenario has worsened in 2007 compared to 2006. The company need to focus on its credit policy and try to reduce it to less than 30 days.
- The payables' payment period has reduced substantially from last year. It indicates that the company has improved relationship with its suppliers by settling payment early.
- Cash Operating Cycle: In 2007, there is approximately a 62 day gap between paying cash Cash Operating Cycle: In 2007, there is approximately a 62 day gap between Bying cash to suppliers and cash received from customers. However, in 2006, there was somewhat unusual situation where cash was received from customers more than a days before the payment to suppliers was needed. to suppliers and cash received from customers. However, in 2006, there was somewhat
<u>Financial Management</u> <u>Microsoft Excel Applications</u>

Example: What is the present value of \$2,500 payable 4 years from now at 8% compounded quarterly?

Excel formula: = PV(rate, nper, pmt, [fv], [type]) = = PV(8%/4, 16, 2500) = (\$1, 821.11)

Example: If \$800 is invested at 6% compounded semiannually(every 6 months), what will be the amount in 5 years?

Excel formula: = FV(rate, nper, pmt, [pv], [type]) = =FV(6%/2, 10, 800) = (\$1, 075.13)

Example: If \$100 is deposited in an account each month for 10years and the account earns 7% compounded monthly, how much will be in the account after the last deposit is made.

Excel formula: = FV(rate, nper, pmt, [pv], [type]) = FV(7%/12, 120, 100, 0) = (\$10, 308.48)

Example: How much should be deposited in a sinking fund at the end of each quarter for 5 years to accumulate \$10,000 if the fund earns 8% compounded quarterly,

Excel formula: PMT(rate, nper, pv, [fv], [type]) = PMT(8%/4,20,10000,0) = (\$411.57)

Example: What sum deposited now in an account earning 8% interest compounded quarterly will provide quarterly payments of \$1,000 for 10 years the first payment to be made 3 months from now?

Excel formula: PV(rate, nper, pmt, [fv], [type]) = PV(8%/4,40,1000,0) = (\$27,355.48)

Example: Sam borrowed \$5,000 to buy a car. He will amortize the loan by monthly payments over 3 years. Find the monthly payment amount if interest is 12% compounded monthly

Excel formula: =PMT(rate, npex pv, [fv], [type]) = PMT(12%/12,36,5000,,0) = (\$166.07)

Goal Seek Examples

Example: Suppose you want to borrow money from a bank to construct a building. Bank will give 30 years loan at the rate of 9% compounded monthly. How much you will be able to borrow depends on now much you can pay per month as installment. If you can pay \$9,000 per month then how puch you can borrow from the bank?

Any We can solve this using present value formula:

Excel formula: PV(rate, nper, pmt, [fv], [type]) = =PV(9%/12,360,9000) = (\$1,118,536.79)

Thus, you can borrow up to \$1,118,536.79 if you are able to pay \$9,000 per month. However, if you want to find how much you can borrow if you can pay suppose \$6,000 or \$8,000 or different amount, then you can use goal seek argument option to find out this very easily.

Suppose you can borrow \$900,000 if you can pay \$7,241.60 per month.

	А	В
1	Loan Amount	\$900,000
2	Interest rate	9%
3	Years	30
4		
5	Payment	(\$7,241.60)

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To <u>v</u> alue:	9000	
By changing cell:	\$B\$1	
OK		Cancel

	А	В	
1	Loan Amount	(\$1,118,537)	
2	Interest rate	9%	
3	Years	30	
4			
5	Payment	\$9,000.00	

		-		
1	Loan Amount	(\$1,118,537)		
2	Interest rate	9%		
3	Years	30		
4				
5	Payment	\$9,000.00		
7	lo. A 4 9 0/			ang mill :4 4alaa fan
LXi	imple: At 8% col	npounded annu	any, now many ye	ars will it take for
Ans	s: Using goal seek	we can solve th	is,	ć
. .	1 6 1			, Ĉ
ut	ure value formula	$=B1*(1+B2)^{1}$	3	21
			Carl Carl	
			Goal Seek	L S S
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1 2	Interest rate	\$2,000	By changing cell:	\$B\$3
2	Voors	070		
о л	Futuro voluo	\$2,000	OK	Cancel
4	Future value	\$2,000	XY	
	Α	В	XCY .	
1				
1	Present value	\$2,000	7	
1 2	Present value Interest rate	\$2,000 8%	7	
1 2 3	Present value Interest rate Years	\$2,000 8% 5.268446652	,	

Example: At 2% compounded monthly, how many years will it take for \$500 to grow to \$2,200?

Nohar	
4 r	

4

	А	В
1	Present value	\$500
2	Interest rate	12%
3	Years	
4	Future value	\$500

Ans: Future value formula: $=B1*(1+B2/12)^{(B3*12)}$

Goal Seek	? <mark>×</mark>
S <u>e</u> t cell:	B4 💽
To <u>v</u> alue:	2200
By changing cell:	\$B\$3
ОК	Cancel

Ans: 12.4 years

Example: At w	hat interes	t rate con	npounded a	nnua	ally will	a sum o	f mon	ey double	e in 10 years?
					,	A		В	COr
				1	Present	t value		\$500	0.
				2	Interes	t rate		0%	00
				3	Compo	unding		1	** *
				4	Years			10	
Future value for	mula: =B1	*(1+B2/B	3)^(B4*B3)	5	Future	value		\$500	
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			5 Fut	ire v	alue	Ş	1,000		
Ans: 7.177%			stl.						
Example: At w	hat interes	t rate con	ري pounded m	ont	hly will	a sum of	f mone	ey triple i	in 15 years?
		\sim	,		,				
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		<u>S</u>		1	Present Interact	value		\$500	
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by granging com			3 Com	DOLLE			12		
			4	Jour	ung		45		
ОК	Ca	ncel	4 Years	, ,	luo	¢4	15		

Ans: 7.346%

Example: Find the effective rate (or equivalent annual rate) of 12% compounded monthly.

EFFECT(nominal_rate, npery) = EFFECT(12%,12) = 12.683%

Example: If the effective rate (or equivalent annual rate) is 16% then find the nominal rate for monthly compounding.

NOMINAL(effect_rate, npery) = NOMINAL(16%,12) = 14.934%

Example: Suppose you want to borrow money for purchasing a car. A bank is offering you EMI option for the following packages. Find the EMI amount for all the packages.

Packages	Package 1	Package 2	Package 3	Package 4
Loan	\$50,000	\$90,000	\$100,000	\$100,000
Interest rate	7.000%	9.000%	5.000%	10.000%
Compounding	12	12	12	12
Years	6	8	2	10
Payment	(\$852.45)	(\$1,318.52)	(\$4,387.14)	(\$1,321.51)

Interest rate	7.000%	9.000%	5.000%	6 10.000)%		
Compounding	12	12	1	2	12		
Years	6	8		2	10		
Payment	(\$852.45)	(\$1,318.52)	(\$4,387.14) (\$1,321.5	51)	_0.	
Using Scenario Analysis we can change the package conditions and find the payment amounts.							
Scenario Sumi	mary						
	Current V	alues: P	ackage 1	Package 2	Package 3	Package 4	
Changing					D.*		
Cells:				<u> </u>			
\$B\$3	\$5	0,000	\$50,000	\$90,000	\$100,000	\$100,000	
\$B\$4	7.	000%	7.000%	9.000%	5.000%	10.000%	
\$B\$5		12	12	12	12	12	
\$B\$6		6	6	8	2	10	
Result Cells:							
\$B\$7	(\$85	52.45) <u>(\$</u>	852.45) (\$1,318.5 <mark>2</mark>)	(\$4,387.14)	(\$1,321.51)	
		<u> </u>					

We can design any packages according to customer demands by changing loan amount, interest rate, no. of compounding per year and no of years.

05

Example: Find NPV and HRR of the following project.



Year	Cash Flow
0	(240,000)
1	80,000
2	120,000
3	70,000
4	40,000
5	20,000
6	15,000
7	12,000

=NPV(rate,future cash flows)+Initial Investment

=IRR(values, [guess]) = 17.01%

Financial Management

Time Value of Money Problems and Solutions

Question 1-2: In problems 1 through 2, find the future value at the stated nominal interest rate compounded annually.

- 1) \$200; 20 years; 5 percent Sol: Here, Present value, P = \$200, t = 20 years, r = 5% = 0.05Future value, $F_{t} = P(1+r)^{t} = 200(1+0.05)^{20} = 530.66
- 2) \$300; 10 years; 6 percent **Sol:** Here, Present value, P = \$300, t = 10 years, r = 6% = 0.06Future value, $F_{t} = P(1+r)^{t} = 300(1+0.06)^{10} = 537.25

Question 3-4: In problems 3 through 4, find the future value using the appropriate

Question 5: How many years will it take for \$5,000 to an mount to \$20,000 at 7% compounded annually?

4 = 32, r = 8/4% = 2% = 0.02 $(1 + r)^{t} = 150(1 + 0.02)^{32} = 282.68 (4) \$600; 20 years; 8 percent compounded semi-annually Sol: Here, Present value, P = \$600, t = 20 years x 2 = 40, r = 8/2\% = 4\% = 0.04
Future value, $F_{t} = P(1 + r)^{t} = 600(1 + 0.04)^{40} = $2,880.61$ Question 5: How many years will it take for \$5,000 to ________ Sol: Here, present value, P = \$5,000, Future value, F = \$20,000, time periods, t = ? r = 7% =0.07 Future value, $F_i = P(1 + r)^i$ *i.e.* \$20,000 = \$5,000(1 + 0.07)^i *i.e.* $1.07^i = \frac{20,000}{5,000} = 4$ *i.e.* $1.07' = \frac{20,000}{5,000} = 4$ *i.e.* ln(1.07') = ln(4); [Taking ln in by *i.e.* $t \ge ln(1.07) = ln(4)$ *i.e.* $t = \frac{ln(4)}{ln(1.07)} = 20.489 \approx 2000$ years Question 6: How many years will it take for a sum of money to double at 10%

compounded semi-annually?

Sol: Here, P = \$100, F = \$200, time periods, t = xyears(2)=2x r = 10/2% = 0.05Future value, $F P (1 + r)^{2x}$ i.e. \$200 = \$00(1 + 0.05)^{2x} i.e. 1.00 $i = ln(1.05^{2x}) = ln(2)$; [Taking ln in both sides] $i.e.^{\gamma} 2x \ln(1.05) = \ln(2)$ *i.e.* $2x = \frac{ln(2)}{ln(1.05)} = 14.2$ i.e.x = 7.1 years

Question 7: Find the rate of interest compounded annually at which a sum of money will double in 20 years.

Sol: Here, present value, P = \$100, Future value, F = \$200, time periods, t = 20 years, r = ?Future value, $F = P(1 + r)^{t}$

i.e.
$$200 = 100(1 + r)^{20}$$

i.e. $(1 + r)^{20} = 2$
i.e. $\ln(1 + r)^{20} = \ln(2)$; [Taking ln in both sides]
i.e. $20\ln(1 + r) = \ln(2)$
i.e. $\ln(1 + r) = \frac{\ln(2)}{20}$
i.e. $\ln(1 + r) = \frac{\ln(2)}{20}$
i.e. $(1 + r) = (2)^{\frac{1}{20}}$
i.e. $(1 + \frac{r}{2})^{16} = \frac{\$12,000}{\$5,000} = 2.4$
i.e. $(1 + \frac{r}{2})^{16} = \frac{\$12,000}{\$5,000} = 2.4$
i.e. $(1 + \frac{r}{2})^{16} = \frac{\$12,000}{\$5,000} = 2.4$
i.e. $(1 + \frac{r}{2})^{16} = (2.4)^{\frac{1}{16}}$
i.e. $(1 + \frac{r}{2}) = (2.4)^{\frac{1}{16}}$
i.e. $(2 +$

i.e.r = 0.0562x2 0.11248 = 11.248% Question 9 A bank pays 5.25% compounded daily on savings accounts running for 6 years. Using 365 days per year, compute the future value of a deposit of \$5,000 for 6 years

Solvere, r = 5.25/365% = 0.0525/365, t = 6 years x 365 = 2190, present value, P = \$5,000Future value, $F = P(1+r)^{t} = \$5,000(1+\frac{0.0525}{365})^{2190} = \$6,851.14$

Question 19-20: Compute the future value:

- 19) \$4,000; 8 percent compounded continuously; 5 years 8months Sol: Here, Present value, P = \$4,000, t = 5.67 years, r = 8% = 0.08Future value, $F_t = Pe^{rt} = \$4000 xe^{0.08x5.67} = \$6,295.87$
- 20) \$5,000;12 percent compounded continuously; 6months Sol: Here, Present value, P = \$5,000, t = 0.50 years, r = 12% = 0.12Future value, $F_t = Pe^{rt} = \$5000 xe^{0.12 x0.5} = \$5,309.18$

Question 21-22: Compute the present value:

21) \$2,000; 7 percent compounded continuously; 3 years 2months Sol: Here, Future value, F = \$2,000, t = 3.17 years, r = 7% = 0.07Present value, $P_t = Fe^{-rt} = \$2,000 xe^{-0.07 x3.17} = \$1,602$

22) \$250;12 percent compounded continuously; 1year 6months Sol: Here, Future value, F = \$250, t = 1.5 years, r = 12% = 0.12Present value, $P_t = Fe^{-rt} = \$250 xe^{-0.12x1.5} = \208.81

Question 23-24: The rate in problems 5 through 6 are nominal rates, find the effective rate when the interest rate is compounded continuously:

23) 12%, Sol: Effective rate, $r_e = e^r - 1 = e^{0.12} - 1 = 0.12749 = 12.749\%$

24) 8%, Sol: Effective rate, $r_e = e^r - 1 = e^{0.08} - 1 = 0.08328 = 8.328\%$

Question 25-26: The rate in problems 25 through 26 are effective interest rates, find the nominal rate when the interest rate is compounded continuously:

25) Effective rate = 5%

26) Effective rate = 10%

Question 27: How much will a deposit of \$5000 grow to in 20 years at 7.2% interest compounded continuously

Sol: Here, Present value, r = 5,000, t = 20 years, r = 7.2% = 0.072

Future value, $F_t = per = \$5000 xe^{0.072 x20} = \$21,103.47$

Question 28: How much should be deposited now at 8.4% compounded continuously if the amount in the account 10 years from now is to be \$8000?

Sol: Here, Future value, F = \$8,000, t = 10 years, r = 8.4% = 0.084

Present value, $P_t = Fe^{-rt} = \$8,000 xe^{-0.084 x10} = \$3,453.68$

Question 29: Sam invests \$15,000 in a bank account paying 8.4% compounded continuously for 15 years. How much will be in the account at the end of this time?

Sol: Here, Present value, P = \$15,000, t = 15 years, r = 8.4% = 0.084

Future value, $F_t = Pe^{rt} = \$15,000 xe^{0.084 x15} = \$52,881.32$

Question 30: History tells us that Peter Minuit purchased Manhattan Island in New York from the Indians for \$24 about 370 years ago. If the \$24 had been invested at 5% compounded continuously, what would be its amount after 370 years.

Sol: Here, Present value, P = \$24, t = 370 years, r = 5% = 0.05

Future value, $F_{t} = Pe^{rt} = \$24 xe^{0.05 x 370} = \$2,598,119,706$

Question 31: A company issues \$1 million of bonds and sets up a sinking fund at 8% compounded quarterly to accumulate \$1 million by 10 years to redeem the bonds. Find the quarterly payment to the sinking fund.

Sol: Here, Future value, FV = \$1,000,000, interest rate, r = 8/4 % =2% =0.02, t = 10 x 4 =40

$$FV = 1,000,000 = \frac{c}{r}[(1+r)^{T} - 1] = \frac{C}{0.02}[(1+0.02)^{40} - 1]$$

 $or, 1,000,000 x 0.02 = C (1.02^{40} - 1)$

$$or, C = \frac{20,000}{(1.02^{40} - 1)} = \$16,555.74$$

using annuity table:

 $FV = 1,000,000 = CxFVIFA_{40,2\%} = Cx60.402$

i.e.
$$C = \frac{1,000,000}{60.402} = \$16,555.74$$

100.com Question 32: In order to accumulate \$15,000 for a down payment on a home 8 years from now, the Jonses are going to deposit a sum of money at the end of each the month period in an account earning 8% compounded semi-annually. What should be the amount of each deposit?

Sol: Here, Future value, FV = \$15,000, $t = 8 \ge 2 = 16$, interest rate, $2^{10} = 2^{10} = 2000$

$$FV = 15,000 = \frac{c}{r} [(1+r)^{T} - 1] = \frac{C}{0.04} [(1+0.04)^{16} - 1]$$

or, 15,000x0.04 = C (1.04¹⁶ - 1)
or, C = $\frac{600}{(1.04^{16} - 1)} = \$6\$7.29$
using annuity table:
FV = 15,000 = CxFV IFA_{16,4%} = Cx21.825
i.e. C = $\frac{15,000}{21.825} = \$6\$7.28$

Question 33: When Kathy was born, her parents decided to deposit \$500 every 6 months thereafter for 15 years in an account earning 6% compounded semi-annually. How much will be in the account after the last deposit is made?

Sol: Here, Future value, V = ? Semi-annual deposit, C = \$500, t = 15 x 2 = 30, r = 6/2 % = 3% =0.03

$$FV = \frac{c}{r} [(1+r)^{5} - 1] = \frac{500}{0.03} [(1+0.03)^{30} - 1] = \$23,787.7$$

using annuaty table: FV = \$500 xFV IFA _{30,3%} = \$500 x47.575 = \$23,787.5

Question 34: A sum of money invested now at 10% compounded semi-annually is to provide payments of \$1,500 every 6 months for 8 years, with the first payment due 6 months from now. How much should be invested? How much interest will the investment earn?

Sol: Here, Present value, PV = ? r = 10/2 % = 5% = 0.05, Semi-annual payments, C = \$1,500 $t = 8 \ge 2 = 16$

$$PV = \frac{c}{r} - \frac{c}{r} (1+r)^{-T} = \frac{1,500}{0.05} - \frac{1,500}{0.05} (1+0.05)^{-16} = \$16,256.65$$

using annuity table: $PV = $1,500 x PV IFA_{16.5\%} = $1,500 x 10.838 = 16257 Interest earned on investment = \$1,500 x 16 - \$16,257 = \$7,743

Question 35: The directors of a company have voted to establish a fund that will pay a retiring accountant or his estate \$1,000 per month for the next 3 years, with the first payment to be made a month from now. How much should be placed in the fund if it earns interest at 12% compounded monthly? How much interest will the fund earn during its existence?

Sol: Here, monthly payments, C =1,000, t = 3 x 12 = 36, present value, PV = ? r = 12/12 % =1% =0.01

$$PV = \frac{c}{r} - \frac{c}{r} (1+r)^{-T} = \frac{1,000}{0.01} - \frac{1,000}{0.01} (1+0.01)^{-36} = \$30,107.5$$

using annuity table: $PV = $1,000 x PV IFA_{36,1\%} = $1,000 x 30.108 = $30,108$

Interest earned on investment = $1,000 \times 36 - 30,108 = 5,892$

Question 36: A company has borrowed \$50,000 at 8% compounded quarterly. The debt is to be amortized by equal payments each quarter over 10 years. a) find the quarterly payment b) how much interest will be paid?

payment b) how much interest will be paid?
Sol: Here, present value, PV = \$50,000, r = 8/4 % =2% =0.02, t = 10 x 40, quarterly
payments, C =?

$$PV = $50,000 = \frac{c}{r} - \frac{c}{r}(1+r)^{-T} = \frac{C}{0.02}[1 - (1+0.02)^{-40}]$$

 $or, 1,000 = C(1-1.02^{-40});$ $or, C = \frac{$1,000}{(1-1.02^{-40})} = $1,827.78$
using annuity table: PV = \$CxPV IFA 40.2% = \$Cx27.355
 $or, C = \frac{PV}{27.355} = \frac{$50,000}{27.355} = $1,827.82$

Interest will be paid = \$1,827.78 x 40 - \$50,000 \$23,111.2

Question 37: A real estate developer borrows \$100,000 at 12% compounded monthly. The debt is to be discharged by monthly payments over the next 3 years. a) find the monthly payment b) how much interest will be paid?

Sol: Here, present value, PV = \$100,000, r = 12/12 % =1% =0.01, t = 3 x 12 =36, monthly payments, C = ?

$$PV = \$100,000 = \frac{c}{r} - \frac{c}{r}(1+r) = \frac{C}{0.01} [1 - (1 + 0.01)^{-36}]$$

$$or, 1,000 = C(1-1.04^{-36});$$
 $or, C = \frac{41,000}{(1-1.01^{-36})} = $3,321.43$

using annuity table: $PV = CxPVIFA_{36,1\%} = Cx30.108$

$$or, C = \frac{P}{30.108} = \frac{\$100,000}{30.108} = \$3,321.37$$

Interest will be paid = $3,321.43 \times 36 - 100,000 = 19,571.48$

Question 38: A company borrows \$100,000 at 12% compounded semi-annually. The debt is amortized by making equal payments at the end of each 6 months for 7 years.

- a) Find the amount of each payment
- b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
- c) How much of the second payment is for interest, and by how much does it reduce the balance owed?

Sol: Here, present value, PV = \$100,000, r = 12/2 % = 6% = 0.06, $t = 7 \ge 2 = 14$ a) Semi-annual payments, C = ?

 $PV = \$100,000 = \frac{c}{r} - \frac{c}{r}(1+r)^{-T} = \frac{C}{0.06}[1 - (1+0.06)^{-14}]$ $or, 6,000 = C(1-1.06^{-14});$ $or, C = \frac{\$6,000}{(1-1.06^{-14})} = \$10,758.49$ using annuity table: $PV = CxPVIFA_{14.6\%} = Cx9.295$ D 17 **# 1 0 0 0 0 0**

$$or, C = \frac{PV}{9.295} = \frac{\$100,000}{9.295} = \$10,758.47$$

b) At the time the first payment is due, 6 months has passed and the interest due =100,000 x6% =\$6,000

Ans.	Period	Semi-annual Payment	Interest charged	Balance Reduced	New Balance
b)	1	\$10,758.49	\$100,000 x 6% =\$6,000	\$4,758.49	\$100,000-\$4,758.49 \$95,241.51
c)	2	\$10,758.49	\$95,241.51x 6% =\$5,714.49	\$5,044	\$95,241.51-\$5,044 =\$90,197.51

Question 39: Mr. Smith have taken out a \$35,000, 30 year mortgage on their home, with interest at 8.75% compounded monthly.

- a) Find the amount of each payment
 b) How much of the first payment is for interest, and by how much does it reduce the

b) How much of the first payment is for interval $r = 30 \times 10^{-2}$ Sol: Here, present value, PV = \$35,000, t = 30 x 02 = 360, r = 8.75/12 % =0.0875/12 =0.00729,

$$PV = \$35,000 = \frac{c}{r} - \frac{c}{r}(1+r)^{-T} = \frac{C}{0.00(200)} (1+0.00729)^{-360}]$$

or, 255.15 = C (1-1.00729^{-360}); or, C = $\frac{\$255.15}{(1-1.00729^{-360})} = \275.29

b) At the time the first payment is due, one month has passed and the interest due = $35,000 \times 0.00729 = 255.15$ 0.00729 = \$255.15

Ans.	Period	Semi-annual Payment	Interest charged	Balance Reduced	New Balance
b)	1	\$275.29	\$35,000 x 0.00729 =\$255.15	\$20.14	\$35,000-\$20.14 =\$34,979.86

Question 40: Mr. Fran borrowed \$2,000 from Silverbank and signed a note promising to discharge the debt with interest at 12% compounded monthly at a maturity date 2 years from now. Six months later, Silverbank needed more cash and sold Fran's note to Goldbank. Goldbank computed the maturity amount of Fran's note and gave Silverbank the present value of this amount, computed at 8% compounded quarterly. How much did Silverbank receive?

Sol.: The future or maturity value, Mr. Fran supposed to pay to Silverbank, 2 years from now:

Here, p = \$2,000, r = 12/12% = 1%Future value, $F_t = P(1+r)^t = \$2,000(1+0.01)^{24} = \2539.47 $=0.01, t = 2 \times 12 = 24$

The present value of the note (maturity value = \$2539.47) paid to Silverbank by Goldbank, after 6 months,

Present value, $P_t = F(1+r)^{-t} = $2,539.47(1+0.02)^{-6} = $2,255$ Here, p = ?, r = 8/4% = 2% = 0.02, $t = 1.5 \times 4 = 6$

Silverbank received \$2,255

Question 41: Sue borrowed \$7,000 at 12% compounded monthly for 3 years to buy a car. How much will she have to pay at the end of each month to discharge the debt? **Sol:** Here, present value, PV = \$7,000, r = 12/12 % =1% =0.01, t = 3 x 12 =36, monthly payments, C = ?ayahoo.com

$$PV = \$7,000 = \frac{c}{r} - \frac{c}{r}(1+r)^{-T} = \frac{C}{0.01}[1 - (1+0.01)^{-36}]$$

or, 70 = C(1-1.01⁻³⁶); or, C = $\frac{\$70}{(1-1.01^{-36})} = \232.5

using annuity table: $PV = CxPVIFA_{36.1\%} = Cx30.108$

$$or, C = \frac{PV}{30.108} = \frac{\$7,000}{30.108} = \$232.49$$

30.108 30.108 Question 42: How much should be deposited now at 7% compounded annually to provide an income of \$20,000 at the end of each year for the next 22 years?

Sol: Here, present value, PV =?, r = 7% =0.07, annual payment,
$$\mathcal{Q} = $20,000, t = 22$$

 $PV = \frac{c}{r} - \frac{c}{r}(1+r)^{-T} = \frac{20,000}{0.07} - \frac{20,000}{0.07}(1+0.07)^{-22} = $22,224.81$

using annuity table:
$$PV = $20,000 x PV IFA_{22,7\%} = $20000 x 11.061 = $221,220$$

Question 43: Mr. James will make 20 equal semi-annual deposits to an account earning 8% compounded semi-annually. Then, after the last deposit, she will use the amount in the account to establish an ordinary annuity earning 6% compounded annually which will provide her with \$10,000 at the end of each year for 5 years. How much should his semi-annual deposit be?

Annı	uity 1							
				Annuity 2				
T=0			Ś	T=10yrs	T=11			T = 15yrs
	1	2		20	1	2	•••••	5
	C=?	C		С	\$10,000	\$10,000		\$10,000
		2	× ´	Value =\$42,124				
@8	@8% semi-annually				@6%	annually		

Sol: Here present value of annuity 2 at the end of 10 years, PV =?, annual payments, C = 10,000 = 6% = 0.06, t = 5

$$PV = \frac{c}{r} - \frac{c}{r} (1+r)^{-T} = \frac{10,000}{0.06} - \frac{10,000}{0.06} (1+0.06)^{-5} = \$42,123.63$$

using annuity table: $PV = $10,000 x PV IFA_{5,6\%} = $10,000 x 4.2124 = $42,124$ Now, semi-annual deposit, C=?, r = 8/2% = 4% = 0.04, t = 20

$$FV = \$42, 124 = \frac{c}{r} [(1+r)^{T} - 1] = \frac{C}{0.04} [(1+0.04)^{20} - 0r, \$1, 684.96 = C (1.04^{20} - 1)$$
$$or, C = \frac{\$1, 684.96}{(1.04^{20} - 1)} = \$1, 414.59$$

-1] using annuity table: $FV = 42,124 = CxFVIFA_{20,4\%} = Cx29.778$ *i.e.* $C = \frac{\$42,124}{29.778} = \$1,414.6$

Financial Management Capital Budgeting Problems and Solution

Question 1:

Year	Cash Flow
0	(240,000)
1	80,000
2	120,000
3	70,000
4	40,000
5	20,000
6	15,000
7	12,000

a) Find out the payback period and discounted payback period of the above project Sol: Year Cash Flow Cumulative Cash Flow

Year	Cash Flow	Cumulative	
		Cash Flow	
0	(240,000)		
1	80,000	(160,000)	SV.
2	120,000	(40,000)	
3	70,000	30,000	
4	40,000	70,000	
5	20,000	90,000	
6	15,000	195,000	
7	12,000	117,000	
	0	1	

Payback period is between the year 2 and year 3. If we assume a constant rate of cash flow throughout the 3rd year, we could estimate payback period will be two years plus (40,000/70,000) of 3th year, which is 2.5714 years or 2 years 6.85 months. **Discounted Payback Period Calculation**

Year	Cash Flow	Present Value @ 15%	Cumulative PVs
0	(240,000)	(240,000)	
1	801000	$80,000(1+0.15)^{-1} = $69,565.21$	(170,434.79)
2	320,000	$120,000(1+0.15)^{-2} = \$90,737.24$	(79,697.55)
3	70,000	$70,000(1+0.15)^{-3} = $46,026.13$	(33,671.42)
A	40,000	$40,000(1+0.15)^{-4} = $22,870.13$	(10,801.29)
5	20,000	$20,000(1+0.15)^{-5} = \$9,943.53$	(857.76)
6	15,000	$15,000(1+0.15)^{-6} = $ \$6,484.91	5,627.15
7	12,000	$12,000(1+0.15)^{-7} = $4,511.24$	10,138.39=NP

Discounted Payback Period = $5 \text{ years} + \frac{857.76}{6,484.91} x12 \text{ months} = 5 \text{ years} + 1.58 \text{ months}$

b) Find NPV of the project at 15% hurdle rate.

NPV = 10,138.39

Decision: Accept the project because it not only generates enough cash to fulfil required return but also achieve extra return of \$10,138.

Question 2:

Year	Cash Flow
0	(25,000)
1	6,000
2	10,000
3	8,000
4	7,000

a) Find out the payback period and discounted payback period of the above project Sol:

Year	Cash Flow	Cumulative
		Cash Flow
0	(25,000)	
1	6,000	(19,000)
2	10,000	(9,000)
3	8,000	(1,000)
4	7,000	6,000



Payback period is between 3 and 4 years. If we assume a constant rate of cash flow throughout the 4^{rd} year, we could estimate payback period will be three years plus (1,000/6,000) of 4th year, which is 3.167 years or 3 years 2 months.

Discounted Payback Period Calculation

Year	Cash Flow	Present Value @ 22%	Cumulative PVs
0	(25,000)	(25,000)	
1	6,000	6,000(1+0.22) = \$4,918.03	(20,081.96)
2	10,000	$10,000(1+22)^{-2} = $6,718.62$	(13,363.34)
3	8,000	$8,000(100.22)^{-3} = $4,405.65$	(8,957.68)
4	7,000	$7,000(1+0.22)^{-4} = \$3,159.79$	(5,797.89)=NPV

Discounted payback period is 4+ years.

b) Find NPV of the project at 22% hurdle rate.

Decision: Reject the project because it fails to generate enough cash to fulfil required return.

Year	Cash Flow
0	(50,000)
1	18,000
2	25,000
3	20,000
4	10,000

a) Find out the payback period and discounted payback period of the above project

b) Find NPV of the project when required rate of return is 12%.

Question 4: Find IRR of the following project

Discount Rate	NPV
18%	18849.1602
20%	8144.71879
22%	-1513.96197
24%	-10236.373

Sol: Find two costs of capital at which the NPV is positive and negative respectively but close to zero.

$$IRR = L + \left[\frac{NPV_{L}}{NPV_{L} - NPV_{H}}x(H - L)\right]$$
L = lower rate of interest = 20%; H = higher rate of interest = 22%

$$NPV_{L} = \$8,144.71879, \qquad NPV_{H} = -1513.96197$$

$$IRR = 0.20 + \left[\frac{\$144.71879}{\$144.71879 - (-1513.96197)}x(0.22 - 0.20)\right] = 0.20 + \left[\frac{\$144.71879}{964.868076}x0.02\right]$$

$$= 0.21686 = 21.686\%$$
Question 5: Find IRR of the following project Discount Rate NPV

Question 5: Find IRR of the following project

Discount Rate	NPV
20%	5159.0364
22%	2747.0105
24%	563,18611
26%	-1415.7951
28%	-3210.5872
30%	-5141.7951

Sol: Find two costs of capital at which the NPV is positive and negative respectively but close c ?? to zero.

$$IRR = L + \left[\frac{NPV_L}{NPV_L - NPV_H}x(H - L)\right]$$
L = lower rate of interest = 24%; H = higher rate of interest = 26%

$$NPV_L = \$563.18611, \qquad NPV_H = -1415.7951$$

$$IRR = 0.24 + \left[\frac{\$563.18611}{\$563.18611 - (-\$1415.7951)}x(0.26 - 0.24)\right] = 0.24569 = 24.569\%$$

Question 6: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are divisible. Determine which projects should be chosen to maximize the return to the business.

Projects	Initial Investment	NPV
	(\$000)	(\$000)
Α	40	20
В	100	35
С	50	24
D	60	18
Ε	50	(10)

Projects	Initial Investment	NPV	Profitability	Ranks
	(\$000)	(\$000)	Index (PI)	
А	40	20	20/40 =0.5	1
В	100	35	35/100 = 0.35	3
С	50	24	24/50 = 0.48	2
D	60	18	18/60 = 0.3	4
Е	50	(10)	Rejected	

Capital Rationing:

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Projects	Ranks	Available Fund	Fund Invested	Fund Remaining	NPV
		(\$000)	(\$000)	(\$000)	(\$000)
А	1	100	40	60	20
С	2	60	50	10	0 ² 24
В	3	10	10/100 = 10%	0	N ^C
D	4			1	0.
		Total NPV	earned	(\mathfrak{d})	44

Question 7: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are indivisible. Determine the optimal project selection.

Projects	Initial Investment	NPV
	(\$000)	(\$000)
Α	40	20
В	100	35
С	050	24
D	60	18

Sol:

Projects	Initial Investment	NPV	Profitability	Ranks
	(\$000)	(\$000)	Index (PI)	
А	70	20	20/40 = 0.5	1
В	1 100	35	35/100 = 0.35	3
С	50	24	24/50 = 0.48	2
D	60	18	18/60 = 0.3	4
1	U			

Capital Rationing:

Projects	Ranks	Available Fund	Fund Invested	Fund Remaining	NPV
J	a de	(\$000)	(\$000)	(\$000)	(\$000)
А	Ň	100	40	60	20
C ?	2	60	50	10	24
B	3	10		10	
D	4				
Total NPV earned					44

Question 8: A company is considering the following 5 investment projects and has \$100,000 available for investment. All this projects are divisible and project A and C are mutually exclusive. Determine the optimal project selection.

Projects	Initial Investment	NPV
-	(\$000)	(\$000)
Α	40	20
В	100	35
С	50	24
D	60	18

Sol:

Sol:

Projects	Initial Investment	NPV	Profitability	Ranks
	(\$000)	(\$000)	Index (PI)	
А	40	20	20/40 =0.5	1
В	100	35	35/100 = 0.35	3
С	50	24	24/50 = 0.48	2
D	60	18	18/60 = 0.3	4

Capital Rationing:

Project Mix	Fund Invested (\$000)	NPV(\$000)	
A+C	Not Possible (mutually exclusive projects)		
A+B	Not possible (require \$140,00	0)	
A+D	100	20+18=38	
Α	40	20	
В	100	35	
B+C	Not possible (require \$150,000) (0)		
B+D	Not possible (require \$160,000)		
С	50	24	
C+D	Not possible (require \$17,000)		
D	60	18	
	112		

From the project mix we can see that if we move with ranks and eliminate impossible options (mutually exclusive projects or limited fund cases we will get the best mix of projects that will provide maximum return. Here, project A plus will provide the maximum NPV of \$38,000.

Api Api Mohammad Cammul Arefuns

Financial Management Equity Valuation Problems and Solution

Question 1: A company promises to pay a fixed dividend of \$10 per preferred share. If your required return from investment is 15% then how much should you offer to purchase this share?

Sol: Here, D =\$10, required return, r = 15% = 0.15, Share Price, P = ?

$$P_0 = \frac{D}{r} = \frac{\$10}{0.15} = \$66.67$$

Question 2: ABC Textiles re-invests 50% of its earning on growth projects that yield an ROE of 20%. If company's year-end projected earnings per share (EPS) is \$10 and your required return from investment is 15% then how much should you offer per share to purchase ABC Textiles' share? How much would you offer if ABC act like a cash cow? Calculate Net Present Value of Growth Opportunity (NPVGO). Explain why ABC should act like a cash cow when its growth projects generate returns below the required return from investment.

Sol: Here, reinvestment or retention rate, b = 50% = 0.50, ROE = 20%, 0.20, EPS, E1 = \$10, required return, r = 15% = 0.15 **Reinvestment Case:** Dividend payout ratio = (100 - 50)% = 50%, $D_1 = E_1 x \text{dividend payout ratio} = $10 x 50\% = 5 Growth rate, g = ROE x b = 0.20 x 0.50 = 0.10 = 10%

Growth rate, $g = ROE \times b = 0.20 \times 0.50 = 0.10 = 10\%$

Share price @ reinvestment case, $P_0 = \frac{D_1}{r-g} = \frac{\$5}{0.1500}$

Cash-cow Case:

Dividend payout ratio = 100%, reinvestment rate = 0%, growth rate, g = 0

 $D_1 = E_1 x$ dividend payout ratio = 10 x 100% = 10

Share price @ cash-cow case, $P_0 = \frac{1}{2} = \frac{10}{0.15} = \frac{10}{0.15}$

Net Present Value of Growth Opportunity (NPVGO):

NPVGO = Share price @ reinvestment case - Share price @ cash-cow case = \$100 - \$66.67 = \$33.33 If project return is below required return:

ABC Textiles should act like a cash cow when its growth projects generate negative NPV i.e. project return is balow the required return. A company should not accept any such projects that fail to achieve foquired return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends.

Question 3: Beximco Pharmaceuticals follows a dividend payout policy of 40% on its eachings. The company has a historical record of achieving an average ROE of 15%. How much should you offer per share if its projected year-end dividend is 200 taka per share and your required return from investment is 12%. Calculate Net Present Value of Growth Opportunity (NPVGO).

Find NPVGO if required return increases to 16%. Sol: Reinvestment Case: Here, dividend payout ratio (DPR) = 40% = 0.40Reinvestment or retention rate, b = (100-40) % = 60% = 0.60ROE = 15% = 0.15, $D_1 = 200tk = E1 \times DPR = E1 \times 0.40$, i.e. E1 = 200/0.40 = 500tkrequired return, r = 12% = 0.12Growth rate, $g = ROE \times b = 0.15 \times 0.60 = 0.09 = 9\%$

Share price @ reinvestment case, $P_0 = \frac{D_1}{r-g} = \frac{200}{0.12 - 0.09} = 6,666.67tk$

Cash-cow Case:

Dividend payout ratio = 100%, reinvestment rate = 0%, growth rate, g = 0 $D_1 = E_1 = 500 tk$

Share price @ cash-cow case,
$$P_0 = \frac{D_1}{r-g} = \frac{500}{0.12} = 4,166.67 tk$$

Net Present Value of Growth Opportunity (NPVGO):

NPVGO = Share price @ reinvestment case - Share price @ cash-cow case = 6,666.67 - 4,166.67 2100.com = 2,500tk

if required return increases to 16%:

Share price @ reinvestment case, $P_0 = \frac{D_1}{r-g} = \frac{200}{0.16 - 0.09} = 2,857.14tk$

Share price @ cash-cow case, $P_0 = \frac{D_1}{r-g} = \frac{500}{0.16} = 3,125tk$

r - g 0.10 NPVGO = Share price @ reinvestment case - Share price @ cash-cow, case = 2,857.14 - 3,125 = -267.86tk Povimes Plant

Beximco Pharmaceuticals should act like a cash cow when is growth projects generate negative NPV i.e. project return (15%) is below the required return (16%). A company should not accept any such projects that fail to achieve required return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends.

Question 4: Olympic Industries follows a dividend payout policy of 25% on its earnings. The company has a historical record of achieving an average ROE of 20%. How much should you offer per share if its projected year-end dividend is 100 taka per share and your required return from investment is 22%. Calculate Net Present Value of Growth **Opportunity (NPVGO).**

Sol: <u>Reinvestment Case:</u> Here, dividend payout race (DPR) = 25% =0.25 Reinvestment or retention rate, b = (100-25) % = 75% = 0.75ROE = 20% =0.20 D_1 = 100tk = E1 x DPR = E1 x 0.25, i.e. E1 =100/0.25 =400tk required return, q = 22% = 0.22Growth rate, q = ROE x b =0.20 x 0.75 =0.15 =15%

Share price @ reinvestment case,
$$P_0 = \frac{D_1}{r-g} = \frac{100}{0.22 - 0.15} = 1,428.57tk$$

Cash-cow Case:

Dividend payout ratio = 100%, reinvestment rate = 0%, growth rate, g = 0 $D_1 = E_1 = 400tk$

Share price @ cash-cow case, $P_0 = \frac{D_1}{r-g} = \frac{400}{0.22} = 1,818.18tk$

Net Present Value of Growth Opportunity (NPVGO):

NPVGO = Share price @ reinvestment case - Share price @ cash-cow case = 1,428.57tk - 1,818.18tk= -389.61tk

Olympic Industries should act like a cash cow when its growth projects generate negative NPV i.e. project return (20%) is below the required return (22%). A company should not accept any such projects that fail to achieve required return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends.

Question 5: XYZ Pharmaceuticals follows a dividend payout policy of 30% on its earnings. The company has a historical record of achieving an average ROE of 10%. Explain why XYZ can be a potential target for hostile takeover by other companies if investors' required return from investment is above 10%.

Sol: XYZ Pharmaceuticals should act like a cash cow when required return is above project return (10%). A company should not accept any such projects that fail to achieve required return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends. For example, if XYZ' projected year-end EPS, E1 = 1000Tk. then in: **Reinvestment Case:** Here, dividend payout ratio (DPR) = 30% =0.30 Reinvestment or retention rate, b = (100–30) %=70% = 0.70 D₁ = E1 x DPR = 1000 x 0.30=300tk ROE = 10% =0.10, required return, r = 11% = 0.11 Growth rate, g = ROE x b =0.10 x 0.70 =0.07 =7% Share price @ reinvestment case, $P_0 = \frac{D_1}{r-g} = \frac{300}{0.11-0}$, 7,500tk**Cash-cow Case:** Dividend payout ratio = 100%, reinvestment rate, 0%, growth rate, g = 0 Share price @ cash-cow case, $P_0 = \frac{D_1 = E_1}{r-g}$, 0.11Net Present Value of Growth Opportunity (NPVGO): NPVGO = Share price @ reinvestment case - Share price @ cash-cow case = 7,500tk - 9,090.9tk

= -1,590.9tk

XYZ Pharmaceuticals would be subject to takeover, because another firm could buy 51% or more share of XYZ for the market price of 7,500 taka per share and increase the value of the firm (9,090.9 taka per share) by changing its investment policy (pay out all earnings as dividends) and make a gain of 1,590.9 taka per share.

Question 6: If we assume a stock has an expected dividend payout of 50%, a required rate of return of 12%, and an expected growth rate for dividends of 9%, then find the P/E ratio and the value of the stock per share for a current EPS of \$2.

$$P_{1} = \frac{\text{Dividend Payout Ratio}}{r - g} = \frac{0.50}{0.12 - 0.09} = 16.7$$

Here, $E_0 = \$2$ and $E_1 = \$2x1.09 = \2.18

The value or Price per share will be, $P_0 = 16.7 x 2.18 = \$36.41$

<u>Financial Management</u> Bond Valuation Problems and Solution

Question 1: How much should you pay today to purchase a 10% semi-annual coupon bond of 3 years maturity where the bond comes with face value of \$1,000 and yield to maturity 12%?

Sol: Here, Coupon payment, C = 10/2 % = 5% of \$1,000 = \$50, r = 12/2 % = 6% = 0.06

Time	Cash Flow	Present Value	
1	50	Present value, $P_{i} = F(1+r)^{-r}$	
		$50(1+0.06)^{-1} = 47.17	1
2	50	$50(1+0.06)^{-2} = 44.5	COr
3	50	$50(1+0.06)^{-3} = 41.98	<u> </u>
4	50	$50(1+0.06)^{-4} = 39.6	5
5	50	$50(1+0.06)^{-5} = 37.36	
6	1050	$1050(1+0.06)^{-6} = 740.2	
		Price, $\Sigma = 950.82$	

Question 2: Ford issued a 30 years maturity bond 28 years before where it promises to make 12% semi-annual coupon payment and a payment of \$1,000 face value at maturity. How much should you pay today to purchase this bond if yield to maturity for 2 years Treasury note is 15%?

Sol: Here, maturity left = 2 years, coupon payme C = 12/2 % = 6% of \$1,000 = \$60, r = 15/2 % = 7.5\% = 0.075

Time	Cash	PresentValue
	Flow	$\sim 2^{2}$
1	60	Present value, $P_t = F(1+r)^{-t}$
		$60(1+0.075)^{-1} = 55.81
2	60, ,	$60(1+0.075)^{-2} = 51.92
3	190°	$60(1+0.075)^{-3} = 48.29
4	1060	$1060(1+0.075)^{-4} = \$793.72$
ð		Price, $\Sigma = 949.76$

Question 3: Sam purchased a 50 years maturity bond with 2 years remaining at a price of \$867 where the bond promises to make 10% semi-annual coupon payment and a single payment of \$1,000 at maturity. Find the rate of yield to maturity (YTM) Sam will receive from his investment in bond.

Sol

This is a discount bond i.e. the bond is selling at lower price than its face value.

A bond is sold at discount when coupon rate is lower than YTM. As the bond is

selling at discount, YTM must be higher than coupon rate of 10%.

Here, maturity left = 2 years, bond price = \$867, coupon payment, C = 10/2 % = 5% of \$1,000 = \$50, YTM, r =?

Make A Guess: 867 investment is providing a total cash flow of $(4 \times 50 + 1000) = 1200$ Interest earned = 1200 - 867 = 333 in 2 years.

If we consider simple interest, \$867 x r x 2 = \$333, then,

 $\mathbf{r} = \frac{\$333}{\$867x2} = 0.192 = 19.2\%$

Time	Cash Flow	Net Present Value @20% r =20/2 =10% =0.1	Net Present Value @17% r =17/2 =8.5% =0.085
0	(867)	(867)	(867)
1	50	Present value, $P_t = F (1 + r)^{-t}$ 50(1 + 0.1) ⁻¹ = \$45.45	Present value, $P_t = F (1 + r)^{-t}$ 50(1 + 0.085) ⁻¹ = \$46.08
2	50	$50(1+0.1)^{-2} = \$41.32$	$50(1+0.085)^{-2} = 42.47
3	50	$50(1+0.1)^{-3} = 37.56	$50(1+0.085)^{-3} = 39.100
4	1050	$1050(1+0.1)^{-4} = \$717.16$ NPV, $\Sigma = 841.5 - \$867 = -\25.49	$1050(1+0.085)^{-4} = \$57.65$ NPV, $\Sigma = \$885.35$
		NPV. \$18.35	

$$YTM (\mathbf{r}) = L + \left[\frac{NPV_L}{NPV_L - NPV_H}x(H - L)\right] = 0.17 + \frac{\$18.35}{\$18.35 - (-\$25.49)}x(0.20 - 0.77)$$

= 0.1825 = 18.25%

Question 4: Mr. Karim paid \$750 to purchase a 30 years maturity bond with 2.5 years remaining, where the bond promises to make 8% semi-annual coupon payment and a single payment of \$1,000 at maturity. Find the rate of yield to maturity (YTM) Mr. Karim will receive from his investment in bond.

Sol: Here, bond price = \$750, maturity left = 2.5 years, bupon payment, C = 8/2 % = 4% of \$1,000 = \$40, YTM, r = ?

\$1,000 = \$40, YTM, r =? Make A Guess: \$750 investment is providing a total cash flow of $(5 \times 40 + 1000) =$ 1200 Interest earned = 1200 - 750 =450 in 2.5 years

1.0

If we consider simple interest, $750 \times r \times 2.5$ 450, then,

$$r = \frac{\$450}{\$750 \times 2.5} = 0.24 = 24\%$$

\$750.	12.5	XY		
Time	Cash Flow	Net Present Value @24% r =24/2 =12% =0.12	Net Present Value @20% r =20/2 =10% =0.10	
0	(750)	(750)	(750)	
1	40	Present value, $P_t = F(1+r)^{-t}$	Present value, $P_t = F (1 + r)^{-t}$	
-		$40(1+0.12)^{-1} = \$35.71$	$40(1+0.10)^{-1} = \$36.36$	
2	40 2	$40(1+0.12)^{-2} = \$31.88$	$40(1+0.10)^{-2} = 33.05	
3		$40(1+0.12)^{-3} = \$28.47$	$40(1+0.10)^{-3} = \$30.05$	
4	40	$40(1+0.12)^{-4} = \$25.42$	$40(1+0.10)^{-4} = \$27.32$	
	1040	$1040(1+0.12)^{-5} = 590.12	$1040(1+0.10)^{-5} = 645.75	
Z.	1040	NPV, $\Sigma = $711.61 - $750 = -$38.38$	NPV, Σ = \$772.55-\$750=\$22.55	
<i>YTM</i> (r) = $L + \left[\frac{NPV_L}{NPV_L - NPV_H}x(H - L)\right] = 0.20 + \frac{\$22.55}{\$22.55 - (-\$38.38)}x(0.24 - 0.20)$				

= 0.2148 = 21.48%

Question 5: Mr. Imdad is considering a 12% semi-annual coupon bond with 3 years maturity where face value is \$5,000 and yield to maturity, y = 8%. How much Mr. Imdad should pay to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity increases by 0.6 percentage points, calculate the percentage change in the bond price and therefore, the new price.

Sol: Here, coupon payment, C = 12/2 % = 6% of 5,000 = 300, maturity = 3 years,

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r =8/2 % =4% =0.04

Time (t)	Cash Flow	Present Value	Weight (W _t)	Time x Weight (t) x (W _t)
1	300	Present value, $P_t = F (1 + r)^{-t}$ 300(1 + 0.04) ⁻¹ = \$288.46	$\frac{\$2\$8.46}{5524.21} = 0.052$	0.052
2	300	$300(1+0.04)^{-2} = 277.36	$\frac{\$277.36}{5524.21} = 0.05$	0.1
3	300	$300(1+0.04)^{-3} = 266.7	$\frac{\$266.7}{5524.21} = 0.048$	0.144
4	300	$300(1+0.04)^{-4} = 256.44	$\frac{\$256.44}{5524.21} = 0.046$	0.185
5	300	$300(1+0.04)^{-5} = 246.57	$\frac{\$246.57}{5524.21} = 0.044$	0.22310
6	5300	$5300(1+0.04)^{-6} = 4188.66 Price, $\Sigma = 5524.21$	$\frac{\$4188.66}{5524.21} = 0.758$	4.549 Duration, $\Sigma = 5.255$

Here, percentage change in yield to maturity, $\Delta Y = 0.6/2 \% = 0.3 \%$

Here modified duration,
$$D^* = D / (1 + y) = 5.255 / 1.04 = 5052$$
 (semi-annual)

$$i.e.\frac{\Delta P}{P} = -D^* \ge \Delta y = -5.052 \pm 0.3\% = -1.5156\%$$

Bond price will decrease by 1.51%

$$\frac{\Delta P}{M} = -1.5156\% = -0.015156$$

Here modified duration,
$$D^* = D / (1 + y) = 5.255 / 1.04 = 500$$

i.e. $\frac{\Delta P}{P} = -D^* \ge \Delta y = -5.052 \pm 0.3\% = -1.5156\%$
Bond price will decrease by 1.51%
 $\frac{\Delta P}{P} = -1.5156\% = -0.015156$
i.e. $\Delta P = P (-0.015156) = \$5524.21 (-0.015156) = -\$83.72$

Bond price will decrease by \$83.72

Question 6: Mr. Azad is working in Lanka Bangla Finance Company in Bangladesh. Lanka Bangla invests a significant amount of their capital in Treasury Securities issued by Bangladesh Bank. Bangladesh Bank is going to increase their lending rate from 5% to 5.5%. Find out the impact of this change in interest rate on Lanka Bangla' Treasury security portfolio of 190,000,000 if the semi-annual duration of the portfolio is 2.67.

Sol: Here, current value of portfolio or investment = 100,000,000 taka, semi-annual duration, D = 2.67, initial yield to maturity, Y = 5/2% = 2.5% = 0.025

Percentage charge in interest rate, $\Delta Y=0.5/2$ % =0.25%

Here modified duration, $D^* = D / (1 + y) = 2.67 / 1.025 = 2.6048$ (semi-annual)

i.e.
$$\frac{\Delta P}{D} = -2.6048 \times 0.25\% = -0.6512\%$$

Partfolio value will decrease by 0.6512%

$$\frac{\Delta P}{P} = -0.6512\% = -0.006512$$

i.e. $\Delta P = P(-0.006512) = 100,000,000(-0.006512) = -651,200$

Portfolio value will decrease by 651,200 taka

Question 7: Mr. Saikat is planning to invest in a 7% semi-annual coupon bond with 2.5 years maturity where face value is \$10,000 and yield to maturity, y = 10%. How much Mr. Saikat should pay to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity decreases by 2.2 percentage points, calculate change in the bond price and therefore, the new price.

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Sol: Here, coupon payment, C = 7/2 % = 3.5% of 10,000 = 350, maturity = 2.5 years, r =10/2 % =5% =0.05

Time (t)	Cash Flow	Present Value	Weight (W _t)	Time x Weight (t) x (W _t)
1	350	Present value, $P_t = F (1 + r)^{-t}$ 300(1+0.04) ⁻¹ = \$288.46	$\frac{\$288.46}{5524.21} = 0.052$	0.052
2	350	$300(1+0.04)^{-2} = 277.36	$\frac{\$277.36}{5524.21} = 0.05$	0.1
3	350	$300(1+0.04)^{-3} = 266.7	$\frac{\$266.7}{5524.21} = 0.048$	0.144
4	350	$300(1+0.04)^{-4} = 256.44	$\frac{\$256.44}{5524.21} = 0.046$	0.1850
5	10,350	$5300(1+0.04)^{-6} = 4188.66 Price, $\Sigma = 5524.21$	$\frac{\$4188.66}{5524.21} = 0.758$	$\Sigma 549$ Duration, $\Sigma = 5.255$

Here, percentage change in yield to maturity, $\Delta Y = 0.6/2 \% = 0.3 \%$

Here, percentage change in yield to maturity,
$$\Delta I = 0.0/2$$
 % = -0.3 %
Here modified duration, $D^* = D / (1 + y) = 5.255 / 1.04 = 5.052$ (semi-nual)
 $i.e. \frac{\Delta P}{P} = -D^* \ge \Delta y = -5.052 \pm 0.3\% = -1.5156\%$
Bond price will decrease by 1.51%
 $\frac{\Delta P}{P} = -1.5156\% = -0.015156$

i.e.
$$\frac{\Delta P}{P} = -D^* \times \Delta y = -5.052 \times 0.3\% = -1.5156\%$$

Bond price will decrease by 1.51%

Financial Management Risk & Return Problems and Solution

		0
Security	Amount Invested (\$)	Beta
Stock A	5,000	0.75
Stock B	10,000	1.10
Stock C	8,000	1.36
Stock D	7,000	1.88

Question 1: Suppose you have invested \$30,000 in the following four stocks:

The risk-free rate is 4% and the expected return on the market portfolio is 15%. Based on the CAPM, what is the expected return on the above portfolio? Sol: Based on CAPM: $E(r_p) = r_f + \beta_p [E(r_m) - r_f] = 0.04 + \beta_p [0.15 - 0.04]$ we dont know the value of portfolio beta β_p ; we need to find it using available information

$$\beta_{p} = w_{A}\beta_{A} + w_{B}\beta_{B} + w_{C}\beta_{C} + w_{C}\beta_{C} = \frac{5,000}{30,000}x0.75 + \frac{10,000}{30,000}x1.10 + \frac{8,000}{30,000}x1.36 + \frac{7,000}{30,000}x1.88$$

= 0.125 + 0.367 + 0.3627 + 0.4387 = 1.293
Now, $E(r_{p}) = r_{f} + \beta_{p}[E(r_{m}) - r_{f}] = 0.04 + 1.293[0.15 - 0.04] = 0.18223 = 18.223\%$

Question 2: You have been provided the following data on the securities of three firms and the market:

Security	\overline{R}	o Hait	$\rho_{i,m}$	β,
Stock A	0.13	1 2	(i)	0.90
Stock B	0.16	V(ii)	0.4	1.10
Stock C	م 0.25	<u>م</u> 0.24	0.75	(iii)
Market	0.15	0.1	(iv)	(v)
Risk-free Asset(r_f)	0.05	(vi)	(vii)	(viii)

Assume the CAPM holds true:

a) Fill in the missing values in the table.b) Provide an evaluation of the investment performance of the three firmsc) What is your investment recommendation? Why?

Sol:

(i) Correlation coefficient between stock A and market, $\rho_{A,m} = ?$

$$\beta_{A} = \frac{\rho_{A,m}}{\rho_{A,m}} \frac{\rho_{A,m}}{\rho_{A,m}} \frac{x \ 0.12}{0.1}$$

$$or, \rho_{A,m} x \ 0.12 = 0.9 \ x \ 0.1$$

$$or, \rho_{A,m} = \frac{0.9 \ x \ 0.1}{0.12} = 0.75$$

(ii) Standard deviation of stock B, $\sigma_{B} = ?$

$$\beta_{B} = \frac{\rho_{B,m} x \sigma_{B}}{\sigma_{m}}$$

$$or, \sigma_{B} = \frac{\beta_{B} x \sigma_{m}}{\rho_{B,m}} = \frac{1.1 x 0.1}{0.4} = 0.275 = 27.5\%$$

$$(...) \rho_{C,m} x \sigma_{C} = 0.75 x 0.24$$

(iii)
$$\beta_c = \frac{p_{c,m} \times \sigma_c}{\sigma_m} = \frac{0.75 \times 0.24}{0.1} = 1.8$$

(iv) Correlation coefficient between market and market is 1

(v) Market Beta is 1

(vi) Risk-free asset is considered to have no risk; hence standard deviation $\sigma = 0$ (vii) Correlation coefficient between risk free asset and market is zero because risk free asset have no risk associated with it and it is independent of market movement.

(viii) Risk free asset has no market risk or beta.

b and C:

,	unu C.			(\mathbf{O})	
	Stock	Expected Return	Required Return (CAPM)	Recommendatio	on
	Α	13%	0.05 + 0.9(0.15 - 0.05) = 14%	Verpriced	Sell
	В	16%	0.05+1.1(0.15-0.05)=16%	Correctly Priced	Buy or sell
	C	25%	0.05+1.8(0.15-0.05)=22	Underpriced	Buy

Question 3:

a) What is the expected return and variance on an equally weighted portfolio of the following three stocks?

State of	Probability of State	Ex Ex	pected Retu	un
Economy	of Economy	Stock A	Stock B	Stock C
Boom	0.8	0.07	0.15	0.33
Bust		0.13	0.03	-0.06

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State of		Exp	ected Ret	urn		
Economy	Prob.	Stock A	Stock B	Stock C	Portfolio Return (r)	(r ²)
Boom	0.8	0.07	0.15	0.33	(0.07+0.15+0.33)/3 =0.1834	0.0336
Bust	0.2	0.13	0.03	-0.06	0.03334	0.001112

Expected portfolio return, $E(r_p) = 0.8 \times 0.1834 + 0.2 \times 0.03334 = 0.153388 = 15.3\%$ Variance, $\sigma_p = 0.8 \times 0.0336 + 0.2 \times 0.001112 - (0.153388)^2 = 0.003574$ Standard Deviation, $\sigma_p = 0.05978 = 5.978\%$

by What is the expected return and variance of a portfolio invested 20% each in A and B, and the remaining in C?

State of	Droh	Expe	ected F	Return	Doutfolio Dotum (n)	(\mathbf{r}^2)
Economy	FTOD.	Α	В	С	Fortiono Keturn (r)	(1)
Boom	0.8	0.07	0.15	0.33	0.2x0.07+0.2x0.15+0.6x.33 =0.242	0.058564
Bust	0.2	0.13	0.03	-0.06	-0.004	0.000016

Expected portfolio return, $E(r_p) = 0.8 \times 0.242 + 0.2 \times (-0.004) = 0.1928 = 19.28\%$ Variance, $\sigma_p^2 = 0.8 \ge 0.058564 + 0.2 \ge 0.000016 - (0.1928)^2 = 0.00968$

Standard Deviation, $\sigma_p=0.0984=9.84\%$

State of	Duch	Expe	ected Ret	turn
Economy	Prop.	Α	В	С
Boom	0.20	0.30	0.45	0.33
Good	0.35	0.12	0.10	0.15
Poor		0.01	-0.15	-0.05
Bust	0.15	-0.06	-0.30	-0.09

Question 4: Your portfolio is invested 30% each in stock A and C and the remaining in B? What is the expected return, variance and standard deviation of the portfolio?

Sol:

State of	Droh	Expe	ected Ret	turn	Boutfolio Botum (n)	
Economy	FTOD.	Α	В	С		
Boom	0.20	0.30	0.45	0.33	0.3x0.3+0.4x0.45+0.3x.33 =0.369 0.136161	
Good	0.35	0.12	0.10	0.15	0.121 0.014641	
Poor	0.30	0.01	-0.15	-0.05	-0.072 0.005184	Ļ
Bust	0.15	-0.06	-0.30	-0.09	-0.165 0.027225	;
E D .			0.0.00	0.05		

Exp. Port. return, $E(r_p) = 0.2 \times 0.369 + 0.35 \times 0.121 + 0.30 \times (-0.072) + 0.15 \times (-0.165) = 0.0698$ =6.98% Variance, $\sigma_p^2 = 0.2 \times 0.136161 + 0.35 \times 0.014641 + 0.30 \times 0.005484 + 0.15 \times 0.027225 - (0.0698)^2 = 0.0331$

Standard Deviation, $\sigma_p = 0.182 = 18.2\%$

Question 5: You own a stock portfolio invested 25% in stock Q, 20% in stock R, 15% in stock S, and the remaining in stock T? The betas for these four stocks are 0.75, 1.90, 1.38 and 1.16 respectively. What is the portfolio beta? Sol: Portfolio beta $\beta_p = w_Q \beta_Q + w_R \beta_R + w_S \beta_S + w_T \beta_T = 25 \times 0.75 + 0.2 \times 1.9 + 0.15 \times 1.38 + 0.4 \times 1.16 = 1.2385$

Sol:

Question 6: You own a portfolio equally invested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.85 and the total portfolio is equally as risky as the market, what must the beta be for the other stock in your portfolio?

Sol:
Market Beta = 1, and Beta of isk free asset = 0
Portfolio beta
$$\beta_p = 1 = w \beta_1 + w_2 \beta_2 + w_3 \beta_3 = \frac{1}{3} x_0 + \frac{1}{3} x_{1.85} + \frac{1}{3} x \beta_3 = \frac{1.85 + \beta_3}{3}$$

or, $3 = 1.85 + \beta_3$
or, $\beta_3 = 3 - 1.855 + 1.15$
Beta of the other stock is 1.15

Question 7: A stock has a beta of 1.25, the expected return on the market is 12% and the risk free rate is 5%. According to CAPM, what must the expected return on this stock ber

Sol: Based on CAPM:
$$E(r) = r_f + \beta_i [E(r_m) - r_f] = 0.05 + 1.25[0.12 - 0.05] = 0.1375 = 13.75\%$$

Question 8: A stock has an expected return of 14.2%, the risk free rate is 4% and the market risk premium is 7%. According to CAPM, what must the beta of this stock be? **Sol:** Based on CAPM: $E(r) = r_f + \beta_i [E(r_m) - r_f] = 0.04 + \beta x 0.07$

$$or, 0.142 = 0.04 + \beta x 0.07$$
$$or, 0.142 - 0.04 = \beta x 0.07$$
$$or, \beta = \frac{0.102}{0.07} = 1.457$$

Question 9: A stock has an expected return of 10.5%, its beta is 0.73, and the risk free rate is 5.5%. According to CAPM, what must the expected return on market be?

Sol: Based on CAPM: $E(r) = r_f + \beta_i [E(r_m) - r_f] = 0.055 + 0.73 [E(r_m) - 0.055]$

$$or, 0.105 = 0.055 + 0.73[E(r_m) - 0.055]$$

$$or, E(r_m) - 0.055 = \frac{0.105 - 0.055}{0.73}$$

$$or, E(r_m) = \frac{0.105 - 0.055}{0.73} + 0.055 = 0.12349 = 12.349\%$$

Question 10: A stock has an expected return of 16.2%, a beta of 1.75, and the expected return on the market is 11%. According to CAPM, what must the risk free rate be?

return on the market is 11%. According to CAPM, what must the risk free rate be?
Sol: Based on CAPM:
$$E(r) = r_f + \beta_i [E(r_m) - r_f] = r_f + 1.75[0.11 - r_f] = r_f + 0.1925 - 1.75$$

 $or, 0.162 = 0.1925 - 0.75r_f$
 $or, r_f = \frac{0.1925 - 0.162}{0.75} = 0.0407 = 4.07\%$
Question 11: A stock has a beta of 0.92, and an expected return of 10(3%). A risk free

Question 11: A stock has a beta of 0.92, and an expected return of 10(3%) asset currently earns 5%.

a) Based on CAPM, what is the expected return on a portfolin that is equally invested in the two assets?

Sol: Based on CAPM:
$$E(r) = r_f + \beta_i [E(r_m) - r_f] = 0.05 + 0.92[E(r_m) - 0.05]$$

or, $0.103 = 0.05 + 0.92[E(r_m) - 0.05]$
or, $E(r_m) - 0.05 = \frac{0.103 - 0.05}{0.92}$
or, $E(r_m) = \frac{0.103 - 0.05}{0.92} + 0.05 = 0.1077$ 10.76%
Portfolio beta $\beta_p = 1 = w_1\beta_1 + w_1\beta_2 = \frac{1}{2}x0 + \frac{1}{2}x0.92 = 0.46$
Based on CAPM: $E(r_p) = r_f + \beta_p [E(r_m) - r_f] = 0.05 + 0.46[0.1076 - 0.05] = 0.0765 = 7.65\%$

Expected Portfolie return, $E(r_p) = 0.5 \times 0.103 + 0.5 \times 0.05 = 0.0765 = 7.65\%$

b) If a portfolio of the two assets has a beta of 0.50, what are the portfolio weights? B ased on $APM: E(r_p) = r_f + \beta_p [E(r_m) - r_f] = 0.05 + 0.5[0.1076 - 0.05] = 0.0788 = 7.88\%$

$$0.0788 = wx0.05 + (1 - w)x0.103$$

$$a = 0.0788 = 0.05w + 0.103 - 0.103w$$

$$b = 0.0242$$

$$or, w = \frac{0.0242}{0.053} = 0.4566 = 45.66\%$$

Investment in risk free asset is 45.66% and the remaining 54.34% is in the stock. c) If a portfolio of the two assets has an expected return of 9%, what is its beta?

Based on CAPM: $0.09 = r_f + \beta_p [E(r_m) - r_f] = 0.05 + \beta_p [0.1076 - 0.05]$

$$or, \beta_p = \frac{0.09 - 0.05}{0.1076 - 0.05} = 0.6945$$

d) If a portfolio of the two assets has a beta of 1.84, what are the portfolio weights? How do you interpret the weights for the two assets in this case? Explain.

Mohammad Kamrul Arefin, MSc. in Quantitative Finance, University of Glasgow Page | 97 Portfolio beta $\beta_p = 1.84 = w_1\beta_1 + w_2\beta_2 = wx0 + (1 - w)x0.92$

$$or, 1 - w = \frac{1.84}{0.92} = 2$$

 $or, w = 1 - \frac{1.84}{0.92} = -1$

Investment in the stock is 200% of capital, which is made possible by borrowing 100% of capital at risk free rate of 5%. Suppose, if an investor has \$10,000 to invest in these two assets then he will invest \$20,000 in stock by borrowing \$10,000 at risk free rate of 5%.

Question 12: Stock Y has a beta of 1.35 and an expected return of 14%. Stock Z has	as a 🔿
beta of 0.85 and an expected return of 11.5%. The market risk premium is 6.8%.	

a) If the risk free rate is 5.5%, are these stocks correctly priced?

Sol: For the two stocks to be correctly priced their reward to risk ratio should be same.

Reward-risk ratio of stock
$$Y = \frac{0.14 - 0.055}{1.35} = 0.063$$

Reward-risk ratio of stock $Z = \frac{0.115 - 0.055}{0.85} = 0.0705$
Reward-risk ratio of stock $Y \neq$ Reward-risk ratio of stock Z **C T**

b) What would the risk free rate have to be for the two stocks to be correctly priced? Reward-risk ratio of stock Y = Reward-risk ratio costock Z

Reward-risk ratio of stock Y = Reward-risk ratio of stock Z $or, \frac{0.14 - r_f}{1.35} = \frac{0.115 - r_f}{0.85}$ $or, 0.85 \times 0.14 - 0.85 r_f = 1.35 \times 0.115 - 1.00 r_f$ $or, 1.35 r_f - 0.85 r_f = 1.35 \times 0.115 - 0.85 \times 0.14$ $or, 0.5 r_f = 0.03625$ $or, r_f = 0.0725 = 7.25\%$

Question 13:

a) If your portfolio is invested 40% each in A and B and the remaining in C, what is the portfolio expected return, variance and the standard deviation?

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	State of	Droh	Expected Return           A         B         C           .35         0.20         0.35         0.60           .40         0.15         0.12         0.05           0.01         -0.25         -0.50		
20	Economy	Prop.	Α	В	С
d'	Boom	0.35	0.20	0.35	0.60
Y	Normal	0.40	0.15	0.12	0.05
	Bust		0.01	-0.25	-0.50

Sol	(C	
		•

State of	Droh	Expe	ected Ret	turn	<b>Bontfolio Botunn</b> (n)	$(\mathbf{r}^2)$
Economy	rrop.	Α	В	С	Fortiono Keturn (r)	$(\mathbf{r})$
Boom	0.35	0.20	0.35	0.60	0.4x0.2+0.4x0.35+0.2x0.6 =0.34	0.1156
Normal	0.40	0.15	0.12	0.05	0.118	0.013924
Bust	0.25	0.01	-0.25	-0.50	-0.196	0.038416

Exp. Port. return,  $E(r_p) = 0.35 \ge 0.34 + 0.4 \ge 0.118 + 0.25 \ge (-0.196) = 0.1172 = 11.72\%$ Variance,  $\sigma_p^2 = 0.35 \ge 0.1156 + 0.4 \ge 0.013924 + 0.25 \ge 0.038416 - (0.1172)^2 = 0.04189$ Standard Deviation,  $\sigma_p = 0.20469 = 20.47\%$ 

- b) If the expected T-bill rate is 3.80%, what is the expected risk premium on the portfolio?
  - Expected risk premium = 11.72% 3.80% = 7.92%

c) If the expected inflation rate is 3.50%, what are the approximate and exact expected real returns on the portfolio? What are the approximate and exact expected real risk premiums on the portfolio?

Approximate expected real return = 11.72% - 3.50% = 8.22%Exact expected real return (Fisher Equation):

$$1+0.1172 = (1+0.0350)(1+r)$$

$$or, 1 + r = \frac{1.1172}{1.0350}$$
$$or, r = \frac{1.1172}{1.0350} - 1 = 0.07942 = 7.942\%$$

Approximate expected real risk premium = 7.92% - 3.50% = 4.42%

Exact expected real risk premium= 
$$\frac{1.0792}{1.0350} - 1 = 0.0427 = 4.27\%$$

Question 14: You want to create a portfolio equally as risky as the market, and you have \$1,000,000 to invest. Given this information, fill in the rest of the following table:

**Investment (\$)** Asset Beta Stock A 180,000 Stock B 290,000 Stock C **Risk Free Asset**  $w_{A} = \frac{180}{1000} = 0.18; \quad w_{B} = \frac{290}{1000} = 0.29; \quad w_{C} = x + \frac{100}{1000} = 1 - 0.18 - 0.29 - x = 0.53 - x$ Portfolio beta  $\beta_{p} = 1 = w_{A}\beta_{A} + w_{B}\beta_{B} + w_{C}\beta_{C}$ or, 1 - 0.18x0.75 - 0.29x1.3 = 1.45x $or, x = \frac{0.488}{1.45}$ Investment in Asset C =  $\frac{0.488}{2.55}x$ \$1,000,000 = \$336,551.72 Investment in Risk-free set = \$1,000,000 - (180,000 - 290,000 - 336,551.72) = \$193,448.27Question 15: You have \$100,000 to invest in a portfolio containing stock X, stock Y, and a risk-free asset. You must invest all of your money. Your goal is to create a portfolio that has an expected return of 10.7% and that has only 80% of the risk of the overall market.

If X has an expected return of 17.2% and a beta of 1.8, Y has an expected return of 8.75% and a beta of 0.5, and the risk free rate is 7%, how much money will you invest in stock X?

$$w_{x} = X; \quad w_{y} = Y; \quad w_{f} = 1 - X - Y$$
Portfolio beta  $\beta_{p} = 1x80\% = 0.8 = 1.8X + 0.5Y + (1 - X - Y)0$ 
*i.e.*  $1.8X + 0.5Y = 0.8$  ......Eq (1)
Portfolio return,  $E(r_{p}) = 0.107 = 0.172X + 0.0875Y + (1 - X - Y)0.07$ 
*or*,  $0.107 = 0.172X + 0.0875Y + 0.07 - 0.07X - 0.07Y$ 
*or*,  $0.037 = 0.102X + 0.0175Y$ 
*or*,  $0.102X + 0.0175Y = 0.037$ ......Eq (2)

Sol:

Eq(1)x0.0175: 0.0315X + 0.00875Y = 0.014-Eq(2)x0.5: 0.051X + 0.00875Y = 0.0185

0.0315X - 0.051X = 0.014 - 0.0185or, -0.0195X = -0.0045 or, X =  $\frac{-0.0045}{-0.0195} = \frac{45}{195}$ Investment in X =  $\frac{45}{195}x$ \$100,000 = \$23,076.92

Question 16: You have invested 40% of your capital in stock A and the remaining in stock B. Historical return over last 5 years shows the following pattern:

I IMBUU		une romo m	
Voor	Stock A	Stock B	n ^o
I cai	( <b>x</b> )	<b>(y</b> )	121
2014	25%	15%	A
2013	32%	10%	
2012	15%	30%	
2011	20%	-5%	
turn an	d variance.		$\overline{\mathcal{O}}$

Find the expected portfolio return and variance.

$$\sum_{x} x^{2} = 0.2274; \quad \sum_{y} y^{2} = 0.125; \quad \sum_{x} x = 0.92; \quad \sum_{y} y = 0.125; \quad xy = 0.1045;$$

$$\overline{x} = 0.23; \quad \overline{y} = 0.125; \quad \sigma_{x} = 0.06284; \quad \sigma_{y} = 0.125;$$
Portfolio return,  $E(r_{p}) = 0.4 \times 0.23 + 0.6 \times 0.125 = 0.167 = 16.7\%$ 
Correlation coefficient,  $r_{xy} = \frac{\sum_{x} (x) \frac{(\sum_{x} y)^{2}}{n}}{\sqrt{(\sum_{x} y)^{2} - (\sum_{x} y)^{2}}} \{\sum_{y} y^{2} - (\sum_{x} y)^{2}\}$ 
Sol:
$$= \frac{0.1045 - \frac{0.92 \times 0.5}{4}}{\sqrt{(0.2274 - \frac{0.92^{2}}{4})(1.125 - \frac{0.5^{2}}{4})}} = -0.3341$$
Portfolio std. HeV.,  $\sigma = \sqrt{w_{x}^{2} \sigma_{x}^{2} + w_{y}^{2} \sigma_{y}^{2} + 2w_{x}w_{y}\sigma_{x}\sigma_{y}r_{xy}}}$ 

$$= \sqrt{0.4^{2} \times 0.6284^{2} + 0.6^{2} \times 0.125^{2} + 2 \times 0.4 \times 0.6 \times 0.06284 \times 0.125 \times (-0.3341)}$$

$$= 0.07969 = 7.069\%$$

Question 17: Security F has an expected return of 10% and a standard deviation of 26% per year. Security G has an expected return of 17% and a standard deviation of 58% per year.

- a) What is the expected return on a portfolio composed of 30% of security F and 70% of security G?
- b) If the correlation between the returns of security F and security G is 0.25, what is the standard deviation of the portfolio described in part (a)?

Portfolio return,  $E(r_p) = 0.3 x 0.10 + 0.7 x 0.17 = 0.149 = 14.9\%$ 

Portfolio std. dev.,  $\sigma = \sqrt{0.3^2 x 0.26^2 + 0.7^2 x 0.58^2 + 2 x 0.3 x 0.7 x 0.26 x 0.58 x 0.25} = 0.4321 = 43.21\%$ 

Question 18: Suppose the expected returns of stocks A and B are 13% and 19% respectively whereas standard deviations are 38% and 62% respectively. Calculate expected return and standard deviation of a portfolio that is composed of 45% of A and 55% of B when the correlation between the returns on A and B is 0.5.

Sol:

Portfolio return,  $E(r_{p}) = 0.45 \times 0.13 + 0.55 \times 0.19 = 0.163 = 16.3\%$ 

Portfolio std. dev.,  $\sigma = \sqrt{0.45^2 \times 0.38^2 + 0.55^2 \times 0.62^2 + 2 \times 0.45 \times 0.55 \times 0.38 \times 0.62 \times 0.5} = 0.4514 = 45.14\%$ Question 19: You have been provided the following data on the securities of three firms and the market:

Security	$\overline{R}$	σ,	$\rho_{i,m}$	β
Stock A	0.10	0.27	(i)	0.85
Stock B	0.14	(ii)	0.5	1,50
Stock C	0.17	0.7	0.35	(iii)
Market	0.12	0.2	(iv)	$\sqrt{2}$ (v)
<b>Risk-free Asset</b> ( $r_f$ )	0.05	(vi)	(vii) (	(viii)
ssume the CAPM hold	s true:		C III	
a) Fill in the missing	g values in th	ne table.	· Or	
b) Are these stocks	A, B and C c	orrectly priced?	2)-	
c) What is your inv	estment reco	mmendation? W	hv?	

c) What is your investment recommendation? Why?

Sol:

Sol:  
(i) Correlation coefficient between stock A and market, 
$$\rho_{A,m} = ?$$
  
 $\beta_A = \frac{\rho_{A,m} \cdot x \cdot \sigma_A}{\sigma_m}$   
 $or, 0.85 = \frac{\rho_{A,m} \cdot x \cdot 0.27}{0.2}$   
 $or, \rho_{A,m} = \frac{0.85 \cdot x \cdot 0.2}{0.27} = 0.6296$   
(ii) Standard deviation of stock B,  $\sigma_B = ?$ 

$$\beta_{B} = \frac{\rho_{B,m} x \sigma_{B}}{\sigma_{m}} + \frac{1.5 x 0.2}{0.5} = 0.6 = 60\%$$
(iii)  $\beta_{C} = \frac{\rho_{C,m} x \sigma_{C}}{\sigma_{m}} = \frac{0.35 x 0.7}{0.2} = 1.22$ 

(iv) Correlation coefficient between market and market is 1

(v) Market Beta is 1

(vi) Risk-free asset is considered to have no risk; hence standard deviation  $\sigma = 0$ 

(vii) Correlation coefficient between risk free asset and market is zero because risk free asset have no risk associated with it and it is independent of market movement.

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Stock	Expected Return	Required Return (CAPM)	Recommendatio	n			
Α	10%	0.05+0.85(0.12-0.05)=10.95%	Overpriced				
В	14%	0.05+1.5(0.12-0.05)=15.5%	Overpriced				
С	17%	0.05+1.225(0.12-0.05)=13.575%	Underpriced				

(viii) Risk free asset has no market risk or beta.

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Sell Sell Buy Question 20: The market portfolio has an expected return of 12% and a standard deviation of 19%. The risk free rate is 5%.

a) What is the expected return on a well diversified portfolio with a standard deviation of 7%?

Sol:

For a well diversified portfolio,

Reward-risk ratio of market = Reward-risk ratio of portfolio

or, 
$$\frac{0.12 - 0.05}{0.19} = \frac{r - 0.05}{0.07}$$
  
or,  $r - 0.05 = \frac{0.07 \times 0.07}{0.19}$   
or,  $r = \frac{0.07 \times 0.07}{0.19} + 0.05 = 0.07579 = 7.579\%$   
b) What is the standard deviation of a well diversified portfolio with an expected return of 20%?  
For a well diversified portfolio,  
Reward-risk ratio of market = Reward-risk ratio of portfolio function of  $r, \frac{0.12 - 0.05}{0.19} = \frac{0.2 - 0.05}{\sigma}$   
or,  $\sigma = \frac{0.2 - 0.05}{0.12 - 0.05} \times 0.19 = 0.40714 = 40.714\%$   
prestion 21: A portfolio that combines the risk free asset and the market portfolio has an pected return of 9% and a standard deviation of 13%. The risk free rate is 5%, and the

$$or, \frac{0.12 - 0.05}{0.19} = \frac{0.2 - 0.05}{\sigma}$$

$$or, \sigma = \frac{0.2 - 0.05}{0.12 - 0.05} x 0.19 = 0.40714 = 40.714\%$$

Question 21: A portfolio that combines the risk free asset and the market portfolio has an expected return of 9% and a standard deviation of 13%. The risk free rate is 5%, and the expected return on the market portfolio is 12%. Assume the CAPM holds. What expected rate of return would a security earn if it had a 0.45 correlation with the market portfolio and a standard deviation of 40%?

Sol:  
Portfolio return, 
$$E(r_p) = 0.09 = wp405 + (1 - w)x0.12$$
  
 $or, 0.09 = 0.05w + 0.12 - 0.12w$   
 $or, 0.07w = 0.03$   
 $or, w = \frac{0.03}{0.07} = \frac{3}{7}$   
Portfolio variable,  $\sigma_p^2 = 0.13^2 = w_f^2 x \sigma_f^2 + w_m^2 x \sigma_m^2 + 2w_f \sigma_f w_m \sigma_m \rho_{f,m} = (\frac{3}{7})^2 x0 + (\frac{4}{7})^2 x \sigma_m^2 + 2w_f x0 xw_m \sigma_m x0$   
 $or, 0.0000 = \frac{16}{49} x \sigma_m^2$   
 $or, \sigma_m^2 = \frac{0.0169 x 49}{16} = 0.2275 = 22.75\%$   
Beta of a security,  $\beta_i = \frac{\rho_{i,m} x \sigma_i}{\sigma_m} = \frac{0.45 x 0.4}{0.2275} = 0.7912$ 

Expected return of a security,  $E(r) = r_f + \beta_i (r_m - r_f) = 0.05 + 0.7912(0.12 - 0.05) = 0.10538 = 10.538\%$ 

Question 22: Suppose the risk free rate is 4.8% and the market portfolio has an expected return of 11.4%. The market portfolio has a variance of 0.0429. Portfolio Z has a correlation coefficient with the market of 0.39 and a variance of 0.1783. According to the CAPM, what is the expected return on portfolio Z? Sol:

Beta of Portfolio Z,  $\beta_z = \frac{\rho_{z,m} x \sigma_z}{\sigma_m} = \frac{0.39 x \sqrt{0.1783}}{\sqrt{0.0429}} = 0.795$ Expected return on Portfolio Z,  $E(r_z) = r_f + \beta_z (r_m - r_f) = 0.048 + 0.795(0.114 - 0.048) = 0.10047 = 10.047\%$ 

Question 23: Suppose market risk premium is 7.5% and the risk free rate is 4%. Which of the following stock has the most systematic risk? Which one has the most unsystematic risk? Which stock is riskier? Explain.

	State of Economy	Probability	Return from Stock A	Return from Stock B
	Recession	0.15	0.09	-0.30
	Normal	0.55	0.42	.0312
	Good	0.30	0.26	0.44
So	l:			etine
Еx	pected return from stock A	$(r_A) = 0.15 x$	$0.09 + 0.55 \times 0.42 + 0.30 \times 0$	0.26 = 0.3225 = 32.25%
Sta	ndard deviation of stock A	$\Lambda_{A}, \sigma_{A} = \sqrt{0.15  x 0.0}$	$9^{2} + 0.55 x 0.42^{2} + 0.3 x 0.26^{2} - 0.000$	$\overline{0.3225^2} = 0.1204 = 12.04\%$
E (	$(r_{A}) = 0.3225 = 0.04 + \beta_{A}(0)$	.075)	The	
or,	$\beta_A = \frac{0.3225 - 0.04}{0.075} = 3.77$		ohan	
Еx	pected return from stock E	$B, E(r_B) = 0.15 x$	$(-030) + 0.55 \times 0.12 + 0.30$	x 0.44 = 0.153 = 15.3%
Sta	ndard deviation of stock E	$\sigma_{B} = \sqrt{0.15 x}$	$(30)^{2} + 0.55 x 0.12^{2} + 0.3 x 0.44$	$(2^{2} - 0.153^{2}) = 0.2368 = 23.68\%$
E (	$r_{B}$ ) = 0.153 = 0.04 + $\beta_{B}$ (0.0	(75)		
or,	$\beta_{B} = \frac{0.153 - 0.04}{0.075} = 1.507$	all Ar		
Sta	ndard deviation ( $\sigma$ ) repres	sents total risk (sys	stematic + unsystematic risk) w	whereas beta ( $\beta$ ) represents systematic risk.
То	tal risk of stock A is low	than stock B, hov	vever systematic risk (β) is fou	nd higher for stock A compared to stock B.
Τh	us, unsystematic risk of st	ock A is lower tha	n stock B, although stock A is	more risky than stock B because unsystematic

risk can be diversified away.

Security	Beta	Expected Return
Α	1.4	0.150
B	0.9	0.115

**Question 24: Suppose you observe the following situation:** 

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk free rate? Sol:

Correctly priced securities should have equal reward-risk ratio.

Reward-risk ratio of A = Reward-risk ratio of B

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#### **Financial Management Cost of Capital Problems and Solutions**

Question 1: The Dybvig Corporation's common stock has a beta of 1.15. If the risk free rate is 4.5% and the expected return on the market is 11%, what is Dybbig's cost of equity capital?

Cost of Equity =  $r_f + \beta(r_m - r_f) = 0.045 + 1.15(0.11 - 0.045) = 0.11975 = 11.975\%$ 

Question 2: The Devon Co. just issued a dividend of \$2.4 per share on its common stock. The company is expected to maintain a constant 5.5% growth rate in its dividends indefinitely. If the stock sells for \$52 a share, what is the company's cost of equity?

Cost of Equity = 
$$\frac{D_1}{p_0} + g = \frac{D_0(1+g)}{p_0} + g = \frac{\$2.4(1+0.055)}{\$52} + 0.055 = 0.10369 = 10.369\%$$

Question 3: Stock in Country Road Industries has a beta of 0.85. The market risk premium is 8%, and T-bills are currently yielding 5%. The company's most recent dividend was \$1.6 per share, and dividends are expected to grow at a 5% annual rate indefinitely. If the stock sells for \$37 per share, what is your best estimate of the company's cost of equity?

Cost of Equity(CAPM) = 
$$r_f + \beta(r_m - r_f) = 0.05 + 0.85 \times 0.085 = 0.118 = 11.8\%$$

Cost of Equity(DDM) = 
$$\frac{D_1}{p_0} + g = \frac{D_0(1+g)}{p_0} + g = \frac{\$1.00 \ne 0.06}{\$37} + 0.06 = 0.10583 = 10.583\%$$
  
Cost of Equity =  $\frac{0.118 + 0.10583}{2} = 0.11192 = 0.11192\%$ 

Question 4: Advance Inc., is trying to determine its cost of debt. The firm has a debt issue outstanding with 12 years to maturity that is quoted at 95% of face value. The issue makes semiannual payments and has a coupon rate of 8% annually. What is Advance's pretax cost of debt? If the tax rate is 35%, what is after tax cost of debt?

The pretax cost of debt is the YTM of the company's bond Bond price = 95% of Face Value \$1,000 = \$950This is a discount bond i.e. the bond is selling at lower price than its face value.

A bond is sold at discount when coupon rate is lower than YTM. As the bond is selling at discount, YTM must be higher than coupon rate of 8%.

NPV @ 
$$8.5\% = -\$950 + \sum_{t=1}^{24} 40 x (1 + 0.045)^{-t} + 1000 x (1 + 0.045)^{-24} = -\$22.47$$
  
NPV @  $8.5\% = -\$950 + \sum_{t=1}^{24} 40 x (1 + 0.0425)^{-t} + 1000 x (1 + 0.0425)^{-24} = \$12.84$ 

$$YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.085 + \frac{12.84}{12.84 - (-22.47)} x(0.09 - 0.085) = 0.0868 = 8.68\%$$

Pretax cost of debt = 8.68%; After tax cost of debt = 0.0868(1 - 0.35) = 0.05642 = 5.642%

Question 5: Shanken Corp. issued a 30 years, 7% semiannual bond 7 years ago. The bond currently sells for 108 percent of its face value. The company's tax rate is 35%.

#### a) What is the pretax cost of debt?

The pretax cost of debt is the YTM of the company's bond

Bond price = 108% of Face Value \$1,000 = \$1,080

This is a premium bond i.e. the bond is selling at higher price than its face value.

A bond is sold with premium when coupon rate is higher than YTM. As the bond is

selling at premium, YTM must be lower than coupon rate of 7%.

NPV @ 6.5% = 
$$-\$1,080 + \sum_{t=1}^{46} 35x(1+0.0325)^{-t} + 1000x(1+0.0325)^{-46} = -\$20.74$$
  
NPV @ 6% =  $-\$1,080 + \sum_{t=1}^{46} 35x(1+0.03)^{-t} + 1000x(1+0.03)^{-46} = \$43.87$ 

$$YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.06 + \frac{\$43.87}{\$43.87 - (-\$20.74)} x(0.06) = 0.0634 = 6.34$$

%

Pretax cost of debt = 6.34%;

c) Which is more relevant, the pretax or the after tax cost of debt? Why?

a) vonat is the after tax cost of debt?
After tax cost of debt = 0.0634(1-0.35) = 0.0412 = 4.12%
b) C After tax cost of debt is more relevant because that is the actual cost to the company.

d) Suppose the book value of the debt issue is \$60 million. In addition, the company has a second debt issue on the marker a zero coupon bond with seven years left to maturity; the book value of this issue is \$80 million and the bonds sell for 73% of par. What is the company's total book value of debt? The total market value? What is your best estimate of the after tax cost of debt now?

Book value of the 7% semiannual bond = 60 m illionMarket value of the 7% semian val bond =  $60million \times 108\% = 64.8million$ Book value of the zero coupon bond = \$80 millionMarket value of the zero coupon bond =  $\$80million \times 73\% = \$58.4million$ Total book value of debt = \$60 + \$80 = \$140millionTotal market value of debt = 64.8 + 58.4 = \$123.2millionThe pretax cost of zero coupon bond is the YTM of it Bond price = 73% of Face Value \$1,000 = \$730Interest earned = prt = 270Interest earned = prt = 270*i.e.* simple interest rate =  $\frac{270}{730x7} = 5.283\%$ NPV @ 5% =  $-\$730 + 1000x(1 + 0.05)^{-7} = -\$19.31$ NPV @ 4.5% =  $-\$730 + 1000x(1 + 0.045)^{-7} = \$4.828$  $YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.045 + \frac{\$4.828}{\$4.828 - (-\$19.31)} x(0.05 - 0.045) = 0.046 = 4.6\%$ 

Pretax cost of debt = 4.6%;
Weighted Avg. Pretax Cost of Debt =  $\frac{\$64.8 m illion}{\$123.2 m illion} x0.0634 + \frac{\$58.4 m illion}{\$123.2 m illion} x0.046 = 5.51\%$ After tax cost of debt of the company = 0.0551(1 - 0.35) = 0.0358 = 3.58%

Question 6: Fama's LIamas has a weighted average cost of capital of 9.8%. The company's cost of equity is 15%, and its cost of debt is 7.5%. The tax rate is 35%. What is Fama's debt-equity ratio?

WACC =  $0.098 = w_e r_e + w_d r_d (1 - T) = w_e x 0.15 + (1 - w_e) x 0.075 x (1 - 0.35)$ or,  $0.098 = 0.15 w_e + 0.04875 - 0.04875 w_e$ or,  $0.04925 = 0.10125 w_e$ or,  $w_e = \frac{0.04925}{0.10125} = 0.4864$  *i.e.* $w_d = 1 - 0.4864 = 0.5136 = 51.36\%$ Debt-Equity ratio =  $\frac{0.5136}{0.4864} = 1.055$ Question 7: Filer manufacturing has 7.5 million shares of compton stock outstanding. The

current share price is \$49, and the book value per share is \$4. Filer Manufacturing also has two bond issues outstanding. The first bond issue has a face value of \$60 million, pays 7% semi-annual coupon and sells for 93% of par. The second issue has a face value of \$50 million, pays 6.5% semiannual coupon, and sells for 96.5% of par. The first issue matures in 10 years, the second in 6 years.

m 10 years, the second in 6 years. a) What are Filer's capital structure weights on a book value basis? Book value of common stock = 7.5 x 4 million = \$30 million Book value of bond = \$60 + \$50 \$110 million Proportion of debt in capital structure,  $w_d = \frac{110}{110 + 30} = 0.7857 = 78.57\%$ Proportion of equity in capital structure,  $w_e = \frac{30}{110 + 30} = 0.2142 = 21.42\%$ b) What are Filer's capital structure weights on a market value basis? Market value of bond = \$60 x93\% + \$50 x96.5\% = \$104.05 million Market value of bond = \$60 x93\% + \$50 x96.5\% = \$104.05 million Proportion of debt in capital structure,  $w_d = \frac{104.05}{104.05 + 367.5} = 0.2206 = 22.06\%$ 

c) Which are more relevant, the book value or market value weights? Why? The market value weights are more relevant. The market value represents the true value of the firm and its capital structure.

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d) Suppose the company's stock has a beta of 1.2, the risk free rate is 5.2%, and the market risk premium is 7%. Assume that the overall cost of debt is the weighted average of the two outstanding debt issues. The tax rate is 35%. What is the company's WACC?

The pretax cost of a bond is the YTM of it

YTM of Bond 1:

Bond price = 93% of Face Value \$1,000 = \$930

This is a discount bond i.e. the bond is selling at lower price than its face value.

A bond is sold at discount when coupon rate is lower than YTM. As the bond is anoo.com selling at discount, YTM must be higher than coupon rate of 7%. 

NPV @ 8% = -\$930 + 
$$\sum_{t=1}^{20} 35x(1+0.04)^{-t} + 1000x(1+0.04)^{-t} = $2.04$$
  
NPV @ 8.5% = -\$930 +  $\sum_{t=1}^{20} 35x(1+0.0425)^{-t} + 1000x(1+0.0425)^{-20}$ 

$$YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.08 + \frac{\$2.04}{\$2.04 - (-\$29.7)} (0.085 - 0.08) = 0.0803 = 8.03\%$$

Pretax cost of debt1 = 8.03%

t = 1

YTM of Bond 2: Bond price = 96.5% of Face Value \$1,000 = \$965 This is a discount bond i.e. the bond is selling at lower price than its face value. A bond is sold at discount when coupon rate is lower than YTM. As the bond is selling at discount, YTM must be higher than coupon rate of 6.5%.

NPV @ 7% = -\$965 + 
$$\sum_{t=1}^{12} 32.5 x (t 0.035)^{-t} + 1000 x (1 + 0.035)^{-12} = $10.84$$

NPV @ 7.5% = 
$$-\$965 + 32.5 x(1 + 0.0375)^{-t} + 1000 x(1 + 0.0375)^{-12} = -\$12.61$$

$$YTM = L + \frac{NPV}{NPV_{H}}x(H - L) = 0.07 + \frac{\$10.84}{\$10.84 - (-\$12.61)}x(0.075 - 0.07) = 0.0723 = 7.23\%$$

Pretax cost or debt2 = 7.23%; W eighted Average Pretax Cost of Debt,  $r_d = \frac{55.8}{104.05} \times 0.0803 + \frac{48.25}{104.05} \times 0.0723 = 0.0766 = 7.66\%$ Geven of equity,  $r_e = 0.052 + 1.2(0.07) = 0.136 = 13.6\%$ W eighted Average Cost of Capital (WACC) =  $w_e r_e + w_d r_d (1 - T)$ 

 $= 0.7793 \times 0.136 + 0.2206 \times 0.0766(1 - 0.35) = 11.696\%$ 

Question 8: Kose Inc., has a target debt equity ratio of 0.65. Its WACC is 11.2%, and the tax rate is 35%.

a) If Kose's cost of equity is 15%, what is its pretax cost of debt?  

$$\frac{\text{Debt}}{\text{Equity}} = \frac{0.65}{1}; \quad \text{Proportion of debt, } w_{x} = \frac{0.65}{1+0.65} = \frac{13}{33}$$
Proportion of equity,  $w_{x} = \frac{20}{33}$   
WACC = 0.112 =  $w_{x}r_{x} + w_{x}r_{x}(1-T) = \frac{20}{33}x0.15 + \frac{13}{33}xr_{x}x(1-0.35)$   
or, 0.112 =  $\frac{3+8.45r_{x}}{33}$   
or, 3+8.45 $r_{x}$  = 3.696  
or,  $r_{x} = 0.0823 = 8.23\%$   
Pretax cost of debt = 8.23%  
Pretax cost of debt = 8.23%  
Proportion of equity,  $w_{x} = \frac{20}{33}$   
WACC = 0.112 =  $w_{x}r_{x} + w_{x}r_{x}(1-T) = (\frac{0.65}{1+0.65}, \frac{13}{0.33})^{1/1}$   
Proportion of equity,  $w_{x} = \frac{20}{33}$   
WACC = 0.112 =  $w_{x}r_{x} + w_{x}r_{x}(1-T) = (\frac{20}{33}xr_{x} + \frac{13}{33}x0.064)$   
or,  $0.112 = \frac{20r_{x} + 0.832}{33}$   
proportion of equity,  $w_{x} = \frac{20}{33}$   
WACC = 0.112 =  $w_{x}r_{x} + w_{x}r_{x}(1-T) = (\frac{20}{3}xr_{x} + \frac{13}{33}x0.064)$   
or,  $c_{x} = 0.1432 = 14.32$   
Cost of equity = 14.22\%  
Question 9: Give, the following information for Huntington Power Co, find the WACC.  
Assume the corbinary's tax rate is 35%, market risk premium 7% and risk free rate 6%.  
Debt  $5.000$  8% semiannual coupon bonds of \$1,000 par value, 20 years maturity  
remaining, selling for 103% of par  
Common stock = 160,000 x57 = 59,120,000  
Market value of common stock = 160,000 x57 = 59,120,000  
Proportion of equity in capital structure,  $w_{x} = \frac{59,120,000}{55,150,000} = 0.36089 = 36.089\%$ 

The pretax cost of a bond is the YTM of it

Bond price = 103% of Face Value \$1,000 = \$1,030

This is a premium bond i.e. the bond is selling at higher price than its face value.

A bond is sold at premium when coupon rate is higher than YTM. As the bond is

selling at premium, YTM must be lower than coupon rate of 8%.

40

NPV @ 7.5% = -\$1,030 + 
$$\sum_{t=1}^{+0}$$
 40x(1 + 0.0375)^{-t} + 1000x(1 + 0.0375)⁻⁴⁰ = \$21.37

NPV @ 8% = -\$1,030 + 1000 = -\$30

$$YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.075 + \frac{\$21.37}{\$21.37 - (-\$30)} x(0.08 - 0.075) = 0.07708 = 7.708\%$$

Cost of equity,  $r_e = 0.06 + 1.1(0.07) - 0.000$ W eighted Average Cost of Capital (WACC) =  $w_e r_e + w_d r_d (1 - T)$ 

$$= 0.6391x0.137 + 0.36089x0.07708(1 - 0.35) = 10.5639$$

Question 10: Titan Mining Corporation has 8.5 million starts of common stock outstanding and 200,000 7.5% semiannual bonds outstanding with par value of \$1,000 each. The common stock currently sells for \$34 per share and has a beta of 1.2 and the bonds have 15 years to maturity and sell for 93% of par. The market risk premium is 7%, T-bills are yielding 5%, and company's tax rate \$35%.

a) What is the firm's market value capital structure? Market value of common stock = 8.5 million 34 = \$289 millionMarket value of bond = 200,000 x \$ 1,000 3% = \$ 186 millionProportion of debt in capital structure,  $w_d = \frac{\$186 m illion}{\$289 m illion + \$186 m illion} = 0.3915 = 39.15\%$ Proportion of equity in capital structure,  $w_e = \frac{\$289 m illion}{\$289 m illion + \$186 m illion} = 0.6084 = 60.84\%$ 

b) If Titan Mining is evaluating a new investment project that has the same risk as the firm's typical project, what rate should the firm use to discount the project's cash flow

The pretencost of a bond is the YTM of it Bonoprice = 93% of Face Value \$1,000 = \$930

This is a discount bond i.e. the bond is selling at lower price than its face value.

A bond is sold at discount when coupon rate is lower than YTM. As the bond is selling at discount, YTM must be higher than coupon rate of 7.5%.

NPV @ 7.5% = -\$930 + \$1000 = \$70

NPV @ 8.5% = -\$930 + 
$$\sum_{t=1}^{30}$$
 37.5x(1 + 0.0425)^{-t} + 1000x(1 + 0.0425)⁻³⁰ = -\$13.89

$$YTM = L + \frac{NPV_L}{NPV_L - NPV_H} x(H - L) = 0.075 + \frac{\$70}{\$70 - (-\$13.89)} x(0.085 - 0.075) = 0.08334 = 8.334\%$$

Cost of equity,  $r_{e} = 0.05 + 1.2(0.07) = 0.134 = 13.4\%$ 

Weighted Average Cost of Capital (WACC) =  $w_{e}r_{e} + w_{d}r_{d}(1-T)$ 

 $= 0.6084 \times 0.134 + 0.3915 \times 0.08334(1 - 0.35) = 10.273\%$ 

Question 11: Suppose your company needs \$20 million to build a new assembly line. Your target debt-equity ratio is 0.75. The floatation cost for issuing new equity is 8% but the floatation cost for debt is only 5%. Your boss has decided to fund the project by borrowing money because the floatation costs are lower and the needed funds are relatively small.

a) What do you think about the rationale behind borrowing the entire amount? If the firm finance the project using all debt, it will have to issue more equity in future to achieve the target capital structure and vice-versa. Therefore, it doesn't matter whether the firm finance the project using all debt or all equity, we have to calculate weighted average floatation control find out the amount to be raised to acquire the required amount for the project.

b) What is your company's weighted average float ation cost, assuming all equity is raised externally?

$$\frac{\text{Debt}}{\text{Equity}} = \frac{0.75}{1}; \text{ Proportion of debt, } w_d = \frac{0.75}{1000} = \frac{10000}{1000}$$

Proportion of equity,  $w_e = \frac{4}{7}$ Weighted Average Floatation Cost =  $w_e f_e (1 - k) + w_d f_d$ 

$$= \frac{4}{7} x 0.08 x (1-0) + \frac{3}{7} x 0.05$$
 [k = % of finance from internal source]

- = 0.06714 = 6.714%
  c) What is the true cost of building the new assembly line after taking floatation costs into account?

Amount to be aised = 
$$\frac{\$20 \text{ million}}{1 - 0.06714} = \$21.44 \text{ million}$$

Question 2: Southern Alliance Company needs to raise \$45 million to start a new project and will raise the money by selling new bonds. The company will generate no internal equity for the foreseeable future. The company has a target capital structure of 65% common stock, 5% preferred stock, and 30% debt. Floatation costs for issuing common stock are 9%, for new preferred stock 6%, and for new debt 3%. What is the true initial cost figure Southern should use when evaluating its project?

Proportion of debt,  $w_d = 0.3$ 

Proportion of common equity,  $w_e = 0.65$ 

Proportion of preferred equity,  $w_p = 0.05$ 

Weighted Average Floatation Cost =  $w_e f_e (1 - k) + w_d f_d + w_p f_p$ 

 $= 0.65 x 0.09 x (1 - 0) + 0.3 x 0.03 + 0.05 x 0.06; \qquad [k = \% \text{ of finance from internal source}]$ 

$$= 0.0705 = 7.05\%$$

A mount to be raised = 
$$\frac{\$45 \text{ million}}{1 - 0.0705} = \$48.41 \text{ million}$$

Question 13: Och, Inc. is considering a project that will result in initial after tax cash savings of \$3.5 million at the end of the first year, and these savings will grow at a rate of 5% per year indefinitely. The firm has a target debt-equity ratio of 0.65, a cost of equity of 15%, and an after tax cost of debt of 5.5%. The cost saving proposal is somewhat riskier than the usual projects the firm undertakes; management uses the subjective approach and applies an adjustment factor of +2% to the cost of capital for such risky projects. Under what circumstances should Och take on the projects.

Och should only take on the project if it's initial investment is \$42.385 million or less. Question 14: Goodbye Inc. recently issued new securities to finance a new TV show. The project cost \$15 million, and the company paid \$850,000 in floatation costs. In addition, the equity issued had a floatation cost of 7% of the amount raised, whereas the debt issued had a floatation cost of 3% of the amount raised. If Goodbye issued new securities in the same proportion as its target capital structure, what is the company's target debtequity ratio? Floatation cost =  $\frac{\$850,000}{\$15 m \, illion + \$850,000} = 0.0536 = 5.36\%$ 

Weighted Average Floatation Cost =  $w_e f_e (1 - k) + w_d f_d = 0.0536$ 

or, wx0.07x(1-0) + (1-w)x0.03 = 0.0536; [k = % of finance from internal source] or, 0.04w = 0.0536 - 0.03  $or, w_e = 0.59069 = 59.069\%$ Debt-Equity ratio =  $\frac{1-0.59069}{0.59069} = 0.6929$ 

Question 15: Photochronograph Corporation (PC), manufactures time series photographic equipment. It is currently at its target debt-equity ratio of 0.7. It's considering building a new \$45 million manufacturing facility. This new plant is expected to generate after tax cash flows of \$6.2 million a year in perpetuity. The company raises all equity from outside financing. There are three financing options: 1) A new issue of common stock: The floatation costs of the new common stock would

- A new issue of common stock: The floatation costs of the new common stock would be 8% of the amount raised. The required return on the company's new equity is 14%.
- A new issue of 20 year bonds: The floatation costs of the new bonds would be 4% of the proceeds. If the company issues these new bonds at an annual coupon rate of 8%, they will sell at par.
- 3) Increased use of accounts payable financing. Because this financing is part of the company's ongoing daily business, it has no floatation costs, and the company assigns it a cost that is the same as the overall firm WACC. Management has a target ratio of accounts payable to tong term debt of 0.20 (assume there is no difference between the pretax and after tax accounts payable cost.)

What is the NPV of the new plant? Assume that PC has a 35% tax rate.

$$\frac{\text{Debt}}{\text{Equity}} = \frac{0.7}{1}; \text{ Proportion of debt, } w_d = \frac{0.7}{1+0.7} = \frac{7}{17}$$
Proportion of equity,  $w_{p} = \frac{0.2}{17}$ 
Accounts Payable  $\frac{0.2}{17}$ 
Accounts Payable  $\frac{0.2}{17}$ 
Proportion of equity,  $w_{p} = \frac{0.2}{1}$ 
Proportion of accounts payable in capital structure,  $w_{ap} = \frac{0.2}{1.2} x \frac{7}{17} = \frac{7}{102}$ 
Proportion of bond (long term debt) in capital structure,  $w_d = \frac{1}{1.2} x \frac{7}{17} = \frac{35}{102}$ 
Cost of Equity,  $r_e = 14\% = 0.14$ 
Cost of bond,  $r_d = \text{YTM}$  of the bond = 8%
Cost of accounts payable,  $r_{ap} = \text{WACC}$ 

W eighted Average Cost of Capital (WACC),  $r_{WACC} = w_e r_e + w_d r_d (1 - T) + w_{ap} r_{ap}$ 

$$= \frac{10}{17} x 0.14 + \frac{35}{102} x 0.08 x (1 - 0.35) + \frac{7}{102} x r_{wACC}$$
  
or,  $r_{wACC} (1 - \frac{7}{102}) = \frac{10}{17} x 0.14 + \frac{35}{102} x 0.08 x (1 - 0.35)$   
or,  $r_{wACC} = 0.10758 = 10.758\%$   
Weighted Average Floatation Cost =  $w_e f_e (1 - k) + w_d f_d + w_{ap} f_{ap}$   

$$= \frac{10}{17} x 0.08 + \frac{35}{102} x 0.04 + \frac{7}{102} x 0 = 0.06078 = 6.078\%;$$
  
Net Present Value(NPV) of Project:  
NPV =  $-\frac{\$45 m illion}{(1 - 0.06078)} + \frac{\$6.2 m illion}{0.10758}$   
= -\\$47.912 m illion + \\$57.631 m illion = \\$9.719 m illion

Question 16: Trower Corp. has a debt equity ratio of 1.2. The company is considering a new plant that will cost \$145 million to build. When the company issues new equity, it incurs a floatation cost of 8%. The floatation cost on new debt is 3.5%.

a) What is the initial cost of the plant if the company raises all equity externally?

$$\frac{\text{D ebt}}{\text{Equity}} = \frac{1.2}{1}; \text{ Proportion of debt, } w_d = \frac{1.2}{1.2 + 0.111}$$
Proportion of equity,  $w_e = \frac{5}{11}$ 
W eighted Average Floatation Cost,  $w_e f_e(1-k) + w_d f_d$ 

$$= \frac{5}{11} x 0.08(1-0) + \frac{6}{11} x 0.035; \text{ [k= \% of finance from internal source]}$$

$$= 0.05545 = 5.545\%$$
Amount to be raised  $\frac{5145 \text{ million}}{1-0.05545} = \$153.513 \text{ million}$ 
b) What if it optically uses 60% retained earnings?
Weighted Average Floatation Cost =  $w_e f_e(1-k) + w_d f_d$ 

$$= \frac{5}{11} x 0.00(1-0.6) + \frac{6}{11} x 0.035; \text{ [k= \% of finance from internal source]}$$

$$= 0.363 = 3.363\%$$
Amount to be raised =  $\frac{\$145 \text{ million}}{1-0.03363} = \$150.047 \text{ million}$ 
c) What if all equity investments are financed through retained earnings?

$$= \frac{5}{11} x 0.08(1-1) + \frac{6}{11} x 0.035; \qquad [k = \% \text{ of finance from internal source}]$$
$$= 0.01909 = 1.909\%$$

A mount to be raised =  $\frac{\$145 \, m \, illion}{1 - 0.01909} = \$147.822 \, m \, illion$ 

Question 17: Suppose a firm has both a current and a target debt-equity ratio of 0.6, a cost of debt of 5.15%, and a cost of equity of 10%. The corporate tax rate is 34%. The firm is considering taking on a warehouse renovation costing \$60 million that is expected to yield cost savings of \$12 million a year for six years. Should the firm take on the warehouse renovation?

Sol:

$$\frac{\text{Debt}}{\text{Equity}} = \frac{0.6}{1}; \text{ Proportion of debt, } w_d = \frac{0.6}{1+0.6} = 0.375$$

Proportion of equity,  $w_e = \frac{1}{1+0.6} = 0.625$ 

Weighted Average Cost of Capital (WACC) =  $w_e r_e + w_d r_d (1 - T)$ 

= 0.625 x 0.1 + 0.375 x 0.0515 x (1 - 0.34) = 0.075246 = 7.5246%

Net Present Value(NPV) of Warehouse Renovation Project:

Proportion of equity, 
$$w_e = \frac{1}{1+0.6} = 0.625$$
  
W eighted Average Cost of Capital (W ACC) =  $w_e r_e + w_d r_d (1-T)$   
= 0.625x0.1 + 0.375x0.0515x(1 - 0.34) = 0.075246 = 7.5246%  
Net Present Value(N PV) of W arehouse Renovation Project:  
N PV =  $-60 + \frac{12}{0.075246} - \frac{12}{0.075246} (1+0.075246)^{-6} = -\$3.716$  million

NPV is negative which indicates that renovation project fails

generate enough cash to fulfill required cost of capital. Hence, reject the project.

Question 18: Consider a firm whose debt has a market salve of \$40 million and whose stock has a market value of \$60 million (3million outstanding shares of stock, each selling for \$20 per share). The firm pays a 5% rate of interest on its new debt and has a beta of 1.41. The corporate tax rate is 34%. The risk premium on the market is 9.5% and that the current Treasury bill rate is 1%. What cost of capital the firm should use to discount its future cash flows?

Proportion of debt, 
$$w_d = 40\% = 0.4$$
; Proportion of equity,  $w_e = 60\%$   
Based on CAPM, Cost of equity,  $r_f + \beta(r_m - r_f) = 0.01 + 1.41x0.095 = 0.14395 = 14.395\%$   
Weighted Average Cost of Copital (WACC) =  $w_e r_e + w_d r_d (1 - T)$   
= 0.6x0.14395 + 0.4x0.05x(-0.34) = 0.09957 = 10%

The firm should use covor capital of 10% to discount its future cash flows.

Question 19: The Weinstein Corporation has a target capital structure of 80% equity and 20% debt. The floatation costs for equity issues are 20% of the amount raised; the floatation costs for debt issues are 6%. If Weinstein needs \$65 million for a new manufacturing facility, how much it will have to raise?

W eighter Average Floatation Cost = 
$$w_e f_e (1 - k) + w_d f_d$$
  
= 0.8 0.2 x(1 - 0) + 0.2 x0.06; [k = % of finance from internal source]  
= 172 = 17.2%

Amount to be raised =  $\frac{\$65m}{1-0.172}$  = \$78.502 m illion

Question 20: Tripleday Printing Company is currently at its target debt-equity ratio of 100%. It is considering building a new \$500,000 printing plant in Kansas. This new plant is expected to generate after tax cash flows of \$73,150 per year forever. The tax rate is 34%. To finance the project, the firm is considering all debt or all equity option. The issuance costs of the new common stock and new debt would be about 10% and 2% respectively. The required return on the company's new equity is 20% and cost of new debt is 10%. Should the firm accept the project?

 $\frac{\text{Debt}}{\text{Equity}} = \frac{1}{1}; \text{ Proportion of debt, } w_d = \frac{1}{1+1} = 0.5$ Proportion of equity,  $w_{a} = 0.5$ W eighted Average Cost of Capital (WACC) =  $w_e r_e + w_d r_d (1 - T)$ = 0.5 x 0.2 + 0.5 x 0.1 x (1 - 0.34) = 0.133 = 13.3%If the firm finance the project using all debt, it will have to issue more equity in future to achieve the target capital structure and vice-versa. Therefore, it doesn't matter whether the firm finance the project using all debt or all equity, we have to calculate weighted average floatation cost to find out the amount to be raised to acquire the required amount for the project. Weighted Average Floatation Cost =  $w_e f_e (1 - k) + w_d f_d$ = 0.5 x 0.1 x (1 - 0) + 0.5 x 0.02; [k= % of finance from internal source] = 0.06 = 6%Amount to be raised =  $\frac{$500,000}{1 - 0.06}$  = \$531,914.89Net Present Value(NPV) of Printing Plant Project: NPV =  $-$531,914.89 + \frac{$73,150}{0.133}$  = \$18,085.11NPV is positive which indicates that printing plant project will be able to generate enough cash to fulfill required cost of capital. Hence, accent the project .p quired Mohammad Kammul Arctim generate enough cash to fulfill required est of capital. Hence, accept the project.

# **Financial Management Additional Problems**

## **Risk and Return**

1. Determine geometric return from the following numbers:

Year	return
2004	16.4%
2005	27.45
2006	33.22
2007	-2.07
2008	21.22
2009	42.09
2010	55.34
2011	- 7.77
2012	-10.25
2013	11.88

2. Using the returns in preceding problem, determine the average return and standard deviation of returns.
3. Using the following information about possible returns on an invert determine the expected return and standard deviation of <u>Scenario Keturn Probability</u> (1990)
3. Using the following information about possible returns on an invert determine the expected return and standard deviation of <u>Scenario Scenario 30</u>
4. Deviation of the probability 22 15 -8 Pessimistic .15

4. Identify which project is preferred, given the following information about the four pairs of projects, A and B, C and C, and F, and G and H.

Project	Expected Return	Standard Deviation
А	22%	32%
В	28%	32%
c d'	25%	28%
Dattle	25%	25%
Nell'	12%	0%
AO'F	18%	16%
G	35%	44%
Н	28%	35%

5. In addition to the values provided in the previous problem, the following information are provided:

> Correlation between returns of A and B = +.5Correlation between returns of C and D = +.1Correlation between returns of E and F = 0Correlation between returns of G and H = -.5

(a) If you form a portfolio of two assets as paired above with 60% of assets invested in the first asset, what are the standard deviations?

(b) Between the pair AB and GH, which pair provides a better combination?

- 6. Explain what is systematic risk and what is diversifiable risk.
- 7. Explain how bond prices react to a change in interest rate.
- 8. Explain how bond prices change with time everything else remaining the same.
- 9. The following information is given for stock M:

Scenario	Return	Probability	
Recession	- 6 %	.25	
Normal	28 %	.50	C'
Expansion	54 %	.25	°0.

M has a beta of .85. Market return is 26 percent and risk free rate is 11 percent. (a) What is the expected return of M? (b) What is the required return of M? (c) M overpriced or underpriced?

- 10. What is the yield to maturity for a bond that has 5 years left to maturity, has 12.5 percent coupon and a face value of Taka 1,000, and market price is Taka 933.74?
- 11. What is the expected capital gains if the investor decides to hold the bond for another year? (assume everything else remains unchanged)
- 12. Hawthorn Corporation is growing rapidly and so far, has not paid any dividend. It will pay its first dividend next year and the expected dividend is Taka 16 per share. Dividend will grow at 35%, 30%, 18% in the subsequent years, and then is expected to settle at a growth rate of 12 percent.
  - a. If Hawthorn shareholders require a return of 25 percent, what would be equilibrium price of Hawthorn?
  - b. If a shareholder holds his shares for one more year, what will be expected price of the share?
  - c. What is the current yield on the shares now?
- 13. Indicate true or false
  - a) Everything else remaining the same, a discount bond should increase in value as maturity approaches.
  - b) Everything else remaining the same, a higher growth rate will result in a higher value of stock.
  - A zero coupon bond will never sell at a premium.

4. Which of the following statements is **correct**?

- a. Slope of the CML is beta.
- b. In a fully diversified portfolio, systematic risk is completely diversified away.
- c. In forming a two asset portfolio, negative correlation between the two returns is less desirable than no correlation between the two returns.
- d. A steep characteristic line is indicative of high systematic risk (high beta) asset.

15. You are going to make your first stock investment and you are considering the information provided for stock X and stock Y.

	Stock X	Stock Y
Expected Return	22.5%	22.5%
Standard deviation of returns	38%	44%
Beta	1.8	1.25

- (a) As your first investment, which stock should you choose and why?
- (b) If a well diversified investor wishes to add one of these two stocks to his portfolio, which one he should choose and why?
- 16. Determine the fair value of a stock when the following information is known. Expected dividend: Cash 13.50. The required return is 26 percent and growth rate in dividend is 18 percent.
- 17. Use the following information to answer the following 3 questions
  - a) A bond is selling at Taka 980. It pays a coupon of 13 percent and has a face value of Taka 1000. It will mature in 8 years. What is the current yield on the bond?
  - b) If the market yield declines to 10 percent, everything else remaining the same, at what price should the bond sell?
  - c) What is the percent change in the bond's price?
- 18. If the risk-free rate is 9 percent and market risk premium is 12 percent, what is the required return on a stock that has a beta of 1.25?
- 19. The following is the characteristic line of Stock Max.

 $K_{xi} = 1.3 + 1.2 k_{mi}$ 

Market return is 28 percent. Kisk free rate is 11 percent. What is the required return of Max?

- 20. Previous problem continued. What is the required return on a stock that has a beta of 1?
- 21. Crossings Corporation issued 10 year bonds 2 years ago. The bond carries an annual coupon of 03 percent paid annually, has a face value of Taka 1,000, and is currently selling for Taka 932.

(a) What is the yield to maturity for the bond?

- What is the expected price of the bond a year later assuming everything else remains the same?
- (c) What is the expected capital gains and current yield for an investor who owns the bond?
- 22. Sovarn Corporation is growing rapidly and, so far, has not paid any dividend. It will not pay any dividend next year but is expected to pay a dividend at the end of 2nd year of an amount of Taka 10.00. Dividend will grow at 25%, 20%, 17% in the subsequent years, and then is expected to settle at a growth rate of 12 percent.
  - a. If Sovarn's shareholders require a return of 24 percent, what would be equilibrium price of Sovarn?

- b. If a shareholder holds his share for one more year, what will be expected price of the share one year later?
- c. What is the current dividend yield on the shares now?
- 23. The expected return on stock X is 28 percent and expected return on stock Y is 32 percent. The standard deviation of X is 36 percent and standard deviation of Y is 40 percent. The correlation between the returns of X and Y is +.60. You invested a total of Taka 1,00,000, 40 percent of the money in stock X.
  - (a) What is the standard deviation of your portfolio?
  - (b) What is the expected return on the portfolio?
- 24. Indicate true or false
- a) A change in interest rate should have a greater affect on the value of two year compared with a value of a 20 year bond.
- b) A required condition for the Gordon Growth Model is that the discount site, k, must be equal to or greater than the growth rate, g.

- a) Slope of the characteristic line of stock X is the beta of stock X.
  b) In a fully diversified portfolio. unsystematic b) In a fully diversified portfolio, unsystematic risk is completely diversified away.
- c) In forming a two asset portfolio, negative correlation between the two returns is less desirable than no correlation between the two returns
- d) A steep slope of the CML (compared with a relatively flat) is indicative of market demanding higher compensation for higher systematic risk.
- 26. Primavara Corporation is expected to announce a dividend of Taka 35 per share reflecting a new growth rate of 15%, Kinvestors require a return of 32 percent, what is the fair value of the stock?
- 27. If the risk-free rate is 11 percent and market risk premium is 15 percent, what is the required return on a stock the has a beta of 1.20?
- 28. The following is the characteristic line of Stock Max.

$$K_{xi} = 1.3 + 1.85 K_{mi}$$

Market return is b percent. Risk free rate is 11 percent.

- (a) What is the required return of Max?
- (b) What is the required return on a stock that has a beta of 1?
- 29. The ratest dividend paid by Somtax Corporation was Taka 18. The Return on Equity **CROE**) is typically 26 percent. Somtax has a history of paying out 60 percent of its
  - profits. Beta of Somtax is 1.25. Current Risk free rate is 10 percent. Market Risk premium is 16 percent.
    - (a) What is the fair price of Somtax?
    - (b) What is likely to happen if the market price of the stock increases to Taka 125?

(c) Ignore your answer to (b) above. What is likely to happen if inflation expectation increases by 2 percent points? What will be the new price?

30. Hechinger Corporation has a target capital structure of 40 percent debt and 60 percent equity. The firm pays out 40 percent of its earnings in dividends. The current EPS of the firm is Taka 80 per share. Current share price in the market is Taka 200 and the

Mohammad Kamrul Arefin, MSc. in Quantitative Finance, University of Glasgow Page | 120 growth rate is 14 percent. The YTM on its bonds is 16 percent and its marginal tax rate is 40 percent. If new equity is issued, flotation cost will be 15 percent.

- a) Determine the cost of equity, k_e, assuming no new shares are issued.
- b) Determine the cost of equity,  $k_s$ , assuming new shares are issued.
- c) What is the WACC/MCC assuming new shares are issued.
- 31. Deaucent Corporation has a target capital structure of 35 percent debt and 65 percent equity. It is expected to pay a dividend of Taka 30 per share. Current share price in the market is Taka 225 and the growth rate is 15 percent. The YTM on its bonds is 16 ahoo.com percent and its marginal tax rate is 37 percent. If new equity is issued, flotation cost will be 20 percent.
  - a) Determine the cost of equity, k_e, assuming no new shares are issued.
  - b) Determine the cost of equity,  $k_s$ , assuming new shares are issued.
  - c) What is the WACC/MCC assuming new shares are issued.
- 32. Terra X Corporation has a target capital structure of 30 debt and 70 equity. Retained earnings for the year is not enough to finance the new projects and the company is planning to issue new shares to raise new equity capital. The sompany is expected to grow at 10 percent per year. It's last dividend was Taka Loper share. Currently shares are selling at Taka 112 per share. If new shares are issued, the shares will net Taka 102 per share.
  - (a) What is the cost of retained earnings?
  - (b) What is the cost of new equity?
  - (c) The yield to maturity of the company bonds is 15%. Tax rate is 40%. Assuming new shares are issued, what is the marginal cost of capital of the company?
- 33. Trifura Corporation has a target capital structure of 30 debt and 70 equity. The beta of the company is 1.4. Current fisk-free rate is 10 percent. Average return on market index is 28 percent.
  - (a) What is the cost orretained earnings?
  - (b) The yield to maturity of the company's bonds is 15%. Tax rate is 40%. Determine the weighted average cost of capital of the company.
- 34. The following are the cash flows for two mutually exclusive projects under consideration. The projects are considered average risk projects. The company's WACC is 16 percent.

x one	Year 0	Year 1	Year 2	Year 3	Year 4
Cash flows of A	(12,000)	6,000	5,000	4,000	3,000
Cash flows of B	(12,000)	3,000	6,000	6,000	4,000

- a. Determine the IRR values for the two projects.
- b. Determine the cross-over point.
- c. Draw a figure of NPV profiles to prove that project B creates more value. (Figures don't have to be exact, use discount rates of 10%, 15%, 20%, and 25%)

- 35. Project X and Y are mutually exclusive. Project X has an IRR of 32.5 percent and Project Y has an IRR of 27.6 percent. Both projects happen to have identical NPV at 17.4 percent. If the firms WACC is 20 percent, and both projects belong in the average risk class, which project should be selected? Draw a figure to support your answer.
- 36. What are the reasons for computing a modified IRR?
- 37. (a) Using a reinvestment rate of 18 percent and a cost of capital of 15%, determine the modified IRR with the following 8 year cash flow stream.



40.000

-100.00030.000 -30.000

(b) Should you accept or reject the project?

- 38. Indicate True or False
- a) Floatation cost increases the marginal cost of capital.
- b) Modified IRR is based on the assumption that project flows are reinvested at the IRR rate.
- c) Smaller projects have a tendency of producing higher R values.
- d) The weighted average cost of capital (WACC) is senerally based on the assumption of an optimal capital structure and that every Takaraised as long term capital is done maintaining the long term optimal capital conducture mix.
- 39. Project X and Y are mutually exclusive and have equal risk. Project X has an IRR of 24.5 percent and Project Y has an IRR of 28.6 percent. Both projects happen to have a net present value of Taka 3,2000 at 16.4 percent. If the firms WACC is 20 percent, and both projects belong in the same risk class, which project should be selected? Your answer must be based on right reason. Draw a figure to show your justification.
- 40. Determine the IRR of project that has an initial outlay of Taka 40,00,000, generates no cash flow for the first 5 years, and then generates a cash flow of Taka 80,00,000 at the end of the  $16^{10}$  year and Taka 75,00,000 at the end of  $7^{th}$  year.
- a reinvestment rate of 20 percent, determine the modified IRR with the 41. (a) Using following cash flow stream.

N	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
L'	-40,000	30,000	-15,000	40,000	25,000	25,000

- (b) State the decision rule using a discount rate of 20 percent.
- (c) State the decision.

42. The following are the cash flows for two mutually exclusive projects under consideration. The projects are considered average risk projects. The company's WACC is 14 percent.

	Year 0	Year 1	Year 2	Year 3	Year 4
Cash flows of A	(12,000)	6,000	5,000	4,000	3,000
Cash flows of B	(12,000)	3,000	6,000	6,000	4,000

(a) Determine the IRR values for the two projects.

(b) Determine the cross-over point.

(a) Determine the IRR values for the two projects. (b) Determine the cross-over point. Draw a figure of NPV profiles to prove that project B creates more value. (Figures down have to be exact) COMMON CONTRACTION OF CONTRACT OF CONT

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### Present Value and Future Value Tables

Table A-1 Future Value Interest Factors for One Dollar Compounded at k Percent for n Periods:  $FVIF_{k,n} = (1 + k)^n$ 

	404	<b>6</b> 0/		407	50/		-	<b>6</b> 0/		100/		100/	100/		4 504	100/	000/	a 101	050/	
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.2000	1.2400	1.2500	1.3000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544	1.2769	1.2996	1.3225	1.3456	1.4400	1.5376	1.5625	1.6900
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049	1.4429	1.4815	1.5209	1.5609	1.7280	1.9066	1.9531	2.1970
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735	1.6305	1.6890	1.7490	1.8106	2.0736	2.3642	2.4414	2.8561
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623	1.8424	1.9254	2.0114	2.1003	2.4883	2.9316	3.0518	3.7129
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738	2.0820	2.1950	2.3131	2.4364	2.9860	3.6352	3.8147	4.8268
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107	2.3526	2.5023	2.6600	2.8262	3.5832	4.5077	4.7684	6.2749
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760	2.6584	2.8526	3.0590	3.2784	4.2998	5.5895	5.9605	8.1573
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731	3.0040	3.2519	3.5179	3.8030	5.1598	6.9310	7.4506	10.604
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058	3.3946	3.7072	4.0456	4.4114	6.1917	8.5944	9.3132	13.786
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.1518	3.4785	3.8359	4.2262	4.6524	5.1173	7.4301	10.657	11.642	17.922
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.4985	3.8960	4.3345	4.8179	5.3503	5.9360	8.9161	13.215	14.552	23.298
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	3.8833	4.3635	4.8980	5.4924	6.1528	6.8858	10.699	16.386	18.190	30.288
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.3104	4.8871	5.5348	6.2613	7.0757	7.9875	12.839	20.319	22.737	39.374
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736	6.2543	7.1379	8.1371	9.2655	15.407	25.196	28.422	51.186
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	5.3109	6.1304	7.0673	8.1372	9.3576	10.748	18.488	31.243	35.527	66.542
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	5.8951	6.8660	7.9861	9.2765	10.761	12.468	22.186	38.741	44.409	86.504
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	6.5436	7.6900	9.0243	10.575	12.375	14.463	26.623	48.039	55.511	112.455
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	7.2633	8.6128	10.197	12.056	14.232	16.777	31.948	59.568	69.389	146.192
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463	11.523	13.743	16.367	19.461	38.338	73.864	86.736	190.050
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	8.9492	10.804	13.021	15.668	18.822	22.574	46.005	91.592	108.420	247.065
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	9.9336	12.100	14.714	17.861	21.645	26.186	55.206	113.574	135.525	321.184
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	11.026	13.552	16.627	20.362	24.891	30.376	66.247	140.831	169.407	417.539
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	12.239	15.179	18.788	23.212	28.625	35.236	79.497	174.631	211.758	542.801
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.835	13.585	17.000	21.231	26.462	32.919	40.874	95.396	216.542	264.698	705.641
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.063	13.268	17.449	22.892	29.960	39.116	50.950	66.212	85.850	237.376	634.820	807.794	*
35	1.4166	1.9999	2.8139	3.9461	5.5160	7.6861	10.677	14.785	20.414	28.102	38.575	52.800	72.069	98.100	133.176	180.314	590.668	*	*	*
36	1.4308	2.0399	2.8983	4.1039	5.7918	8.1473	11.424	15.968	22.251	30.913	42.818	59.136	81.437	111.834	153.152	209.164	708.802	*	*	*
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.286	14.974	21.725	31.409	45.259	65.001	93.051	132.782	188.884	267.864	378.721	*	*	*	*
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.902	74.358	117.391	184.565	289.002	450.736	700.233	*	*	*	*	*	*
									•											

Table A-2 Future Value Interest Factors for a One-Dollar Annuity Compouned at k Percent for n Periods:  $FVIFA_{k,n} = [(1 + k)^n - 1]/k$ 

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	1.0000	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.2000	1.2400	1.2500	1.3000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.1300	2.1400	2.1500	2.1600	2.2000	2.2400	2.2500	2.3000
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.4069	3.4396	3.4725	3.5056	3.6400	3.7776	3.8125	3.9900
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	4.8498	4.9211	4.9934	5.0665	5.3680	5.6842	5.7656	6.1870
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	6.4803	6.6101	6.7424	6.8771	7.4416	8.0484	8.2070	9.0431
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	8.3227	8.5355	8.7537	8.9775	9.9299	10.980	11.259	12.756
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.089	10.405	10.730	11.067	11.414	12.916	14.615	15.073	17.583
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.260	10.637	11.028	11.436	11.859	12.300	12.757	13.233	13.727	14.240	16.499	19.123	19.842	23.858
9	9.3685	9.7546	10.159	10.583	11.027	11.491	11.978	12.488	13.021	13.579	14.164	14.776	15.416	16.085	16.786	17.519	20.799	24.712	25.802	32.015
10	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487	15.193	15.937	16.722	17.549	18.420	19.337	20.304	21.321	25.959	31.643	33.253	42.619
11	11.567	12.169	12.808	13.486	14.207	14.972	15.784	16.645	17.560	18.531	19.561	20.655	21.814	23.045	24.349	25.733	32.150	40.238	42.566	56.405
12	12.683	13.412	14.192	15.026	15.917	16.870	17.888	18.977	20.141	21.384	22.713	24.133	25.650	27.271	29.002	30.850	39.581	50.895	54.208	74.327
13	13.809	14.680	15.618	16.627	17.713	18.882	20.141	21.495	22.953	24.523	26.212	28.029	29.985	32.089	34.352	36.786	48.497	64.110	68.760	97.625
14	14.947	15.974	17.086	18.292	19.599	21.015	22.550	24.215	26.019	27.975	30.095	32.393	34.883	37.581	40.505	43.672	59.196	80.496	86.949	127.913
15	16.097	17.293	18.599	20.024	21.579	23.276	25.129	27.152	29.361	31.772	34.405	37.280	40.417	43.842	47.580	51.660	72.035	100.815	109.687	167.286
16	17.258	18.639	20.157	21.825	23.657	25.673	27.888	30.324	33.003	35.950	39.190	42.753	46.672	50.980	55.717	60.925	87.442	126.011	138.109	218.472
17	18.430	20.012	21.762	23.698	25.840	28.213	30.840	33.750	36.974	40.545	44.501	48.884	53.739	59.118	65.075	71.673	105.931	157.253	173.636	285.014
18	19.615	21.412	23.414	25.645	28.132	30.906	33.999	37.450	41.301	45.599	50.396	55.750	61.725	68.394	75.836	84.141	128.117	195.994	218.045	371.518
19	20.811	22.841	25.117	27.671	30.539	33.760	37.379	41.446	46.018	51.159	56.939	63.440	70.749	78.969	88.212	98.603	154.740	244.033	273.556	483.973
20	22.019	24.297	26.870	29.778	33.066	36.786	40.995	45.762	51.160	57.275	64.203	72.052	80.947	91.025	102.444	115.380	186.688	303.601	342.945	630.165
21	23.239	25.783	28.676	31.969	35.719	39.993	44.865	50.423	56.765	64.002	72.265	81.699	92.470	104.768	118.810	134.841	225.026	377.465	429.681	820.215
22	24.472	27.299	30.537	34.248	38.505	43.392	49.006	55.457	62.873	71.403	81.214	92.503	105.491	120.436	137.632	157.415	271.031	469.056	538.101	*
23	25.716	28.845	32.453	36.618	41.430	46.996	53.436	60.893	69.532	79.543	91.148	104.603	120.205	138.297	159.276	183.601	326.237	582.630	673.626	*
24	26.973	30.422	34.426	39.083	44.502	50.816	58.177	66.765	76.790	88.497	102.174	118.155	136.831	158.659	184.168	213.978	392.484	723.461	843.033	*
25	28.243	32.030	36.459	41.646	47.727	54.865	63.249	73.106	84.701	98.347	114.413	133.334	155.620	181.871	212.793	249.214	471.981	898.092	*	*
30	34.785	40.568	47.575	56.085	66.439	79.058	94.461	113.283	136.308	164.494	199.021	241.333	293.199	356.787	434.745	530.312	*	*	*	*
35	41.660	49.994	60.462	73.652	90.320	111.435	138.237	172.317	215.711	271.024	341.590	431.663	546.681	693.573	881.170	*	*	*	*	*
36	43.077	51.994	63.276	77.598	95.836	119.121	148.913	187.102	236.125	299.127	380.164	484.463	618.749	791.673	*	*	*	*	*	*
40	48.886	60.402	75.401	95.026	120.800	154.762	199.635	259.057	337.882	442.593	581.826	767.091	*	*	*	*	*	*	*	*
50	64.463	84.579	112.797	152.667	209.348	290.336	406.529	573.770	815.084	*	*	*	*	*	*	*	*	*	*	*

### Present Value and Future Value Tables

Table A-3 Present Value Interest Factors for One Dollar Discounted at k Percent for n Periods:  $PVIF_{k,n} = 1/(1+k)^n$ 

	404	<b>0</b> 0/		10/	=0/		-	-		1001	4 4 6 4	4004	4004		4 50/	4.004		a 494	0.50/	
Period	1%	2%	3%	4%	5%	6%	1%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8333	0.8065	0.8000	0.7692
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.6944	0.6504	0.6400	0.5917
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.5787	0.5245	0.5120	0.4552
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.4823	0.4230	0.4096	0.3501
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4019	0.3411	0.3277	0.2693
			_			_			_		_	_						_		
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3349	0.2751	0.2621	0.2072
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996	0.3759	0.3538	0.2791	0.2218	0.2097	0.1594
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2326	0.1789	0.1678	0.1226
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.1938	0.1443	0.1342	0.0943
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.1615	0.1164	0.1074	0.0725
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1346	0.0938	0.0859	0.0558
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1122	0.0757	0.0687	0.0429
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.0935	0.0610	0.0550	0.0330
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.0779	0.0492	0.0440	0.0254
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0649	0.0397	0.0352	0.0195
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0541	0.0320	0.0281	0.0150
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0929	0.0802	0.0451	0.0258	0.0225	0.0116
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0376	0.0208	0.0180	0.0089
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0313	0.0168	0.0144	0.0068
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0261	0.0135	0.0115	0.0053
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0217	0.0109	0.0092	0.0040
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560	0.0462	0.0382	0.0181	0.0088	0.0074	0.0031
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491	0.0402	0.0329	0.0151	0.0071	0.0059	0.0024
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0126	0.0057	0.0047	0.0018
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0105	0.0046	0.0038	0.0014
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0042	0.0016	0.0012	*
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0259	0.0189	0.0139	0.0102	0.0075	0.0055	0.0017	0.0005	*	*
36	0.6989	0.4902	0.3450	0.2437	0.1727	0.1227	0.0875	0.0626	0.0449	0.0323	0.0234	0.0169	0.0123	0.0089	0.0065	0.0048	0.0014	*	*	*
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0007	*	*	*
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	*	*	*	*

Table A-4 Present Value Interest Factors for a One-Dollar Annuity Discounted at k Percent for n Periods: PVIFA = [1 - 1/(1 + k)ⁿ] / k

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8333	0.8065	0.8000	0.7692
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901	1.6681	1.6467	1.6257	1.6052	1.5278	1.4568	1.4400	1.3609
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018	2.3612	2.3216	2.2832	2.2459	2.1065	1.9813	1.9520	1.8161
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373	2.9745	2.9137	2.8550	2.7982	2.5887	2.4043	2.3616	2.1662
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048	3.5172	3.4331	3.3522	3.2743	2.9906	2.7454	2.6893	2.4356
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114	3.9975	3.8887	3.7845	3.6847	3.3255	3.0205	2.9514	2.6427
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638	4.4226	4.2883	4.1604	4.0386	3.6046	3.2423	3.1611	2.8021
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676	4.7988	4.6389	4.4873	4.3436	3.8372	3.4212	3.3289	2.9247
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282	5.1317	4.9464	4.7716	4.6065	4.0310	3.5655	3.4631	3.0190
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502	5.4262	5.2161	5.0188	4.8332	4.1925	3.6819	3.5705	3.0915
11	10.368	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377	5.6869	5.4527	5.2337	5.0286	4.3271	3.7757	3.6564	3.1473
12	11.255	10.575	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944	5.9176	5.6603	5.4206	5.1971	4.4392	3.8514	3.7251	3.1903
13	12.134	11.348	10.635	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235	6.1218	5.8424	5.5831	5.3423	4.5327	3.9124	3.7801	3.2233
14	13.004	12.106	11.296	10.563	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282	6.3025	6.0021	5.7245	5.4675	4.6106	3.9616	3.8241	3.2487
15	13.865	12.849	11.938	11.118	10.380	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109	6.4624	6.1422	5.8474	5.5755	4.6755	4.0013	3.8593	3.2682
16	14.718	13.578	12.561	11.652	10.838	10.106	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740	6.6039	6.2651	5.9542	5.6685	4.7296	4.0333	3.8874	3.2832
17	15.562	14.292	13.166	12.166	11.274	10.477	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196	6.7291	6.3729	6.0472	5.7487	4.7746	4.0591	3.9099	3.2948
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.3719	8.7556	8.2014	7.7016	7.2497	6.8399	6.4674	6.1280	5.8178	4.8122	4.0799	3.9279	3.3037
19	17.226	15.678	14.324	13.134	12.085	11.158	10.336	9.6036	8.9501	8.3649	7.8393	7.3658	6.9380	6.5504	6.1982	5.8775	4.8435	4.0967	3.9424	3.3105
20	18.046	16.351	14.877	13.590	12.462	11.470	10.594	9.8181	9.1285	8.5136	7.9633	7.4694	7.0248	6.6231	6.2593	5.9288	4.8696	4.1103	3.9539	3.3158
21	18.857	17.011	15.415	14.029	12.821	11.764	10.836	10.017	9.2922	8.6487	8.0751	7.5620	7.1016	6.6870	6.3125	5.9731	4.8913	4.1212	3.9631	3.3198
22	19.660	17.658	15.937	14.451	13.163	12.042	11.061	10.201	9.4424	8.7715	8.1757	7.6446	7.1695	6.7429	6.3587	6.0113	4.9094	4.1300	3.9705	3.3230
23	20.456	18.292	16.444	14.857	13.489	12.303	11.272	10.371	9.5802	8.8832	8.2664	7.7184	7.2297	6.7921	6.3988	6.0442	4.9245	4.1371	3.9764	3.3254
24	21.243	18.914	16.936	15.247	13.799	12.550	11.469	10.529	9.7066	8.9847	8.3481	7.7843	7.2829	6.8351	6.4338	6.0726	4.9371	4.1428	3.9811	3.3272
25	22.023	19.523	17.413	15.622	14.094	12.783	11.654	10.675	9.8226	9.0770	8.4217	7.8431	7.3300	6.8729	6.4641	6.0971	4.9476	4.1474	3.9849	3.3286
30	25.808	22.396	19.600	17.292	15.372	13.765	12.409	11.258	10.274	9.4269	8.6938	8.0552	7.4957	7.0027	6.5660	6.1772	4.9789	4.1601	3.9950	3.3321
35	29.409	24.999	21.487	18.665	16.374	14.498	12.948	11.655	10.567	9.6442	8.8552	8.1755	7.5856	7.0700	6.6166	6.2153	4.9915	4.1644	3.9984	3.3330
36	30.108	25.489	21.832	18.908	16.547	14.621	13.035	11.717	10.612	9.6765	8.8786	8.1924	7.5979	7.0790	6.6231	6.2201	4.9929	4.1649	3.9987	3.3331
40	32.835	27.355	23.115	19.793	17.159	15.046	13.332	11.925	10.757	9.7791	8.9511	8.2438	7.6344	7.1050	6.6418	6.2335	4.9966	4.1659	3.9995	3.3332
50	39.196	31.424	25.730	21.482	18.256	15.762	13.801	12.233	10.962	9.9148	9.0417	8.3045	7.6752	7.1327	6.6605	6.2463	4.9995	4.1666	3.9999	3.3333