## Financial Management



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## Financial Management

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# Financial Management <br> An Overview of Financial Management and Financial Environment 

## Corporate Life Cycle:

- Starting up as a Proprietorship: Many companies begin as a proprietorship, which is an unincorporated business owned by one individual. Starting a business as a proprietor is easy-one merely begins business operations after obtaining any required city or state business licenses.
- The proprietorship has three important advantages: (1) it is easily and inexpentyrly formed, (2) it is subject to few government regulations, and (3) its income is not setrject to corporate taxation but is taxed as part of the proprietor's personal income.
- However, the proprietorship also has two important limitations: (1) it may bafifficult for a proprietorship to obtain the capital needed for growth; (2) the proprigt has unlimited personal liability for the business's debts, which can result in losses thexceed the money invested in the company (creditors may even be able to seize a preprietor's house or other personal property)
- Partnership: Some companies start with more than one owner, and some proprietors decide to add a partner as the business grows. A partnership exists whenever two or more persons or entities associate to conduct a non-corporate business for profit.
- Partnerships may operate under different degree of formality, ranging from informal, oral understandings to formal agreements. Partnegship agreements define the ways any profits and losses are shared between partners.
- Regarding liability, the partners can potentially lose all of their personal assets, even assets not invested in the business, becare under partnership law, each partner is liable for the business's debts. Therefore, in the event the partnership goes bankrupt, if any partner is unable to meet his or her proteata liability then the remaining partners must make good on the unsatisfied claims, drating on their personal assets to the extent necessary.
- However, it is possible to limit the liabilities of some of the partners by establishing a limited partnershipg herein certain partners are designated general partners and others limited partners. Ar a limited partnership, the limited partners can lose only the amount of their investment in the partnership, while the general partners have unlimited liability. The limited pathers typically have no control-it rests solely with the general partners-and their retons are likewise limited.
- In bot regular and limited partnerships, at least one partner is liable for the debts of the penership. However, in a limited liability partnership (LLP), sometimes called a limited
líability company (LLC), all partners enjoy limited liability with regard to the business's liabilities, and their potential losses are limited to their investment in the LLP.
- Many Owners: A Corporation: A corporation is a legal entity created under state laws, and it is separate and distinct from its owners and managers. This separation gives the corporation three major advantages: (1) unlimited life-a corporation can continue after its original owners and managers are deceased; (2) easy transferability of ownership interestownership interests are divided into shares of stock, which can be transferred far more easily than can proprietorship or partnership interests; and (3) limited liability-losses are limited to the actual funds invested.
- The corporate form offers significant advantages over proprietorships and partnerships, but it also has two disadvantages: (1) Corporate earnings may be subject to double taxationthe earnings of the corporation are taxed at the corporate level, and then earnings paid out as dividends are taxed again as income to the stockholders. (2) Setting up a corporation involves preparing a lot of documentation, which is more complex and time consuming than creating a proprietorship or a partnership. 3)For proprietorships, partnerships, and small corporations, the firms owners are also its managers. This is usually not true for a large corporation, which means that large firm's stockholders, who are it's owners, face a serious problem- agency problem, because managers are hired as agents to act on behald ff the owners.
- Agency problems can be addressed by a company's corporate governance, which is the set of rules that controls the company's behavior towards its directors, manager@employees, shareholders, creditors, customers, competitors, and community.
- Shareholders are the owners of a corporation, and they purchase stocks because they want to earn a good return on their investment without undue risk expostre. In most cases, shareholders elect directors, who then hire managers to run the caporation on a day-to-day basis. Because managers are supposed to be working on behalfor shareholders, they should pursue policies that enhance shareholder value. Consequently, management's primary objective is stockholder wealth maximization.
- When we say management's objective should be to aximize stockholder wealth, we really mean it is to maximize the fundamental price or intinsic value of the firm's common stock, not just the current market price.


## Efficient Capital Markets:

- An efficient capital market i@one in which stock markets fully reflect available information.
- Suppose Eastern Housing being a real estate company, got a contract from government to build 10,000 apartments for Dhaka city people. Eastern Housing found a positive NPV for this project which ageans if the project goes ahead it will certainly increase shareholder' wealth. If this information is released in the media on Wednesday morning, the price of the shares of Easten Housing will immediately adjust to this new information.
- The efficient market hypothesis predicts that the price of shares of Eastern Housing on Wednesay afternoon will already reflect the information contained in the Wednesday mpring media release.
- E円ficient market hypothesis states that financial market process all relevant information about securities quickly and efficiently, that is, the security price usually reflects all the information available to investors concerning the value of the security.
- According to this hypothesis, as new information about a security becomes available, the price of the security quickly adjusts so that at any time, the security price equals the market consensus estimate of the value of the security. If this were so, there would be neither underpriced nor overpriced securities.


## Implications of EMH for Investors and Firms:

- Because information is reflected in prices immediately, investors should only expect to obtain a normal rate of return. Awareness of information when it is released does an investor no good. The price adjusts before the investor has time to trade on it.
- Firms should expect to receive fair value for securities that they sell. Fair means that the price they receive from issuing securities is the present value. Thus, valuable financing opportunities that arise from fooling investors are unavailable in efficient capital markets.
- From the figure in the following slide we can see, the solid line represents the path taken by the stock in an efficient market. In this case, the price adjusts immediately to the new information with no further price changes. The dotted line represents a slow reaction. Wene it takes the market 30 days to fully absorb the information. Finally, the broken line illustrates an over-reaction and subsequent correction back to the true price.
- The broken line and the dotted line show the paths that the stock price might take in an inefficient market. If the price of the stock takes several days to adjusto rrading profits would be available to investors who suitably timed their purchases andsates.

- Andrie Shleifer argues that there are three conditions any one of which will lead to efficiency:
- Rationality: If all investors act rationally, with the release of new information in the marketplace, all investors will adjust their estimates of stock prices in a rational way. In my previous example of Eastern Housing, investors will use the information in the media release in conjunction with existing information about the firm, to determine the NPV of
the new project. If the information in the release implies that the NPV of the new project is 200 million taka and there are 10 million shares, investors will calculate that the NPV is 20 taka per share. The price of shares would rise immediately by 20 taka per share.
- Independent Deviations from Rationality: If the new information is unclear, some investors may act irrationally optimistic and while some may act irrationally pessimistic. If we same number of irrationally pessimistic and optimistic investors in the market, prices would likely rise in a manner consistent with market efficiency, even though most investors would be classified as less than fully rational. Thus, market efficiency does not require rational investors- only countervailing irrationalities.
- Arbitrage: Suppose in a market we have irrational amateurs and the rational professionals. The amateurs get caught up in their emotions, at times believing irrationally that a strak is undervalued and at other times believing the opposite. If the passions of the different amateurs do not cancel each other out, they would tend to carry stocks eithe1 above or below the efficient prices.
- However, professionals evaluate information objectively and estimate s soek prices coldly and clearly and act accordingly. If a stock is underpriced, they wo bld buy it and if is overpriced they would sell it. Professionals try to arbitrage by sinentaneous purchase and sale of different, but substitute securities. If the arbitrage of eforssionals dominates the speculation of amateurs, markets would still be efficient.


## Types of Market Efficiency:

- Weak form of Efficiency: A capital market is said to be weakly efficient, if it fully incorporates the information in past sto prices. Weak form of efficiency is mathematically represented as

$$
P t=P_{t-1}+\text { Expected return }+ \text { Random error }{ }_{t}
$$

- The above equation states that the prientoday is equal to the sum of the past observed price plus the expected return on the stek plus a random component occurring over the interval.
- Weak form efficiency is about the weakest type of efficiency that we would expect a financial market to dispry because historical price information is the easiest kind of information about a stre to acquire. If it were possible to make extraordinary profits simply by finding patorns in stock price movements, everyone would do it, and any profits would disappear 1 the scramble.
- Semi-strong And Strong form Efficiency: A market is semi-strong form efficient if prices reflect (inceporate) all publicly available information, including information such as publisheaccounting statements for the firm, as well as historical price information.
- A maflet is strong form efficient if prices reflect all information, public or private.


## Financial Management

Time Value of Money

## Simple Interest and the Future Value:

- $\quad$ Simple Interest $=\mathrm{P}$ x rxt

Here, $\mathrm{P}=$ Present Value or Principal
$r=$ Rate of interest
$\mathrm{t}=$ Time period

- Future Value, $\mathrm{F}=$ Principal + Interest $=\mathrm{P}+\mathrm{Prt}=\mathrm{P}(1+\mathrm{rt})$
- Example: Compute interest amount and the future value of $\$ 600$ invested $29.5 \%$ interest rate for 10 months.
Sol: Here, Principal, $P=\$ 600$, Interest rate, $r=7.5 \%=7.5 / 100=0.0750$
Time periods, $\mathrm{t}=10$ months $=10 / 12$ year $=0.83$ years
Interest $=\operatorname{Prt}=\$ 600 \times 0.075 \times 0.83=\$ 37.5$
Future value $=\$ 600+\$ 37.5=\$ 637.5$


## Simple Interest and Bank Discount:

- In many loans, the interest charge is computed on the amount the borrower receives, but on the amount that is repaid later. The intere charge for a loan computed in this manner is called the bank discount, the amount the borrower receives is called the proceeds or present value of a loan. The future amount to be paid back is F, now called the maturity value of the loan.
- Example: If $\$ 1,000$ is borrowed at $12 \%$ for 6 months, the borrower receives $P$ and pay back $\mathrm{F}=\$ 1,000$ after 6 morths.
Time periods, $\mathrm{t}=6 \mathrm{~m}$ (1)hs $=0.5$ years
Future value, $\mathrm{F}=\mathrm{Pr}$ Frt
Present value, $\mathrm{P}=\mathrm{F}-\mathrm{Frt}=\$ 1000-\$ 1000 \times 0.12 \times 0.5=\$ 940$


## Compound Interest and the Future Value:

- Suap pose, $\$ 5,000$ is invested at $10 \%$ interest compounded each year. The amount at the end Of the first year would be,
- $\mathrm{F} 1=5,000+5,000 \times 0.10 \times 1=\$ 5,500$
- This $\$ 5,500$ becomes the principal at the beginning of the second year, and the amount at the end of the second year is,
- $\mathrm{F} 2=5,500+5,500 \times 0.10 \times 1=\$ 6,050$
- Thus in the second year, interest is earned on not only the $\$ 5,000$ invested, but also on the $\$ 500$ of interest earned in the first year. This common practice of computing interest on interest is called compounding interest.


## Derivation of Compound Interest Formula:

Future value at the end of one year,
$F_{1}=P(1+r x 1)=P(1+r)=P(1+r)^{1}$
Future value at the end of two years,
$F_{2}=P(1+r)(1+r x 1)=P(1+r)(1+r)=P(1+r)^{2}$
Future value at the end of three years,
$F_{3}=P(1+r)^{2}(1+r)=P(1+r)^{3}$
Future value at the end of $n$ years,
$F_{t}=P(1+r)^{t}$

## Compound Interest and the Future Value:

- Example: find the future value of $\$ 1,000$ at $7 \%$ per year for 10

Sol: Here, Present value, $\mathrm{P}=\$ 1,000, \mathrm{t}=10$ years, $\mathrm{r}=7 \%=0.07$
Future value, $F_{t}=P(1+r)^{t}=1000(1+0.07)^{10}=\$ 1967.15$

- Practice question: If $\$ 500$ is invested at $6 \%$ compounded annually, what will be the future value 30 years later?
- Example: Find the future value of $\$ 500$ at $\$ \%$ compounded quarterly for 10 years.

Sol: Here, present value, $\mathrm{P}=\$ 500$,
time periods, $\mathrm{t}=10$ years x 4 quarters/year $=40$ quarters
Yearly interest rate $=8 \%=0.08$
Quarterly interest rate, $\mathrm{r}=0.08 / 4=0.02$
Future value, $F_{t}=P(1+r)=00(1+0.02)^{40}=\$ 1104.019 \approx \$ 1104.02$

- Example: If $\$ 800$ is 1 vested at $\mathbf{6 \%}$ compounded semiannually(every 6 months), what will be the amountin 5 years?
Sol: Here, prest value, $\mathrm{P}=\$ 800$,
time periock $Q=5$ years $\times 2$ semi-annual period/year $=10$ periods
Yearly interest rate $=6 \%=0.06$
Semirqnual interest rate, $r=0.06 / 2=0.03$
F(u) Ure value, $F_{t}=P(1+r)^{t}=800(1+0.03)^{10}=\$ 1075.13$
- Example: Compute the future value of $\$ \mathbf{5 , 0 0 0}$ at $\mathbf{9 \%}$ compounded monthly for 10 years.
Sol:Here, present value, $\mathrm{P}=\$ 5,000, \mathrm{t}=10 \times 12=120$ periods
Monthly interest rate, $\mathrm{r}=0.09 / 12=0.0075$

```
Future value, F}\mp@subsup{F}{t}{}=P(1+r\mp@subsup{)}{}{t}=5000(1+0.0075) (120 = $12,256.7
```

- Example: A bank pays $\mathbf{7 . 2 5 \%}$ compounded daily on $\mathbf{9 0}$ day notice accounts. If $\mathbf{\$ 5 0 0}$ is deposited in such an account, what will be the amount in 90 days? (use 365 days per year).
Sol: Here, present value, $\mathrm{P}=\$ 500$,
time periods, $\mathrm{t}=90$ days
Yearly interest rate $=7.25 \%=0.0725$
Daily interest rate, $\mathrm{r}=0.0725 / 365$
Future value, $F_{t}=P(1+r)^{t}=500\left(1+\frac{0.0725}{365}\right)^{90}=\$ 509.0178 \approx \$ 509.02$
- Practice question: Find the future value of $\mathbf{\$ 2 , 5 0 0}$ invested at
a) $10 \%$ compounded monthly, for next 4 years
b) $\mathbf{8 \%}$ compounded daily, for next 6 months (use 360 days/year)
c) $\mathbf{1 2 \%}$ compounded quarterly, for next 2 years
- Example: At $\mathbf{8 \%}$ compounded annually, how many years will it tafe for $\$ 2,000$ to grow to $\mathbf{\$ 3 , 0 0 0}$ ?
Sol: Here, present value, $\mathrm{P}=\$ 2,000$, Future value, $\mathrm{F}=\$ 3,000$
time periods, $\mathrm{t}=$ ?
Yearly interest rate $=8 \%=0.08$
Future value, $F_{t}=P(1+r)^{t}$
i.e. $3000=2000(1+0.08)^{t}$
i.e. $1.08^{t}=\frac{3000}{2000}=1.5$
i.e. $\ln \left(1.08^{t}\right)=\ln (1.5) \quad ;$ TTaking $\ln$ in bqthrdes]
i.e. $t \times \ln (1.08)=\ln (1.5)$
i.e. $t=\frac{\ln (1.5)}{\ln (1.08)}=5.268 \approx 5.27 \mathrm{ye}$
- Practice question: Hownany years will it take for $\mathbf{\$ 1 , 0 0 0}$ to grow to $\mathbf{\$ 2 , 0 0 0}$ at $\mathbf{9 \%}$ compounded annuahty.
- Example: At yhat interest rate compounded annually will a sum of money double in 10 years?
Sol: Let oresent value, $\mathrm{P}=\$ 1$, and Future value, $\mathrm{F}=\$ 2$, time periods, $\mathrm{t}=10$ years, Yearbonterest rate $=$ ?
Fhere value, $F_{t}=P(1+r)^{t}$
$2=1(1+r)^{10}$
i.e. $(1+r)^{10}=2$
i.e. $\ln (1+r)^{10}=\ln (2) \quad ;$ [Taking $\ln$ in both sides]
i.e. $10 \ln (1+r)=\ln (2)$
i.e. $\ln (1+r)=\frac{\ln (2)}{10}$
i.e. $(1+r)=e^{\frac{\ln (2)}{10}}$
i.e. $r=e^{\frac{\ln (2)}{10}}-1=0.07177=7.17 \%$


## Alternate sol:

Future value, $F_{t}=P(1+r)^{\prime}$
or, $2=1(1+r)^{10}$
or, $(1+r)^{10}=2$
or, $(1+r)=2^{\frac{1}{10}}$
or, $r=2^{\frac{1}{10}}-1=0.07177=7.177 \%$

## Compound Interest and the Present Value:

- Example: What is the present value of $\mathbf{\$ 2 , 5 0 0}$ payable 4 years from now at $\mathbf{8 \%}$ compounded quarterly?
Sol: Here, present value, $\mathrm{P}=$ ? , and Future value, $\mathrm{F}=\$ 2,500$
time periods, $\mathrm{t}=4 \times 4=16$
Yearly interest rate $=8 \%=0.08$
Quarterly interest rate, $\mathrm{r}=0.08 / 4=0.02$
Future value, $F_{t}=P(1+r)^{t}$
i.e. $2500=P(1+0.02)^{16}$
i.e. $P=\frac{2500}{1.02^{16}}=\$ 1821.11$
- Practice question: What is the present value of $\$ 4,000$ payable in 20 rears at $\mathbf{8 \%}$ compounded semi-annually?
- Example: How much must be deposited now in an account paying $99.3 \%$ compounded daily in order to have just enough in the account 3 years frind now to make $\mathbf{\$ 1 0 , 0 0 0}$ available for investment in a business enterprise?
Sol: Here, present value, $\mathrm{P}=$ ? , and Future value, $\mathrm{F}=\$ 10,00 \infty^{~}$
time periods, $\mathrm{t}=3 \times 365=1095$
Yearly interest rate $=7.3 \%=0.073$
daily interest rate, $r=0.073 / 365$
Future value, $F_{t}=P(1+r)^{t}$
i.e. $10000=P\left(1+\frac{0.073}{365}\right)^{1095}$
i.e. $P=\frac{10000}{\left(1+\frac{0.073}{365}\right)^{1095}}=\$ 8033.39$ '9 $^{9}$
- Practice question: How much must be deposited now in an account paying 8\% compounded monthly 40 order to have just enough in the account 5 years from now to make a $\$ 10,000$ down payment on a home?
- Example: Find the effective rate(or equivalent annual rate) of $\mathbf{1 2 \%}$ compounded monthly.
Sol: Herearime periods, $t=12$ months
Yeard 6 interest rate $=12 \%=0.12$
MgQthly interest rate, $r=0.12 / 12=0.01$
Wffective Rate, $\mathrm{r}_{e}=(1+r)^{t}-1=(1+0.01)^{12}-1=0.1268=12.68 \%$
- Practice question: Find the effective rate of $16 \%$ compounded quarterly.
- Example: Find the effective rate(or equivalent annual rate) of $15 \%$ compounded daily.
Sol: Here, time periods, $\mathrm{t}=$ 365days
Yearly interest rate $=15 \%=0.15$
Daily interest rate, $\mathrm{r}=0.15 / 365$
Effective Rate, $\mathrm{r}_{e}=(1+r)^{t}-1=\left(1+\frac{0.15}{365}\right)^{365}-1=0.161798 \approx 16.18 \%$


## Derivation of Continuous Compounding Formula:

- Future value of $\$ 1$ at $100 \%$ compounded daily for one year is,

Future value, $F_{t}=P(1+r)^{t}=1\left(1+\frac{1}{365}\right)^{365}=\$ 2.7145674$
Future value of $\$ 1$ at $100 \%$ compounded hourly for one year (365daysx24hours=8760hrs)is,
Future value, $F_{t}=P(1+r)^{t}=1\left(1+\frac{1}{8760}\right)^{8760}=\$ 2.7181266$
We know,$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots \ldots \ldots \ldots+\frac{1}{n!}=2.718281828$
Future value of $\$ 1$ at $100 \%$ compounded in every minute for ofe year (365daysx24hoursx60mins=525600mins)is,
Future value, $F_{t}=P(1+r)^{t}=1\left(1+\frac{1}{525600}\right)^{525600}=\$ 2.718279 \approx e$
Future value of $\$ 1$ at $100 \%$ compounded $m$ times a year $F_{t}=1\left(1+\frac{1}{m}\right)^{m}$
If $\mathrm{m}>525,600$ times a year, $F_{t}=1\left(1+\frac{1}{m}\right)^{m}=\left(1+\frac{1}{m}\right)^{m}=e$
Future value of $\$ 1$ at $j \%$ rate compounded $m$ times a year for $t$ years is,
Future value, $F_{t}=P(1+r)^{t}=1\left(1+\frac{j}{m}\right)^{m t}=\left(1+\frac{j}{m}\right)^{m t}$
let, $\mathrm{P}=\frac{m}{j}$; so $\frac{j}{m}=\frac{1}{P}$; and $m=P j$
$F_{t}=\left(1+\frac{j}{m}\right)^{m t}=\left(1+\frac{1}{P}\right)^{P_{j t}}=\left[\left(1+\frac{1}{P} r^{p}{ }^{p}\right.\right.$
Now as $m$ becomes larger and rayer, $P()$ also becomes larger and larger
so, $\left(1+\frac{1}{m}\right)^{m} \approx\left(1+\frac{1}{P}\right)^{P}$
hence, $F_{t}=e^{j t}$
For continuous pounding (m very large) the future value $\$ P$ is,


- Exangie: Find the future value of $\$ 500$ at $\mathbf{8 \%}$ compounded continuously for 9 years and 3 months
Sol: Here, Present value, $\mathrm{P}=\$ 500$, interest rate, $\mathrm{r}=8 \%=0.08$
${ }^{7}$ Time periods, $\mathrm{t}=9$ years and 3 months $=9.25$ years
Future value, $F_{t}=P e^{r t}=\$ 500 x e^{0.08 x 9.25}=\$ 1047.97$
- Practice question: Find the future value of $\$ \mathbf{2 , 0 0 0}$ at $\mathbf{1 0 \%}$ compounded continuously for 9 months.
- Example: How much must be deposited now in an account earning $7.5 \%$ compounded continuously if the amount in the account 8 years from now is to be $\mathbf{\$ 1 0 , 0 0 0}$ ?
Sol: Here, Present value, $\mathrm{P}=$ ?, Future value, $\mathrm{F}=\$ 10,000$
Interest rate, $\mathrm{r}=7.5 \%=0.075$, time periods, $\mathrm{t}=8$ years

Future value, $10000=P e^{0.075 \times 8} ; P=\frac{10,000}{e^{0.075 x 8}}=\$ 5,488.11$

- Example: Find the effective rate of 10 percent compounded continuously.

Effective rate, $\mathrm{r}_{e}=e^{r}-1=e^{0.1}-1=0.10517=10.517 \%$

- Example: A bank states that the effective interest on savings accounts that earn continuous interest is $\mathbf{7 \%}$. Find the nominal rate.
Here, Effective rate, $\mathrm{r}_{e}=7 \%=0.07$, find the nominal rate, r
$\mathrm{r}_{e}=0.07=e^{r}-1 ;$ i.e. $e^{r}=1.07$
or, $\ln e^{r}=\ln (1.07)$
or, $r \ln e=0.06765 \quad ;[\ln \mathrm{e}=1]$
or, $r=6.765 \%$
- Practice question: What nominal rate compounded continuously gives an effective rate of 8 percent?


## Perpetuity:

- Perpetuity is a constant stream of cash flows for an infiaite period of time. Example: The British bonds called 'consols'. An investor purchasing consol is entitled to receive yearly interest from the British Government forever.
- Present value of Perpetuity or Consol, $P=\frac{c}{1} \frac{c}{(1+r)^{2}}+\frac{c}{(1+r)^{3}}+\ldots \ldots \infty=\frac{c}{r}$


## Growing Perpetuity:

- $P=\frac{c}{1+r}+\frac{c(1+g)}{(1+r)^{2}}+\frac{c(1+g)^{2}}{(1+r)^{3}}+\cdots \cdots \infty=\frac{c}{r-g}$
- Where, $\mathrm{C}=$ constant streator of cash flows, $\mathrm{g}=$ constant growth in cash flows


## Annuities:

- An annuity is series of periodic payments, usually made in equal amounts.
- Annuity eqtain: An annuity that begins and ends on designated dates is called an annuity certaineg. loan transactions and rent payments.
- Simque Annuity: An annuity whose payment intervals coincide with the interest intervals iscalled a simple annuity, e.g. a transaction whose payments is made monthly and interest
Nis also monthly compounding. When the two interval do not coincide, the annuity is called complex.
- Ordinary annuity: An annuity whose payment is made at the end of each payment interval is called an ordinary annuity.
- Annuity due: An annuity whose payment is made at the beginning of each payment interval is called an annuity due.
- Present value of Ordinary Annuity, $P=\frac{c}{1+r}+\frac{c}{(1+r)^{2}}+\frac{c}{(1+r)^{3}}+\ldots \ldots .+\frac{c}{(1+r)^{T}}$

Ordinary Annuity:


Present value of consol $1=\frac{c}{r}$;
Value of consol2 at time $\mathrm{T}=\frac{c}{c}$
Present value of consol $2=\frac{c}{r} x(1+r)^{-T}$
Present value of Ordinary Annuity, $P=P V$ of consol $1-P$ of consol 2
$=\frac{c}{r}-\frac{c}{r} x(1+r)^{-T}=\frac{c}{r}\left[1-(1+r)^{-T}\right]$
Future value of Ordinary Annuity, $F=c \boldsymbol{\mathcal { C }}(1+r)+c(1+r)^{2}+\ldots \ldots c(1+r)^{T-1}$ Present value of Ordinary Annuity ${\underset{9}{9}}^{P} \frac{c}{r}-\frac{c}{r} x(1+r)^{-T}$

Future value, $\left.F=P(1+r)^{T}-\frac{c}{r} x(1+r)^{-T}\right](1+r)^{T}=\frac{c}{r}(1+r)^{T}-\frac{c}{r}$
Future value, $F={ }_{-}^{c}\left[(1-r)^{T}-1\right]$

Example: If $\$ 100$ is deposited in an account at the end of every quarter for the next 5 years, how myen will be in the account at the time of the final deposit if interest is $\mathbf{8 \%}$ compounded quarterly. What will be the present value?
Sol: Herero. of payments, $T=5 \times 4=20$ payments; $r=0.08 / 4=0.02$

| Periods | Now(0) | 1 | 2 | 3 | $\cdots \cdots$ | T=20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annuity |  | $\$ 100$ | $\$ 100$ | $\$ 100$ | $\ldots \cdots$ | $\$ 100$ |
|  |  |  |  |  |  | FV =? |

$F V=\frac{c}{r}\left[(1+r)^{T}-1\right]=\frac{100}{0.02}\left[(1+0.02)^{20}-1\right]=\$ 2429.74$
using annuity table: $\mathrm{FV}=\$ 100 x$ FV IFA $_{20,2 \%}=\$ 100 x 24.297=\$ 2429.7$

$$
\begin{aligned}
& P V=\frac{c}{r}\left[1-\frac{1}{(1+r)^{T}}\right]=\frac{100}{0.02}\left[1-\frac{1}{(1+0.02)^{20}}\right]=\$ 1635.14 \\
& \text { using annuity table: } \mathrm{PV}=\$ 100 \times \text { PV IFA }_{20,2 \%}=\$ 100 \times 16.351=\$ 1635.1
\end{aligned}
$$

Example: If $\$ 100$ is deposited in an account each month for $10 y e a r s$ and the account earns $\mathbf{7 \%}$ compounded monthly, how much will be in the account after the last deposit is made.
Sol: Here, no. of payments, $\mathrm{T}=10 \times 12=120$ payments; $\mathrm{r}=0.07 / 12$

| Periods | $\mathbf{N o w}(\mathbf{0})$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\ldots \ldots$ | $\mathbf{T}=\mathbf{1 2 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annuity |  | $\mathbf{\$ 1 0 0}$ | $\mathbf{\$ 1 0 0}$ | $\mathbf{\$ 1 0 0}$ | $\ldots .$. | $\$ \mathbf{1 0 0} \mathbf{O}^{\prime}$ |
|  |  |  |  |  |  | 0 |

Practice question: Sums of $\$ 500$ are deposited in an account at the end of each $\mathbf{6}$-month period for 10 years. Find the amount in the account after the last deposit is made if interest is computed at $6 \%$ compounded semi-annually. (Usoboth formula and annuity table)

Example: How much should be deposited in a sinking fund at the end of each quarter for 5 years to accumulate $\$ 10,000$ if the fund earns $8 . \%$ compounded quarterly.
Sol: No. of payments, $\mathrm{T}=5 \times 4=20$ payments; $\mathrm{r}=84=2 \%=0.02 ; \mathrm{FV}=\$ 10,000$;


Practice Question: A company wants to accumulate $\$ 100,000$ to purchase replacement machinery 8 years from now. To accomplish this, equal semi-annual payments are made to a fund that earns $\mathbf{7 \%}$ compounded semi-annually. Find the amount of each payment.

Example: What sum deposited now in an account earning $\mathbf{8 \%}$ interest compounded quarterly will provide quarterly payments of $\$ 1,000$ for 10 years, the first payment to be made 3 months from now?
Sol: Here, No. of periods, $\mathrm{T}=10 \times 4=40 ; \mathrm{C}=\$ 1,000 ; \mathrm{r}=8 / 4=2 \%=0.02 ; \mathrm{PV}=$ ?

| Periods | Now(0) | $\mathbf{1}$ | 2 | 3 | $\cdots \cdots$ | $\mathbf{T}=40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annuity | PV =? | $\mathbf{1 , 0 0 0}$ | $\mathbf{1 , 0 0 0}$ | $\mathbf{1 , 0 0 0}$ | $\cdots \cdots$ | $\mathbf{1 , 0 0 0}$ |

$$
P V=\frac{c}{r}\left[1-\frac{1}{(1+r)^{T}}\right]=\frac{1000}{0.02}\left[1-\frac{1}{(1+0.02)^{40}}\right]=\$ 27,355.48
$$

using annuity table: $\mathrm{PV}=\$ 1000 x$ PVIFA $_{40,2 \%}=\$ 1000 x 27.355=\$ 27355$
Total interest earned $=40 \times \$ 1,000-\$ 27,355.48=\$ 12,644.52$
Example: Sam borrowed $\$ 5,000$ to buy a car. He will amortize dite loan by monthly payments over 3 years. a) find the monthly payment amount if interest is $\mathbf{1 2 \%}$ compounded monthly b) find the total amount sam will pay.
Sol: Here, No. of periods, $T=3 \times 12=36 ; C=? ; r=12 / 12=1 \%=0 . \theta 1 ; P V=\$ 5,000$

| Periods | Now(0) | $\mathbf{1}$ | $\mathbf{2}$ | $0^{2}$ | $\cdots \cdots$ | $\mathrm{~T}=\mathbf{3 6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annuity | $\mathrm{PV}=\$ 5,000$ | C | $\sigma^{2}$ | C | $\cdots \cdots$ | C |

$P V=\$ 5,000=\frac{c}{r}\left[1-\frac{1}{(1+r)^{T}}\right]=\frac{C}{0.01}\left[1-\frac{(1)}{(1 \text { す } 0.01)^{36}}\right]$
or, $50=C\left(1-1.01^{-36}\right) ; \quad$ or,$C=\frac{20}{\left.-1.01^{-36}\right)}=\$ 166.07$
using annuity table: $\mathrm{PV}=\$ C x \mathrm{RV}^{\mathrm{FFA}} \mathrm{YF}_{36,1 \%}=\$ C x 30.108$
or,$C=\frac{P V}{30.108}=\frac{\$ 5,000}{30.10 .8} \$ 166.07$
Total Payment $=36166.07=\$ 5,978.47$

## Amortizatiou Schedules:

- When a mortgage payment is made or debt is amortized, the interest due is subtracted first and the remainder is then applied to the outstanding balance.
Example: A $\mathbf{\$ 7 0 , 0 0 0}$ condominium is to be purchased by paying $\mathbf{\$ 1 0 , 0 0 0}$ in cash and a $\$ 60,000$ mortgage for 30 years at $9.75 \%$ compounded monthly. a) find the monthly payment on the mortgage b) what will be the total amount of interest paid? c) show amortization schedule for the first two payments.
Sol: PV=\$60,000; No. of periods, T=30x12=360; r = 0.0975/12

| Periods | Now(0) | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\ldots \cdots$ | $\mathbf{T}=\mathbf{3 6 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annuity | $\mathrm{PV}=\$ 60,000$ | C | C | C | $\ldots \cdots$ | C |

$P V=\$ 60,000=\frac{c}{r}\left[1-\frac{1}{(1+r)^{T}}\right]=\frac{C}{0.0975 / 12}\left[1-\frac{1}{(1+0.0975 / 12)^{360}}\right]$
or, $487.5=C\left(1-1.008125^{-360}\right) ; \quad$ or, $C=\frac{487.5}{\left(1-1.008125^{-360}\right)}=\$ 515.49$

Total interest paid $=360 \times \$ 515.49-\$ 60,000=\$ 125,577.35$

At the time the first payment is due, one month has passed and the interest on the $\$ 60,000$ $=\$ 60,000 \times 0.0975 / 12=\$ 487.5$

| Period | Monthly <br> Payment | Interest charged | Balance <br> Reduced | New Balance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 515.49$ | $\$ 60,000 \times 0.0975 / 12=\$ 487.5$ | $\$ 27.99$ | $60,000027.99$ <br> $=\$ 59,972.01$ |
| 2 | $\$ 515.49$ | $59,972.01 \times 0.0975 / 12=\$ 487.27$ | $\$ 28.22$ | $\$ 9,972.01-28.22$ <br> $=\$ 59,943.8$ |

## Multi-step Problems:

Example: Sam wants to determine how much he should deposit in a retirement account now at $8 \%$ compounded quarterly so that the amount in the account 10 years from now will provide an income of $\$ 5,000$ every 6 months for 12 years, with the first $\$ 5,000$ to be received in 10.5 years from now. Sam estimates, that 10 years from now he should be able to earn $6 \%$ compounded semi-annually on the account. How much should Sam deposit now?

| Now | Year-10 | $\mathbf{N}^{9}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\ldots .$. | $\mathbf{T}=\mathbf{2 4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\$ 38.350$ | Value $=\$ 84.680$ | $\$ .000$ | $\$ 5.000$ | $\$ 5.000$ | $\ldots .$. | $\$ 5.000$ |
| Interest rate $=6 \%$ compounded semi-annually |  |  |  |  |  |  |

using annuity table: V alue @SYear $10=\$ 5,000 x$ PV IF $_{24.3 \%}=\$ 5,000 x 16.936=\$ 84,680$
Mr. Sam will need $\$ 84680$ in his account after 10 years to receive $\$ 5,000$ thereafter in every 6 months for 12 years.

$$
P V=\$ 84,6881(1+0.02)^{-40}=\$ 38,350.76
$$

Mr. Sam shapd deposit $\$ 38,350.76$ today to receive $\$ 84,680$ after 10 years.

Example: James wishes to provide himself with an income of $\$ 5,000$ every six months, starting 15.5 years from now and continuing for 20 years. He deposits $\$ 25,000$ in the account now, and he has a guaranteed inheritance of $\$ 10,000$, which he will receive 10 years from now and add to the account. He knows these sums will not provide the income he wants, so he plans to make periodic deposits to the account at the end of every 6 months for 15 years to make up the difference. How much should the periodic deposits be if all interest is compounded at $6 \%$ semi-annually?

| Now | Year-10 | Year-15 | Year-15.5 (1) |  | 3 | $\cdots$ | $\mathrm{T}=40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | \$5,000 |  | \$5,000 | .... | \$5,000 |
| \$25,000 |  |  |  |  |  |  |  |
| \$10,000 $\rightarrow$ |  |  |  |  |  |  |  |
| Shortage |  |  |  |  |  |  | D) |
| Year-0.5(T1) |  | Year-1 (T2) | T3 | ..... | Year-15(T=30) ${ }^{\circ}$ |  |  |
| C? |  | C? | C? | ..... | O) |  |  |

@ year 15, Sam need $=\$ 5,000 x$ PVIFA $_{40,3 \%}=\$ 5,000 \times 23.115=\$ 11.55^{7} 5$
Future value of $\$ 25,000 @$ year- $15=\$ 25,000 x(1+0.03)^{30}=\$ 60,681.56$
Future value of $\$ 10,000 @$ year- $15=\$ 10,000 x(1+0.03)^{10}=813,439.16$

| Now | Year-10 | Year-15 | $\begin{gathered} \text { Year-15.5 } \\ (1) \sigma^{\circ} \end{gathered}$ | 2 | 3 | .... | T=40 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\$ 9,000$ | \$5,000 | \$5,000 | ..... | \$5,000 |
|  |  | $\text { Need }=\$ 115,5$ |  |  |  |  |  |
| \$25,000 |  | $\$ 60,68,56$ |  |  |  |  |  |
|  | \$10,000 | $\$ 18,439.16$ |  |  |  |  |  |
| Shortage $\$ 41,454.27$ |  |  |  |  |  |  |  |


| Year-0.5(11) | Year-1 (T2) | T3 | $\ldots .$. | Year-15(T=30) |
| :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\sigma}^{\boldsymbol{\sigma}}$ |  |  |  | $\$ 41,454.27$ |
| $\mathrm{C} ?$ | $\mathrm{C} ?$ | $\mathrm{C} ?$ | $\ldots .$. | $\mathrm{C} ?$ |

Future Value, $\$ 41,454.27=\$ C x \mathrm{FV}^{2} \mathrm{IFA}_{30,3 \%}=\$ C x 47.575$
i.e. $C=\frac{\$ 41,454.27}{47.575}=\$ 871.35$

Mr. Sam should deposit $\$ 871.35$ every 6-month for 15 years to cover the shortage

# Financial Management <br> Time Value of Money Problems 

Question 1-2: In problems 1 through 2, find the future value at the stated nominal interest rate compounded annually.

1) $\$ 200 ; 20$ years; 5 percent
2) $\$ 300 ; 10$ years; 6 percent

Question 3-4: In problems 3 through 4, find the future value using the appropriate interest and number of periods.
3) $\$ 150 ; 8$ years; 8 percent compounded quarterly
4) $\$ 600 ; 20$ years; 8 percent compounded semi-annually

Question 5: How many years will it take for $\$ 5,000$ to amount to $\$ 20,000$ at $7 \%$ compounded annually?
Question 6: How many years will it take for a sum of money to double at $\mathbf{1 0 \%}$ compounded semi-annually?

Question 7: Find the rate of interest compounded annually at which a sum of money will double in 20 years.
Question 8: Find the rate of interest compounded semi-amually at which $\$ 5,000$ will grow to $\$ 12,000$ in 8 years.
Question 9: A bank pays $5.25 \%$ compounded daily oif savings accounts running for 6 years. Using 365 days per year, compute the future value of a deposit of $\$ 5,000$ for 6 years.
Question 10: How many years will it tale for $\$ 5,000$ to grow to $\$ 10,000$ at $\mathbf{9 \%}$ compounded annually?
Question 11: At what rate of interest cempounded annually will $\mathbf{\$ 1 , 0 0 0}$ grow to $\mathbf{\$ 5 , 0 0 0}$ in 10 years?
Question 12-14: In problems 12 through 14, compute the present value:
12) $\$ 1,000$; due in 20 yeas 8 , 8 percent compounded semi-annually
13) $\$ 5,000$; due in 5 yealss; 10 percent compounded quarterly
14) $\$ 1,000$; due in 2vears; 12 percent compounded daily (use 365 days/year)

Question 15: Whatsum of money deposited now at $\mathbf{8 \%}$ compounded quarterly will provide just enqugh money to pay a $\$ 1$ million debt due 7 years from now?
Question 16. What sum of money invested now at $12 \%$ compounded monthly will provide just enought to pay a debt of $\mathbf{\$ 5 0 , 0 0 0}$ due in 15 years?
Question 17: If output per labor-hour increases by $5 \%$ compounded annually and is currently 100 units per labor-hour, what was output per labor-hour 5 years ago?
Question 18: An account bearing interest at $6 \%$ compounded semi-annually was established 10 years ago. The account balance now is $\$ 9,030.55$. What was the initial amount when the account was established?

Question 19-20: Compute the future value:
19) $\$ 4,000$; 8 percent compounded continuously; 5 years 8 months
20) $\$ 5,000 ; 12$ percent compounded continuously; 6 months

Question 21-22: Compute the present value:
21) $\$ 2,000 ; 7$ percent compounded continuously; 3 years 2 months
22) $\mathbf{\$ 2 5 0 ; 1 2}$ percent compounded continuously; 1year $\mathbf{6 m o n t h s}$

Question 23-24: The rate in problems 5 through 6 are nominal rates, find the effective rate when the interest rate is compounded continuously:
23) $\mathbf{1 2 \%}$, Sol:
24) $8 \%$, Sol:

Question 25-26: The rate in problems 25 through 26 are effective interest rates, find the nominal rate when the interest rate is compounded continuously:
25) Effective rate $=5 \%$
26) Effective rate $=10 \%$

Question 27: How much will a deposit of $\$ 5000$ grow to in 20 years at $7.2 \%$ interest compounded continuously?
Question 28: How much should be deposited now at $8.4 \%$ compounded confinuously if the amount in the account 10 years from now is to be $\$ 8000$ ?
Question 29: Sam invests $\$ 15,000$ in a bank account paying $848 \%$ compounded continuously for 15 years. How much will be in the account at the endof this time?
Question 30: History tells us that Peter Minuit purchased Manhattan Island in New York from the Indians for $\$ 24$ about 370 years ago. If the $\$ 244$ had been invested at $\mathbf{5 \%}$ compounded continuously, what would be its amount after 370 years.
Question 31: A company issues $\$ 1$ million of bonds and sets up a sinking fund at $\mathbf{8 \%}$ compounded quarterly to accumulate $\$ 1$ million by 10 years to redeem the bonds. Find the quarterly payment to the sinking fund.
Question 32: In order to accumulate $\mathbf{\$ 1 5 , 0 0 0}$ fara down payment on a home 8 years from now, the Jonses are going to deposit a sum money at the end of each 6 -month period in an account earning $8 \%$ compounded semi-annually. What should be the amount of each deposit?
Question 33: When Kathy was boen, her parents decided to deposit $\$ 500$ every 6 months thereafter for 15 years in an aqceunt earning $6 \%$ compounded semi-annually. How much will be in the account after the last deposit is made?
Question 34: A sum of provide payments of 1,500 every 6 months for 8 years, with the first payment due 6 months from now. Iow much should be invested? How much interest will the investment $_{\text {m }}$ earn?

Question 35 ; Ohe directors of a company have voted to establish a fund that will pay a retiring accountant or his estate $\$ 1,000$ per month for the next 3 years, with the first paymentoto be made a month from now. How much should be placed in the fund if it earnointerest at $12 \%$ compounded monthly? How much interest will the fund earn dewing its existence?
Question 36: A company has borrowed $\$ 50,000$ at $8 \%$ compounded quarterly. The debt is to be amortized by equal payments each quarter over 10 years. a) find the quarterly payment b) how much interest will be paid?
Question 37: A real estate developer borrows $\$ 100,000$ at $12 \%$ compounded monthly. The debt is to be discharged by monthly payments over the next $\mathbf{3}$ years. a) find the monthly payment b) how much interest will be paid?
Question 38: A company borrows $\$ 100,000$ at $12 \%$ compounded semi-annually. The debt is amortized by making equal payments at the end of each $\mathbf{6}$ months for $\mathbf{7}$ years.
a) Find the amount of each payment
b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
c) How much of the second payment is for interest, and by how much does it reduce the balance owed?

Question 39: Mr. Smith have taken out a $\mathbf{\$ 3 5 , 0 0 0}, 30$ year mortgage on their home, with interest at $\mathbf{8 . 7 5 \%}$ compounded monthly.
a) Find the amount of each payment
b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
Question 40: Mr. Fran borrowed $\mathbf{\$ 2 , 0 0 0}$ from Silverbank and signed a note promising to discharge the debt with interest at $12 \%$ compounded monthly at a maturity datez years from now. Six months later, Silverbank needed more cash and sold Fraro note to Goldbank. Goldbank computed the maturity amount of Fran's note and gareSilverbank the present value of this amount, computed at $8 \%$ compounded quarterly 2 How much did Silverbank receive?

Question 41: Sue borrowed $\$ 7,000$ at $12 \%$ compounded monthlyf for 3 years to buy a car. How much will she have to pay at the end of each month to discarge the debt?

Question 42: How much should be deposited now at 7\% compounded annually to provide an income of $\$ 20,000$ at the end of each year for the nexte2 years?
Question 43: Mr. James will make 20 equal semi-annual deposits to an account earning $8 \%$ compounded semi-annually. Then, after theg ast deposit, she will use the amount in the account to establish an ordinary annuity earning $6 \%$ compounded annually which will provide her with $\$ 10,000$ at the end ofreach year for 5 years. How much should his semi-annual deposit be?

## Additional Pime Value of Money Problems

1. Selim has taken a loanof Taka 56,000 at 12.5 percent annual rate for a period of 6 years. How much does he have to pay at the end of six years to fully pay off the loan?
2. Sadek has a fixeddeposit that will mature at Taka $16,00,000$ three years from now. He needs money urgently and he wants to mortgage the fixed deposit. The bank agreed to accept the morygage at an annual rate of $\mathbf{1 1 . 5}$ percent and lend him the maximum. How much will tre receive as loan?
3. Shahid has a fixed deposit that will mature at Taka $24,00,000$ four years from now. He need $D_{\text {money }}$ urgently and he wants to mortgage it. The bank agreed to accept the mertgage at an annual rate of $\mathbf{1 5 . 2 5}$ percent and lend him the present value of the fixed deposit but not to exceed 75 percent of the maturity value of the fixed deposit. How much will he receive as loan?
4. Shofiq made an investment of Taka 68,000 . If the investment grows at an annual rate of 18.5 percent for 7 years, what will be the value of the investment at the end of the seventh year?
5. Siddique had made an investment of Taka $1,25,000$ five years ago. He was promised a return of $\mathbf{1 7 . 5}$ percent per year. He wants to withdraw the investment now. How much should he get?
6. Sajed just purchased two bighas of land for a total price of Taka $38,00,000$. He expects the land value to appreciate by 9.5 percent per year. At what price should the land sell at the end of six years?
7. Solaiman purchased three bighas of land for a total price of Taka $\mathbf{4 8 , 0 0 , 0 0 0}$ six years ago. The land value appreciated at 8 percent during the first 4 years and at 12 percent in the last two years. What is the price of the land today?
8. Shaker made an investment twelve years ago. The amount was Taka 48,000. The investment grew at 9 percent in the first three years, at 11 percent the following three years, at 13.5 percent in the next three years, and at 15 percent in the final three years. What is the value of the investment today?
9. Karim plans to save Taka 20,000 per year and deposit the savings in a bank acconnt at the end of each year. The first deposit will be made at the end of this year. The bank account will pay 11 percent interest per year. How much would be the total accuimulated savings if he does this for 12 years?
10. Kamrul needs a loan of Taka 58,000 . He approaches a bank and the bank agrees to lend him the required amount at $\mathbf{1 2 . 5 \%}$ annual rate if he pays off thedoan in five years in five equal annual installments, the first payment to be made at dhe end of year 1 . How much does he have to pay per year? Build an amortization schedule.
11. Kofil wants to set up a plan so that he will have at legast Taka $50,00,000$ by the end of his twenty fifth year in the job. His plan is to set aside a certain amount every year and deposit it in an account that will pay him 11 percenanannually. How much does he need to set aside per year?
12. Kabir has set up a bank account that pax 12 percent annual interest. Kabir's goal is to have Taka 10,000 at the end of 6 years He will deposit Taka 1,280 per year at the end of each year until his target is achieyed, How much does he need to deposit at the end of the sixth year if his account balanceis to exactly equal Taka 10,000 ?
13. A Taka 80,000 loan was set ipp to be paid off in six equal annual installments. The annual rate is 15 percent. (a) What is the size of required installment? (b) How much is the remaining balance afteriour years?
14. Khaleque wants to set up a bank account for his granddaughter now to cover her for four years of college expenses. The annual college cost next year is expected to be Taka $10,00,000$. This expected to go up by 8 percent per year. How much does he need to place in the account today if the account is credited with 11 percent interest annually?
15. Without doing any math or using a calculator, at 7 percent annual growth rate, how long dogeu think it will take an initial investment of Taka $\mathbf{1 0 , 0 0 , 0 0 0}$ to become Taka $20,00,000$ ?

[^0]account will credit her with an annual interest of 9 percent. (a) How long will she need to accumulate the desired amount? (b) What deposit will she need to make to make the terminal amount exactly Taka $12,00,000$ ?
20. Build an amortization table for a loan of Taka 36,000 to be paid off in four equal annual installments and the interest rate is 12.5 percent per year.
21. Kamaruzzaman has taken a loan of Taka 80,000 for a period of 5 years. He is paying off the loan in five equal annual installment of Taka $24,432.75$ per year. What interest rate is he paying?
22. What is the value of a land that will generate Taka $1,00,000$ per year for ever? The expected return per year is $\mathbf{1 2}$ percent.
23. Helal needs to borrow Taka 72,000 for a period of 5 years. He looked into the following options:
a. $15 \%$ interest with annual compounding
b. $\mathbf{1 4 . 7 5 \%}$ interest with quarterly compounding
c. $\mathbf{1 4 . 6 \%}$ interest with monthly compounding
d. $\mathbf{1 4 . 5} \%$ interest with daily compounding
e. $\mathbf{1 4 . 3 5 \%}$ interest with continuous compounding

Determine the amount needed to pay off the loan at thedend of the 5th year under each option. Which option is the best?
What are the EAR for option (d) and (e)?
24. A piece of land is used to produce vegetabls The net market value of the output this year is expected to be Taka 125,000 . Thaqegetables generally experience an annual inflation of 7 percent. If an investor expectst to earn 18 percent annual return, what is the fair value of the land?
25. If you deposit the following amgents at the end of the indicated years and the account earns 12 percent interest, how miuch will be in the account at the end of the five years? Year 1 Taka 12,000 , year 2 Taka 16,000 , year 3 Taka 18,000, year 4 Taka 20,000, and year 5 Taka 21,000.
26. If a contract is expeeted to generate the following net cash flows and the expected return is 16 percent, what is the present value of the contract? Year 1 Taka 22,000, year 2 Taka 26,000, year 3 Taka 28,000, year 4 Taka 30,000, and year 5 Taka 35,000.
27. A proposed ${ }^{\text {innestment will cost Taka } 1,22,000 \text {. It will generate the following cash }}$ flows: Year Taka 32,000, year 2 Taka 29,000, year 3 Taka 38,000, year 4 Taka 42,000, and years Taka 41,000 . What rate of return is obtained from the investment?
28. IOa business generates Taka $12,00,000$ per year and an investor wants 20 percent antual return, what is the value of the business?
29. What is more valuable - (a) Taka $1,50,000$ per year for ever or (b) Taka 3,00,000 for 4 years? Annual rate is 20 percent.
30. A loan of Taka $12,00,000$ requires 5 equal annual installment at 16 percent with an extra provision that in the 6th year, a lump sum payment of Taka $2,00,000$ will be made. What is the size of annual payments?
31. What is the future value of 7 equal annual deposits of Taka 40,000 at 9 percent if the first deposit is made today?
32. What is the present value of 6 equal annual deposits of Taka 45,000 at 11 percent if the first deposit is made today?
33. A bank says that it will credit depositors with 10 percent annual interest on a continuous basis. If you make a deposit of Taka $\mathbf{1 6 , 0 0 0}$ in the account for 6 years, how much will be the accumulated value in the account?
34. A bank says that it will credit depositors with 12 percent annual interest on a continuous basis. If you deposit Taka 12,000 every year in the account for 4 years, how much will be the accumulated value in the account?
35. You deposited Taka 12,600 per year at the end of each year for 12 years. If you have Taka 2,65,427 in the account today, what annual rate of return did you earn?

Financial Management<br>Bonds and Stocks Valuation

## Bonds Characteristics:

- Bonds represent long term debt securities that are issued by government agencies or corporations. The issuer of bond is obligated to pay interest (coupon) payments periodically such as (annually or semi-annually) and the par value (principal or face value) at maturity. The coupon rate, maturity date, and par value of the bond are part of the bond indenture, which is the contract between the issuer and the bondholder.
- Bonds are often classified according to the type of issuer. Treasury bond are issuedy the treasury, federal agency bonds are issued by federal agencies, municipal bondoare issued by state and local governments, and corporate bonds are issued by corporation.
- Most bonds have maturities between 10 to 30 years. Bonds can also bg Classified by the ownership structure as either bearer bonds or registered bonds. Beate bonds require the owner to clip coupons attached to the bonds and send them to the issuer to receive coupon payments. Registered bonds require the issuer to maintain recedts of who owns the bond and automatically send coupon payments to the owners.


## Different types of Bonds:

- Treasury Bonds and Notes: The federal govt Tgeasury commonly issues treasury notes or treasury bonds to finance federal govt exanditures. The minimum denomination for treasury notes or bonds is $\$ 1,000$. The kepdifference between a note an a bond is that note maturities are usually less than 10 years, whereas bond maturities are 10 years or more. An active over the counter secondary net allows investors to sell treasury notes or bonds prior to maturity. Investors reeive semi-annual interest payments from the treasury. Although the interest is taxedryy the federal govt as ordinary income, it is exempt from state and local taxes.
- Striped Treasury Bonds: The cash flows of bonds are commonly transformed (striped) by securities firms so dat one security represents the principal payment only while a second security represents the interest payments. For example, consider a 10 year treasury bond with a par value of $\$ 100,000$ that has a $12 \%$ semi-annual coupon rate. This bond could be striped into a principal only (PO) security that will provide $\$ 100,000$ upon maturity and an interestenly (IO) security that will provide 20 semi-annual payments of $\$ 6,000$ each.
- Investors who desire a lump-sum payment in the distant future can choose the PO part, and inostors desiring periodic cash inflows can select the IO part.
- Inflation-Indexed Treasury Bonds: Inflation-indexed bonds provide returns tied to the inflation rate. These bonds commonly referred to as TIPS (Treasury inflation protected securities) are intended for investors who wish to ensure that the returns on their investments keep up with the increase in prices over time. The coupon rate offered on TIPS is lower than the rate on typical Treasury bonds, but the principal value is increased by the amount of the inflation rate every six months.
- For example, consider a 10 year inflation indexed bond that has a par value of $\$ 10,000$ and a coupon rate of $4 \%$. Assume that during the first 6 months since the bond was issued, the inflation rate was $1 \%$. The principal of the bond is increased by $\$ 100(1 \%$ of $\$ 10,000)$. Thus the coupon payment after 6 month will be $2 \%$ of the new par value $(\$ 10,100)=\$ 202$.
- Savings Bonds: Savings bonds are issued by the treasury, but can be purchased from many financial institutions. They are attractive to small investors because they can be purchased with as little as $\$ 25$. Large denominations are available as well. Savings bonds have a 30 year maturity and do not have a secondary market.
- Federal Agency Bonds: Federal agency bonds are issued by federal agencies. The bonds are backed both by the mortgages that are purchased with the proceeds and by the federal government.
- Municipal Bonds: State and local govt often issue municipal bonds to finance their budget deficit. Payments on general obligation bonds are supported by the municipal govt's ability to tax, whereas payments on revenue bonds must be generated by revenues of the projects (toll way, toll bridge, etc.). Municipal bonds typically promise semi-annual paynments. Common purchasers of these bonds include financial and non-financial institutions well as individuals. The minimum denomination of municipal bonds is typically $\$ 5,000$. A secondary market exists for them, although it is less active than the one forkersury bonds. Most municipal bonds contain a call provision, which allows the issuer $t$ (or repurchase the bonds at a specified price before the bonds mature. A municipality map exercise the call option to repurchase the bonds it interest rates decline substantialbyecause it can reissue bonds at the lower interest rate and reduce its cost of financing.
- Variable Rate Municipal Bonds: Variable rate municipal bonds have a floating interest rate based on a benchmark interest rate. The coupon payment adjusts to movements in the benchmark interest rate (LIBOR). Some variable rate bonds are convertible to a fixed rate bond until maturity under specified conditions. In general, these bonds are desirable to investors who expect that interest rate will rise.
- Corporate Bonds: When corporations need borrow for longer term periods, they issue corporate bonds which usually promise the owner interest on a semi-annual basis. The minimum denomination is $\$ 1,000$. Larger bond offerings are normally achieved through public offerings, which must first beyegistered with the SEC. The bonds issued by smaller corporations tend to be less liqui $\oplus$ because their trading volume is relatively low.
- Although most corporate bond have maturities between 10 to 30 years. Corporations such as Boeing, Ford and Chetrón have recently issued 50 -year bond. These bonds can be attractive to insurancesompanies that are attempting to match their long term policy obligations.


## Bond Indenture:

- The bond indenture is a legal document specifying the rights and obligations of both issuife firm and the bondholders. Federal law requires that for each bond issue of signficant size a trustee be appointed to represent the bondholders in all matters concerning the bond issue. The trustee's duties include monitoring the issuing firm's activities to ensure compliance with the terms of the indenture. If the terms are violated, the trustee initiates legal action against the issuing firm and represents the bondholders in that action.
- Sinking fund provision: Bond indentures frequently include a sinking fund provision or a requirement that the firm retire a certain amount of the bond issue each year. This provision is considered to be an advantage to the remaining bondholders because it reduces the payments necessary at maturity. For example, a bond with 20 years maturity could have a provision to retire $5 \%$ of the bond issue each year.
- Protective covenants: Bond indentures normally place restrictions on the issuing firm that are designed to protect the bondholders from being exposed to increasing risk during the
investment period. These so called protective covenants frequently limit the amount of dividends and corporate officer's salaries the firm can pay and also restrict the amount of additional debt the firm can issue.
- Call provision: Some corporate bonds are issued with call provisions allowing the issuer to repurchase the bond at a specified call price before the maturity date. For example, if a company issues a bond with higher coupon rate when market interest rates are high, and interest rates later fall, the firm might like to retire the high coupon bond and issue new bonds with lower coupon rate to reduce coupon payments. This is called refunding. Callable bonds typically comes with a period of call protection, an initial time during which the bonds are not callable. Such bonds are referred to as deferred callable bonds. A eall provision normally requires the firm to pay a price above par value when it calls its honds. The difference between the bond's call price and par value is the call promium. Bondholders normally view a call provision as a disadvantage because it can dirisupt their investment plans and reduce their investment returns. As a result, firms manpay slightly higher rates of interest on bonds that are callable, other things being equab $\gamma$


## Bond Collateral:

- Bonds can be classified according to whether they are secayed by collateral and by the nature of that collateral. Usually the collateral is a motgage on real property (land and buildings).
- First Mortgage Bond: A first mortgage bond has first claim on the specified assets.
- Chattel Mortgage Bond: A chattel mortgaggoond is secured by personal property.
- Debentures: Bonds unsecured by specifí property are called debentures (backed only by the general credit of the issuing fint $)$. These bonds are normally issued by large, financially sound firms whose ability to service the debt is not in question.
- Subordinated Debentures: Subordinated debentures have claims against the firms assets that are junior to the claims of both mortgage bonds and regular debentures. Owners of subordinated debentures receive nothing until the claims of mortgage bondholders, regular debenture owners, and secured short term creditors have been satisfied. The main purchasers of subordinated debt are pension funds and insurance companies.

Other types of BOrds:

- Converfinle Bond: Convertible bond allows investors to exchange the bond for a stated number of shares of the firm's common stock. This conversion feature offers investors the @ential for high returns if the price of the firm's common stock rises. Investors are therefore willing to accept a lower rate of interest on these bonds, which allows the firm to obtain financing at a lower cost.
- Puttable bonds: While the callable bond gives the issuer the option to extend or retire the bond at the call date, the extendable or put bond gives this option to the bondholder. If the bond's coupon rate exceeds current market interest rate, the bondholder may choose to extend the bond's life. If the bond's coupon rate is too low, it will be optimal not to extend; the bondholder instead claims principal which can be invested at current yields.
- Inverse Floaters: These bonds are opposite to floating rate bonds; the coupon rate on these bonds falls when the general level of interest rates rises.
- Junk Bonds: Credit rating agencies assign credit ratings to corporate bonds based on their perceived degree of credit risk. Those bonds that are perceived to have high risk are referred to as speculative grade or junk bond. Junk bonds are also known as high-yield bonds.
On the contrary, bonds with good credit rating are called investment-grade bond.


## Preferred Stock:

- Although preferred stock is considered to be equity, it is often included in the fixed income securities like bond. This is because, like bonds, preferred stock promises to pay a fixed stream of dividends. However, unlike bonds, the failure to pay the promised dividend ©oes not result in corporate bankruptcy. Instead, the dividends owed simply accumulates, and the common stockholder may not receive any dividends until the preferred stockholders have been paid in full.
- In the event of bankruptcy, preferred stockholders' claim to the firm's fase have lower priority than those of bondholders, but higher priority than those of comen stockholders.


## Bond Price and Impact of Change in Interest Rate on it

- Yield to Maturity (YTM): Yield to Maturity is definedds the interest rate that makes the present value of future cash flows from the bond equad to its market price. In other words it is the internal rate of return to the bond.

$$
\text { B ond Price, } P=\frac{C_{1}}{(1+y)}+\frac{C_{2}}{(1+y)^{2}}+\frac{C_{3}}{(1+y)} \ldots \ldots \ldots \ldots+\frac{\left(C_{n}+M\right)}{(1+y)^{n}}
$$

- Yield Curve: Yield curve is a plot of yeld to maturity as a function of time to maturity. The yield curve is also called the temastructure of interest rates and it helps us extract the appropriate rates that should be wed to discount cash flows at different maturities. There are various relationship betweefyield and their maturity. Most common patterns:
- Upward sloping
- Downward sloping (inverted)
- Flat
- Hump-shaped (rising and then falling)


## Rules of Interest Rate Sensitivity of a Bond:

- Bonddorices and yields are inversely related; bond prices decrease when yields rise, and that the price curve is convex, meaning that decreases in yields have bigger impacts on price than increases in yields of equal magnitude.
- Prices of long term bonds tend to be more sensitive to interest rate changes than prices of short term bonds. If rates increase, for example, the bond is less valuable as its cash flows are discounted at a now-higher rate. The impact of the higher discount rate will be greater as that rate is applied to more distant cash flows.
- The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increase.
- Interest rate risk is inversely related to the bond's coupon rate. Prices of low coupon bonds are more sensitive to changes in interest rates than prices of high coupon bonds.
- Bonds with higher yield to maturity is less sensitive to changes in interest rates and vice versa, i.e. bond price sensitivity falls with yield to maturity.


## Effective Maturity Concept (Duration):

- Bond price sensitivity falls with yield to maturity. A higher yield reduces the present value of all of the bond's payments, but more so for more distant payments. Therefore, at a higher yield, a higher fraction of the bond's value is derived from its earlier payments, which have lower effective maturity and interest rate sensitivity.
- To deal with the ambiguity of the "maturity" of a bond making many payments, we need a measure of the average maturity of the bond's promised cash flows to serve as a neful summary statistic of the effective maturity of the bond.
- Frederick Macaulay termed the effective maturity concept the duration of bond. Macaulay's duration equals the weighted average of the times to each couper or principal payment made by the bond. The weight associated with each payment tipe clearly should be related to the "importance" of that payment to the value of the bond fact, the weight applied to each payment time is the present value of the payment diaded by the bond price.

$$
w_{t}=\frac{\mathrm{CF}_{t} /(1+y)^{t}}{\text { Bond Price }}
$$

- Macaulay's Duration is given by, $D=\sum_{t=1}^{T} t x w_{t}$
- Duration is a very useful measure of bond portfio's interest rate sensitivity. It is a useful tool for immunization against interest rate ris
- It can be shown that when interest rate, chenges, the proportional change in bond' price is related to the change in its YTM (y).


## Rules for Duration:

1) The duration of a zero-coupon bond equals its time to maturity (since there are no interim cash flows)
2) Holding matyefty constant, a bond's duration is lower when the coupon rate is higher. The higher the oupon payments, the higher the weights on the early payments, and the lower is the weinhted average maturity (duration) of the payments.
3) Helaing the coupon rate constant, a bond's duration generally increases with its time to W urity (because of introducing more payments in the distant future). However, duration
Fheed not always increase with time to maturity. It turns out that for some deep discount coupon bonds, duration may fall with increases in maturity.
4) Ceteris paribus, the duration of a coupon bond is higher, when the bond's yield to maturity is lower. Because at lower yields the more distant payments made by the bond have relatively greater present values and accounts for greater weights of the bond's value.

## Limitation of Duration:

- Bond price sensitivity equation asserts that the percentage change in bond price is directly proportional to the change in the bond's yield. If this were exactly so, there should be a straight line relationship between the percentage change in bond's price and change in its yield. However, the true price-yield relationship is said to be convex and the curvature of the price-yield curve is called the convexity of the bond.
- Duration underestimates the increase in bond price when the yield falls, and it overestimates the decline in price when the yield rises.
- The duration rule is a good approximation for small changes in the bond yield or interest rates, but it is less accurate for larger changes. To improve this approximation, we need a correction term, known as convexity.


## Convexity:

- The convexity of a bond equals the second derivative of the pricetyield curve divided by bond price.

$$
\text { Convexity, } \mathrm{C}=\frac{1}{P} \frac{d^{2} p}{d y^{2}}=\frac{1}{P(1+y)^{2}} \sum_{1}^{T}[\underbrace{C}_{\left(\mathcal{O}^{\prime} y\right)^{\prime}}\left(t^{2}+t\right)]
$$

- The percentage change in the value of a bond after acculiting convexity equals,

$$
\frac{\Delta P}{P}=-\frac{D}{1+y} \times \Delta y+1<\mathrm{x} \times(\Delta y)^{2}
$$

## Immunization:

- Immunization programs aim at protectingbond portfolios against interest rates shifts. They attempt to eliminate the portfolio's sensitivity to shifts in the yield curve by matching the duration of the assets to the duration of the liabilities (duration matching).
- Since duration is a measure sensitivity for bond returns, two bonds (or portfolios of bonds) with the same duration will have their values changed by the same amount due to a common interest rate sherk.
- If duration matching successful, then any rise/ fall in the present value of the portfolios assets due to a shiftin the yield curve will be offset by a fall/ rise of equal magnitude in the present value ff the portfolios liabilities. Therefore, the net position will be immunized against thesechifts. Note: The duration of a bond portfolio is equal to the weighted average of the cortituent bonds" durations (weighted according to their relative value).


## Immunzation Strategies:

- Focused strategy: Find a portfolio of bonds with each bond having a duration close to the duration of the liability.
- Barbell strategy: Use bonds with very different durations and mix them accordingly to construct the desired duration.
- A barbell strategy provides more $\ddagger$ flexibility (necessary as time passes) relative to the focused strategy, but it also incorporates a greater degree of potential inaccuracy.


## Risks or Problems with Immunization:

- Successful immunization depends on the use of a correct measure of duration, which depends on the assumed shape of the yield curve. For an incorrect choice, we won't have a perfectly immunized portfolio.
- Even if a portfolio is perfectly immunized, with the passage of time or small changes in the yields, the portfolio ceases to be immunized. Hence, portfolio rebalancing is necessary to immunize it according to the new conditions (i.e. it is an active strategy!). Only exact cash flow matching implies an always immunized portfolio (this is obviously more difficult and costly to construct).
- Duration provides a rough approximation. Incorporating convexity considerably enhances the approximation. Therefore, most bond portfolio managers engage in immunization by matching both convexity and duration. Convexity matching leads to a veryouccessful immunization, but it also implies a higher cost of constructing the portfoliosidee fewer sets of bonds can achieve both targets
Example: Consider a semi-annual $8 \%$ coupon bond with 2 year matourity i.e. $n=4, C=$ $\$ 40, \mathrm{M}=\$ 1,000$ and yield to maturity, $\mathrm{y}=10 \%$. Calculate the bond 's current price ( $\mathbf{P}$ ), modified duration and convexity. If the bond's yield to madrity increases by 0.02 percentage points, calculate the change in the bond price andfinerefore, the new price.

| Time | Cash Flow | Present Value | Weight | Time*Weight |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 40 | 38.095 | cas35 | 0.0395 |
| 2 | 40 | 36.281 | 0.0376 | 0.0752 |
| 3 | 40 | 34.554 | 0.0358 | 0.1074 |
| 4 | 1040 | $\begin{aligned} & 855.61 \\ & \text { Price, } \Sigma \text { चै } 964.54 \end{aligned}$ | 0.887 | $3.548$ <br> Duration, D $\Sigma=3.77$ |
| $\frac{\Delta P}{P}=-\frac{D}{(1+y)} \times \Delta y=-\frac{3.77}{n 05} 0.01 \%=-0.0359 \%$ <br> nd price will decreasey $0.0359 \%$ |  |  |  |  |



New Price $=\$ 64.54-\$ 0.346=\$ 964.19$

| Wime | Cash Flow | Present Value | $\mathbf{P v ( t + \mathbf { t } )}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 40 | 38.095 | 76.19 |
| $\mathbf{2}$ | 40 | 36.281 | 217.686 |
| $\mathbf{3}$ | 40 | 34.554 | 414.648 |
| $\mathbf{4}$ | 1040 | 855.61 <br> Price, $\Sigma=964.54$ | 17112.2 |

Convexity, $\mathrm{C}=\frac{1}{P(1+y)^{2}} \sum_{1}^{T}\left[\frac{C F_{t}}{(1+y)^{t}}\left(t^{2}+t\right)\right]=\frac{1}{964.54(1.05)^{2}} x 17820.724=16.758$
The percentage change in the value of a bond after accounting convexity equals,

$$
\begin{aligned}
& \frac{\Delta P}{P}=-D^{*} \times \Delta y+1 / 2 \times \mathrm{C} \mathrm{x}(\Delta y)^{2} \\
& =-3.59 \times 0.0001+0.5 \times 16.758 \times(0.0001)^{2}=-0.0003589=-0.0359 \% \\
& \text { i.e. } \Delta P=P(-0.0359 \%)=-\$ 0.346
\end{aligned}
$$

Example: An insurance company must make a payment of $\mathbf{\$ 1 9 , 4 8 7}$ in 7 years. The market interest rate is $10 \%$, so the present value of the obligation is $\$ 10,000$. The company's portfolio manager wishes to fund the obligation using 3-year zero coupon bonds and perpetuities paying annual coupons. How can the manager immunized the obligation?

Sol: Immunization requires that the duration of the portfolio of assets equal the duation of the liability. We can proceed in four steps:

1. Calculate the duration of the liability: It is a single payment obligation duration of 7 years.
2. Calculate the duration of the asset portfolio: The portfolio duratich is the weighted average duration of each component asset, with weights proportional to theefunds placed in each asset. The duration of the zero coupon bond is simple its maturity, 8 years. The duration of the perpetuity is $(1+y) / y=1.10 / 0.10=11$ years.

Asset duration $=\mathrm{wx} 3$ years $+(1-\mathrm{w}) \times 11$ years
Asset duration $=$ Liability duration
Asset duration $=\mathrm{w} \times 3$ years $+(1-\mathrm{w}) \times 11$ years $=7$ years

$$
\text { i.e. } w=0.5
$$

3. The manager should invest half of portfolio in the zero coupon bonds and the rest half in perpetuities. This will result in anasset duration of 7 years.
4. Because the obligation has present value of $\$ 10,000$, the manager must purchase $\$ 5,000$ of the zero coupon bonds an $\$ \$ 5,000$ of the perpetuities. (note that the face value of the zero will be $\$ 5000 \times(1.10)^{3}=\$ 60655$

## Bond Valuation Problems

Question 1: How much should you pay today to purchase a $10 \%$ semi-annual coupon bond of 3 years maturity where the bond comes with face value of $\$ 1,000$ and yield to maturity $\mathbf{1 2 \%}$ ?

Question 2: Ford issued a 30 years maturity bond 28 years before where it promises to make $12 \%$ semi-annual coupon payment and a payment of $\$ 1,000$ face value at maturity. How much should you pay today to purchase this bond if yield to maturity for 2 years Treasury note is $15 \%$ ?

Question 3: Sam purchased a 50 years maturity bond with 2 years remaining at a price of $\$ 867$ where the bond promises to make $10 \%$ semi-annual coupon payment and gingle payment of $\$ 1,000$ at maturity. Find the rate of yield to maturity (YTM) Sam wif receive from his investment in bond.

Question 4: Mr. Karim paid $\$ 750$ to purchase a 30 years maturity bơnd with 2.5 years remaining, where the bond promises to make $8 \%$ semi-annual coupon payment and a single payment of $\$ 1,000$ at maturity. Find the rate of yield 40 maturity (YTM) Mr. Karim will receive from his investment in bond.

Question 5: Mr. Imdad is considering a $12 \%$ semi-anntal/coupon bond with 3 years maturity where face value is $\$ 5,000$ and yield to maturitit, $\mathbf{y}=8 \%$. How much Mr. Imdad should pay to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity increases by 0.6 percentage points bond price and therefore, the new price.

Question 6: Mr. Azad is working in Lâ̂ka Bangla Finance Company in Bangladesh. Lanka Bangla invests a significant amount of their capital in Treasury Securities issued by Bangladesh Bank. Bangladesh Baink is going to increase their lending rate from 5\% to $\mathbf{5 . 5 \%}$. Find out the impact of this change in interest rate on Lanka Bangla' Treasury security portfolio of $\mathbf{1 0 0 , 0 0 0 , 0 0}$ off the semi-annual duration of the portfolio is $\mathbf{2} .67$.

Question 7: Mr. Saikat is planning to invest in a 7\% semi-annual coupon bond with 2.5 years maturity whereface value is $\$ 10,000$ and yield to maturity, $\mathrm{y}=\mathbf{1 0 \%}$. How much Mr. Saikat should day to purchase the bond? Find effective maturity of the bond. If the bond`s yield to nataririty decreases by 2.2 percentage points, calculate change in the bond price and theretore, the new price.

## Equity or Stock Valuation Models:



Discounted Cash Flow Techniques

- Present Value of Dividends (DDM)
- Present Value of Operating Cash Flow
- Present Value of Free Cash Flow

Relative Valuation Techniques

- P/E ratio
- Price-CF Ratio
- P/BV ratio
- Price-Sales ratio


## Dividend Discount Model (DDM):

$$
P_{0}=\frac{D i v_{1}}{1+r}+\frac{D i v_{2}}{(1+r)^{2}}+\frac{D i v_{3}}{(1+r)^{3}}+\ldots \ldots \ldots \ldots .=\sum_{t=1}^{\infty} \frac{D i v_{t}}{(1+r)^{t}}
$$

- The price of a share of common stock to the investor is equale the present value of all of the expected future dividends into perpetuity.
- The DDM can be simplified if dividends are expected follow some basic patterns: 1) Zero growth 2) Constant growth 3) Differential growth
- Case 1 (Zero growth): The price of a share of at

$$
P_{0}=\frac{D i v}{1+r}+\frac{D i v}{(1+r} \frac{O_{i v}}{(1+r)^{3}}+\ldots \ldots \ldots \ldots . . . \quad=\frac{\text { Div }}{r}
$$

- Case 2 (Constant growth): The poe of a share of stock with a constant growth (g) in dividend is given by:

$$
\begin{align*}
& P_{0}=\frac{D_{1}}{1+r}+\frac{D(1+g)}{(1+r)^{2}}+\frac{D_{1}(1+g)^{2}}{(1+r)^{3}}+\ldots \ldots \ldots \ldots .(a)  \tag{a}\\
& \frac{1+r}{1+g} P_{0}=\frac{D_{1}}{1+g}+\frac{D_{1}}{(1+r)}+\frac{D_{1}(1+g)}{(1+r)^{2}}+\frac{D_{1}(1+g)^{2}}{(1+r)^{3}} \ldots \ldots \ldots \ldots \\
& (b)-(a) ; \frac{1+r}{1+g} P_{0}-P_{0}=\frac{D_{1}}{1+g} \quad \text { or, } P_{0}\left[\frac{1+r}{1+g}-1\right]=\frac{D_{1}}{1+g}  \tag{b}\\
& \text { or, } P_{0} \frac{r-g}{1+g}=\frac{D_{1}}{1+g} \quad \text { or, } P_{0}(r-g)=D_{1} \\
& \text { or, } P_{0}=\frac{D_{1}}{r-g}
\end{align*}
$$

## Constant Growth DDM or Gordon Model:

- The constant growth DDM is valid only when " g " is less than " r ". If dividends were expected to grow forever at a rate equal or faster than the discount rate, the value of the stock would be infinite or negative. If an analyst derives an estimate of $g$ that is greater than discount rate (required return), that growth rate must be unsustainable in the long run.
- The appropriate valuation model to use in this case is a multistage DDM.

$$
P_{0}=\frac{D_{1}}{r-g} \text { and } P_{1}=\frac{D_{2}}{r-g}=\frac{D_{1}(1+g)}{r-g}=\frac{D_{1}}{r-g}(1+g)=P_{0}(1+g)
$$

- Therefore, the DDM implies that in the case of constant growth of dividends, the rate of price appreciation in any year will equal that constant growth rate, g.


## Stock Prices and Investment Opportunities:

- A low reinvestment-rate plan allows the firm to pay higher initial dividends, but results in a lower dividend growth rate. Eventually, a high reinvestment-rate plan will provide higher dividends. If the dividend growth generated by the reinvested earnings is high enough, the stock will be worth more under the high reinvestment strategy.
- Suppose total assets of a company (fully equity financed) $=\$ 100$ million
- Current year's earnings $=$ Total equity x $\mathrm{ROE}=\$ 100$ million $\times 15 \%=\$ 15$ millio ${ }^{\circ}$
- Earnings per Share ( 3 million shares outstanding) $=15 / 3=\$ 5$ per share
- If the company pay out all of these earnings as dividends (cash cay, , maintaining a perpetual dividend flow of $\$ 5$ per share, then the value per share will $60=5 / \mathrm{r}$
- i.e if the required return is $12.5 \%$, then the value per share will be $\delta / .125=\$ 40$ per share.
- However, if $60 \%$ of the earnings ( $\$ 15$ million) is reinvested, then the value of the firm's assets will increase by $0.60 \times \$ 15$ million $=\$ 9$ million or by $90 \%$
- Now endowed with $9 \%$ more assets, the company earns $9 \%$ phore income, and pays out $9 \%$ higher dividends, i.e. the growth rate of the dividend $w \sqrt{4} \mathbf{~ b e}$.
- Growth rate of the dividend,
- $\mathrm{g}=$ ROE $\times$ Earnings Retention Ratio $=0.15 \times 0.00=0.09=9 \%$
- Value of the company per share will be, $\mathrm{P}_{0}=\mathbf{8} /(\mathrm{r}-\mathrm{g})=\$ 2 /(.125-0.09)=\$ 57.14$
- When growth prospects decided to redree current dividends and reinvest some of its earnings in new investments, its stock price increased. The increase in the stock price reflects the fact that the planned incoitments provide an expected rate of return greater than the required rate.
- In other words the investmepropportunities should have positive net present value. The value of the firm rises by NPV of these investment opportunities. This net present value is called Net Present Vailue of Growth Opportunities (NPVGO).
- Value or Price per stare, $\mathrm{P}_{0}=$ No-growth value per share + NPVGO
- $\$ 57.14=5 / .125$ 〒PVGO or, $\mathrm{NPVGO}=\$ 17.14$ per share.

Example: $A B C$ company is run by entrenched management that insists on reinvesting $\mathbf{6 0 \%}$ of itsearnings in projects that provide an ROE of $\mathbf{1 0 \%}$, despite the fact that the firm's required rate of return or capitalization rate, $r=15 \%$. The firm's year end dividend will be $\$ 2$ per share, paid out of earnings of $\$ 5$ per share. At what price will the stack sell? What is the net present value of growth opportunities? Why would such a firm be a takeover target for another firm?
Sol: Given current management's investment policy, the dividend growth rate will be, $\mathrm{g}=\mathrm{ROE}$ x Retention Ratio $=0.10 \times 0.60=0.06=6 \%$

- The stock price should be $=\$ 2 /(0.15-0.06)=\$ 22.22$
- $\mathrm{NPVGO}=\$ 22.22-(\$ 5 / 0.15)=\$ 22.22-\$ 33.33=-\$ 11.11$
- NPVGO is negative because the net present value of the firm's projects is negative. The rate of return from these projects is less than the required return or opportunity cost of capital.
- Such a firm would be subject to takeover, because another firm could buy the firm for the market price of $\$ 22.22$ per share and increase the value of the firm by changing its investment policy (pay out all earnings; value $=\$ 5 / 0.15=\$ 33.33$ ).


## Present Value of Operating Cash Flows:

- In this model, we are deriving the value of the total firm because we are discounting the total operating cash flows prior to the payment of interest to the debt holders. Therefore, once we estimate the value of the total firm, we must subtract the value of the debt to arrive at an estimate of the value of the firm's equity.

$$
V_{0}=\sum_{t=1}^{\infty} \frac{O C F_{t}}{(1+W A C C)^{t}}
$$

- If we are dealing with a mature firm whereby its operating cash flows havereched a stage of stable growth, we can adapt the constant growth DDM as follows:

$$
V_{0}=\frac{O C F_{t}}{W A C C-g_{O C F}}
$$

## Free Cash Flow Valuation Method:

- An alternative approach to the dividend discount moded values the firm using free cash flow, that is, cash flow available to the firm or its equiteholders net of capital expenditures. This approach is particularly useful for firms that payo dividends, for which the dividend discount model would be difficult to implementi ut free cash flow model may be applied to any firm and can provide useful insights about firm value beyond the DDM.
- One approach is to discount the free cash for the firm (FCFF) at the weighted average cost of capital to obtain the value of the firm, and subtract the then-existing value of the debt to find the value of equity.
- Another approach is to focus fro (thl the start on the free cash flow to equity holders (FCFE), discounting those directly at cost of equity to obtain the market value of equity.
- The free cash flow to the firm is the after tax cash flow that accrues from the firm's operations, net of investments in capital, and net working capital. It include cash flows available to both debernd equity holders.
- FCFF $=\operatorname{EBIT}\left(11_{2}\right)+$ Depreciation-Capital Expenditures-Increase in NWC
- FCFE $=$ FCFFAlnterest Expense $\left(1-t_{\mathrm{c}}\right)+$ Increase in net debt

$$
\text { Value }=\sum_{t=1}^{T} \frac{F C F F_{t}}{(1+W A C C)^{t}}+\frac{V_{t}}{(1+W A C C)^{t}} ; \text { W here } V_{t}=\frac{F C F F_{t+1}}{W A C C-g}
$$

- To thd equity value, we subtract the existing market value of debt from the derived value Jot the firm. Alternatively, we can discount free cash flow to equity (FCFE) at the cost of equity.

$$
\text { V alue of Equity }=\sum_{t=1}^{T} \frac{F C F E_{t}}{\left(1+r_{e}\right)^{t}}+\frac{V_{t}}{\left(1+r_{e}\right)^{t}} \text {; W here } V_{t}=\frac{F C F E_{t+1}}{r_{e}-g}
$$

- As in the dividend discount model, free cash flow models use a terminal value to avoid adding the present values of an infinite sum of cash flows. The terminal value may simply be the present value of a constant growth perpetuity.


## Earnings Multiplier or P/E Ratio Model:

- Many investors prefer to estimate the value of common stock using an earnings multiplier model.
- We know, Value or Price per share, $\mathrm{P} 0=$ No-growth value per share + NPVGO

$$
\begin{aligned}
& P_{0}=\frac{E P S_{1}}{r}+N P V G O \\
& \text { P/E ratio }=\frac{P}{E P S}=\frac{1}{E P S}\left[\frac{E P S_{1}}{r}+N P V G O\right]=\frac{1}{r}+\frac{N P V G O}{E P S} \\
& =\frac{1}{r}\left[1+\frac{N P V G O}{E P S / r}\right]=\frac{1}{r}\left[1+\frac{N P V G O}{\mathrm{No} \text { growth value of the firm }}\right] \\
& \text { if NPVGO=0 then P/E ratio }=\frac{1}{r} \\
& P_{0} / E_{1}=\frac{D_{1}}{r-g} / E_{1}=\frac{D_{1} / E_{1}}{r-g}=\frac{\mathrm{DividendPayout} \mathrm{R} \mathrm{atio}}{r-g}=\frac{1-b}{r-R O E \times \mathrm{x}}
\end{aligned}
$$

- Here, $\mathrm{b}=$ earnings retention ratio

Example: If we assume a stock has an expected dividend payeut of $50 \%$, a required rate of return of $\mathbf{1 2 \%}$, and an expected growth rate for dividengs of $\mathbf{9 \%}$, then find the $\mathbf{P} / \mathbf{E}$ ratio and the value of the stock per share for a current EBS of \$2.

$$
P_{0} / E_{1}=\frac{\text { DividendPayout Ratio }}{r-g}=\frac{0.50}{0.12-0.09}=16.7
$$

Here, $E_{0}=\$ 2$ and $E_{1}=\$ 2 x 1.09=\$ 2.18$
The value or Price per share will be, $P_{0}=16 / 7 x .18=\$ 36.41$
Price-Book Value Ratio Method: This ratio has been widely used by analysts in the banking industry as a measure of relative alue. The book values of banks are considered good indicators of value because mostbank assets are liquid assets such as bonds and commercial loans.

$$
P / B V=\frac{P_{t}}{B V_{t+1}}
$$

Price-Sales Ratid Yethod: Advocates of this method believe that strong and consistent sales growth is a regelitement for a growing company. Although, they note the importance of an above averas profit margin, the growth process must begin with sales. In addition, given all the datar of the balance sheet and income statement, sales information is subject to less manipy ation than any other data item.

$$
P / S=\frac{P_{t}}{S_{t+1}}
$$

## Equity Valuation Problems

Question 1: A company promises to pay a fixed dividend of $\$ 10$ per preferred share. If your required return from investment is $15 \%$ then how much should you offer to purchase this share?

Question 2: ABC Textiles re-invests $50 \%$ of its earning on growth projects that yield an ROE of $\mathbf{2 0 \%}$. If company's year-end projected earnings per share (EPS) is $\$ 10$ and your required return from investment is $15 \%$ then how much should you offer per share to purchase ABC Textiles' share? How much would you offer if ABC act like a cash cow? Calculate Net Present Value of Growth Opportunity (NPVGO). Explain why ABC shourd act like a cash cow when its growth projects generate returns below the required requrn from investment.

Question 3: Beximco Pharmaceuticals follows a dividend payout policy $\mathbf{0} \mathbf{~} \mathbf{4 0 \%}$ on its earnings. The company has a historical record of achieving an average Rof $15 \%$. How much should you offer per share if its projected year-end dividend is 200 taka per share and your required return from investment is $\mathbf{1 2 \%}$. Calculate Net Present Value of Growth Opportunity (NPVGO). Find NPVGO if required return increases to $\mathbf{1 6 \%}$.

Question 4: Olympic Industries follows a dividend payoûtpalicy of $\mathbf{2 5 \%}$ on its earnings. The company has a historical record of achieving an aterage ROE of $20 \%$. How much should you offer per share if its projected year-end cívidend is 100 taka per share and your required return from investment is $22 \%$. Gaculate Net Present Value of Growth Opportunity (NPVGO).

Question 5: XYZ Pharmaceuticals follows a dividend payout policy of $\mathbf{3 0 \%}$ on its earnings. The company has a historical record of achieving an average ROE of $10 \%$. Explain why XYZ can be a potenfiay target for hostile takeover by other companies if investors' required return from infestment is above $10 \%$.

Question 6: If we assume a tock has an expected dividend payout of $50 \%$, a required rate of return of $\mathbf{1 2 \%}$, and an expected growth rate for dividends of $9 \%$, then find the P/E ratio and the valugg the stock per share for a current EPS of $\$ 2$.

## Financial Management

## Capital Budgeting: Basic Investment Appraisal Techniques

## Investment Appraisal Techniques:

- ROCE (Return on Capital Employed) or Accounting Rate of Return (ARR) Method
- Payback Period Method
- Net Present Value (NPV) Method
- Internal Rate of Return (IRR) Method


## ROCE or ARR method:

- Advantages:

1. Simplicity
2. Links with other accounting measures

- Disadvantages:

1. It ignores time value of money i.e. it fails to take account ofleither the project life or the timing of cash flows.
2. It varies depending on accounting policies
3. It may ignore working capital

Example: A project involves an immediate purchase of an item of machine costing $\mathbf{\$ 1 1 0 , 0 0 0}$. It would generate annual cash flowsof $\$ 24,400$ for five years, starting in year 1 . The machine purchased would have a scrapvalue of $\$ 10,000$ in five years, when the project terminates. Depreciation is on a strarght line basis.

- Depreciation (straight line) $=\left(110,000_{\sigma} 10,000\right) / 5=\$ 20,000$


$R O C E=\frac{(240,000}{\text { Average } C \text { apital investment }=60,000} \%=7.33 \%$
- Decision tand: If the expected ROCE for the investment is greater than the target or require for hurdle rate of return then the project should be accepted.


## Accomiting Profits Vs Cash Flows

- In capital investment appraisal it is more appropriate to evaluate future cash flows than accounting profits, because:
- Profits can not be spent
- Profits are subjective
- Cash is required to pay dividends


## Cash Flows and Relevant Costs:

Capital budgeting decisions must be based on cash flows, not accounting income. The only cash flows that should be taken into consideration in capital investment appraisal are:

1) Cash flows that will happen in the future
2) Cash flows that will arise only if the capital project goes ahead
3) Cash flows are direct revenues from the project and relevant costs are future costs that will be incurred or saved as a direct consequence of undertaking the investment.
4) The relevant cash flow for a project is the additional free cash flow that the company can expect if it implements the project.
5) $\operatorname{FCFF}=\operatorname{EBIT}\left(1-t_{c}\right)+$ Depreciation-net increase in capital expenditures-increase in NWC

We should ignore the following costs:

1) Sunk costs: Costs that have already been incurred are not relevanin investment appraisal decision. For example, a company makes a non-refundable deposit as a down payment for an equipment and then reconsiders whether it wantsede equipment after all. The money that has already been spent can not be recovered and so is not relevant to the current decision about obtaining the equipment.
2) Committed costs: Costs that will be incurred anyway, wether or not a capital project goes ahead, can not be relevant to a decision about investing in the project. Fixed cost expenditures are an example of committed costo for the purpose of investment appraisal, a project should not be charged with anmount for a share of fixed costs that will be incurred anyway.
3) Non-cash expense, such as depreciation, aqnever be relevant to investment appraisal.

## Payback period method:

- The payback period is the time a profect will take to pay back the money spent on it. It is based on expected cash flows an eprovides a measure of liquidity.
- Payback period = Initial investrent / Annual cash flow
- Decision rule: Only select projects which pay back within the specified time period or choose between options the basis of the fastest payback.

Example: Find the payback period of the following project.

| Year | Cash Flow (\$,000) |
| :---: | :---: |
| 0 | $(3,100)$ |
| 1 | 1,000 |
| 2 | 900 |
| 3 | 800 |
| 4 | 500 |
| 5 | 500 |

Sol:

| Year | Cash Flow (\$,000) | Cumulative cash flows |
| :---: | :---: | :---: |
| 0 | $(3,100)$ |  |
| 1 | 1,000 | $(2,100)$ |
| 2 | 900 | $(1,200)$ |
| 3 | 800 | $(400)$ |
| 4 | 500 | 100 |
| 5 | 500 | 600 |

- Payback is between the end of year 3 and end of year 4. If we assume a constant 9 ate of cash flow through the year, we could estimate payback period will be threegears plus (400/500) of 4th year, which is 3.8 years or 3 years 10 months.
- Same way discounted payback period can also be calculated.


## Advantages:

- It is easily understood and easily calculated
- It is useful for equipment with rapidly changing technology; new plant is likely to be scrapped in a short period because of obsolescence, a quick paback is essential.
- It favors quick return because rapid project payback dads to rapid company growth, however, such policy may overlook many profitable investment opportunities because of slow payback period.
- Rapid payback also minimizes risk. It is likely fat earlier cash flows can be estimated with greater certainty.
- Rapid payback maximizes liquidity.
- It uses cash flows, not accounting profith and so is likely to produce an optimistic figure.


## Disadvantages:

- It ignores cash flows after the payback period
- It ignores the timings of the cash flows (time value of money)
- It is subjective -no definitive investment signal: there is no objective measure as to what length of time shoulare set as the minimum payback period. Investment decisions are therefore subjective


## Net Present Value (NPV) method:

- NPV requesents the extra cash flow earned (after deducting initial investment) from the proiecabove the required rate of return.
- NBV involves adding present values of all cash flows associated with the project. In NPV calculation outflows and inflows are treated as negative and positive respectively.
- If NPV is positive, it indicates that the project will earn higher return than the required return and the project is financially viable.
- If NPV is zero, it indicates that the project will break-even at the required rate of return from the project and the project is financially viable.
- If NPV is negative, it indicates that the project will fail to earn required rate of return from the project and it is not financially viable.
- If the company has two or more mutually exclusive projects under consideration then it should choose the one with the highest NPV.


## Example: Find the NPV of the project to assess whether it should be undertaken. (cost of capital is 6\%)

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cash | $(25,000)$ | 6,000 | 10,000 | 8,000 | 7,000 |  |
| PVs | $(25,000)$ | $5,660.37$ | $8,899.96$ | $6,716.95$ | $5,544.65$ | $1,821.95$ |

The project gives excess cash flows of $\$ 1,822$ above the required return. As the project has positive NPV, the project is financially viable and can be accepted.

## Advantages of NPV:

1) It considers time value of money: discounting cash flows to PV takes account of the impact of interest, inflation, and risk over time.
2) It is an absolute measure of return: the NPV of an investment reprents the actual surplus raised by the project
3) This method is based on cash flows not profits
4) It considers the whole life of the project

## Disadvantages of NPV:

1) This method is difficult to explain to managers. To $\mu$ detstand the meaning of the NPV calculated requires an understanding of discountiag. The method is not as intuitive as techniques such as payback period method.
2) It requires knowledge of the cost of capital $?$
3) It is relatively complex.

## Internal Rate of Return (IRR) methodin

- The IRR is another project apprais ${ }^{1}$ method that use discounted cash flow techniques
- The IRR represents the discounf rate at which the NPV of an investment is zero. As such it represents a break-even cost df capital.
- Decision rule: Investmen 0 ype of project (i.e. projects with initial outflow and followed by inflows from the profet): Projects with IRR greater than the cost of capital should be accepted.
- Financing type of projects (i.e. cash inflows precede cash outflows. This type of projects generate reveque first- customers pay in advance.): Accept the project when the IRR is less than the dwcount rate.
- Calcukate two NPVs for the project at two different costs of capital for which the NPV is postive and negative respectively but close to zero.
Find the IRR using the formula: $\operatorname{IR} R=L+\left[\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)\right]$
- Where, $\mathrm{L}=$ lower rate of interest; $\mathrm{H}=$ higher rate of interest

Example: Find the IRR of a potential project with NPV of $\mathbf{\$ 5 0 , 0 0 0}$ at a discount rate of $10 \%$ and $-\$ 10,000$ at a rate of $15 \%$.
$I R R=0.10+\left[\frac{50,000}{50,000-(-10,000)} x(0.15-0.10)\right]=0.10+\left[\frac{50,000}{60,000} x 0.05\right]=14.167 \%$

## Advantages of IRR:

1) It considers the time value of money.
2) IRR gives a percentage and therefore easily understood
3) This method uses cash flows not profits
4) It considers the whole life of the project

## Disadvantages of IRR:

1) It is not a measure of absolute profitability.
2) Interpolation only provides an estimate and an accurate estimate requires the use of a spreadsheet program.
3) It is fairly complicated to calculate
4) Non-conventional cash flows may give rise to multiple IRRs which meras the interpolation method can't be used.

## Modified IRR (MIRR) method:

- Suppose the cash flows from a project are ( $-\$ 100, \$ 230,-\$ 132$ ) Beeause the project has a negative cash flow, a positive cash flow, and another negatived ash flow, we say that the project's cash flows exhibit two changes of sign, or flip-flopso this type of project will have multiple IRR, such as this project have two IRR: $10 \%$ an $20 \%$.
- In a case like this the IRR does not make any sense; woich one of the two IRR should we use? There is no good reason to use one over the othe, IRR simply can not be used here.
- In theory, a project with K changes in sign in the cash flows can have up to K number of IRR.
- This method combine cash flows until ondone change in sign remains.

| Periods | Nowdot | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Invest. rate }= \\ \text { Borrowing rate }=149 \end{gathered}$ | C\$100 | \$230 | -\$132 |
|  | 1 | $-\$ 132 / 1.14=-\$ 115.789$ |  |
|  |  | \$114.21 |  |
|  | \$114.21/1.14=\$100.184 |  |  |
| NPV @, 10\% | \$0.184 |  |  |


| Periods Now(0) $\mathbf{1}$ <br> Invest. rate $=14 \%$   <br> Borrowing rate $=15 \%$   | $-\$ 100$ | $\$ 230$ | $-\$ 132$ |
| :---: | :---: | :---: | :---: |
|  |  | $-\$ 132 / 1.14=-\$ 115.789$ |  |
|  |  | $\$ 114.21$ |  |
| NPV @15\% | $-\$ 0.686$ |  |  |

$I R R=0.14+\left[\frac{0.184}{0.184-(-0.686)} x(0.15-0.14)\right]=14.21 \%$
Excel formula: MIRR(values, finance_rate, reinvest_rate)=MIRR(A1:A3,15\%,14\%) $=14.55 \%$

|  | A |
| :---: | :---: |
| 4 | $(\$ 100)$ |
| 1 | $(\$ 230$ |
| 2 | $\$ 132)$ |
| 3 | $(14.55 \%$ |
| 4 |  |

This method violates the spirit of the IRR approach. The basic rationale behind the IRR methed is that it provides a single number summarizing the merits of a project. This number is internal, intrinsic; it does not depend on discount rate. By contrast, MIRR is clearly a function of the discount rate.

## Capital Rationing using Profitability Index (PI):

- When we have multiple financially viable projects (i.e. projects with rosjitive NPV) but can not accept all due to capital shortage, we can rank projects basedoriphility index (PI) and allocate capital accordingly.
- Profitability Index $(P I)=\frac{N P V}{\text { Initial Investment }}$


## Capital Budgeting,Problems

## Question 1:


a) Find out hat payback period and discounted payback period of the above project
b) Find $X \operatorname{LPV}$ of the project at $15 \%$ hurdle rate.

Question 2

| Year | Cash Flow |
| :--- | :--- |
| $\mathbf{0}$ | $(\mathbf{2 5 , 0 0 0})$ |
| $\mathbf{1}$ | $\mathbf{6 , 0 0 0}$ |
| $\mathbf{2}$ | $\mathbf{1 0 , 0 0 0}$ |
| $\mathbf{3}$ | $\mathbf{8 , 0 0 0}$ |
| $\mathbf{4}$ | $\mathbf{7 , 0 0 0}$ |

a) Find out the payback period and discounted payback period of the above project
b) Find NPV of the project at $22 \%$ hurdle rate.

Question 3:

| Year | Cash Flow |
| :--- | :--- |
| $\mathbf{0}$ | $(\mathbf{5 0 , 0 0 0})$ |
| $\mathbf{1}$ | $\mathbf{1 8 , 0 0 0}$ |
| $\mathbf{2}$ | $\mathbf{2 5 , 0 0 0}$ |


| 3 | 20,000 |
| :--- | :--- |
| 4 | 10,000 |

a) Find out the payback period and discounted payback period of the above project
b) Find NPV of the project when required rate of return is $\mathbf{1 2 \%}$.

Question 4: Find IRR of the following project

| Discount Rate | NPV |
| :--- | :--- |
| $\mathbf{1 8 \%}$ | $\mathbf{1 8 8 4 9 . 1 6 0 2}$ |
| $20 \%$ | $\mathbf{8 1 4 4 . 7 1 8 7 9}$ |
| $22 \%$ | $\mathbf{- 1 5 1 3 . 9 6 1 9 7}$ |
| $24 \%$ | $\mathbf{- 1 0 2 3 6 . 3 7 3}$ |

Question 5: Find IRR of the following project

| Discount Rate | NPV |
| :--- | :--- |
| $20 \%$ | $\mathbf{5 1 5 9 . 0 3 6 4}$ |
| $22 \%$ | $\mathbf{2 7 4 7 . 0 1 0 5}$ |
| $24 \%$ | $\mathbf{5 6 3 . 1 8 6 1 1}$ |
| $26 \%$ | $\mathbf{- 1 4 1 5 . 7 9 5 1}$ |
| $28 \%$ | $\mathbf{- 3 2 1 0 . 5 8 7 2}$ |
| $30 \%$ | $\mathbf{- 5 1 4 1 . 7 9 5 1}$ |

Question 6: A company is considering the following 5 investmen projects and has $\$ 100,000$ available for investment. All this projects are dixisible. Determine which projects should be chosen to maximize the return to the driness.

| Projects | Initial Investment (\$000) | $\begin{gathered} \text { NPV } \\ (\$ 000) \end{gathered}$ |
| :---: | :---: | :---: |
| A | $4{ }^{4}$ | 20 |
| B | 0100 | 35 |
| C | - 50 | 24 |
| D | 60 | 18 |
| E | 50 | (10) |

Question 7: A company is considering the following 5 investment projects and has $\$ 100,000$ available for investment. All this projects are indivisible. Determine the optimal project selection.

| Projects | Initial Investment (\$000) | $\begin{gathered} \hline \text { NPV } \\ (\$ 000) \end{gathered}$ |
| :---: | :---: | :---: |
| A | 40 | 20 |
| B | 100 | 35 |
| C | 50 | 24 |
| D | 60 | 18 |

Questiop 8: A company is considering the following 5 investment projects and has $\$ 100,000$ available for investment. All this projects are divisible and project $\mathbf{A}$ and $\mathbf{C}$ are multually exclusive. Determine the optimal project selection.

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ |
| :--- | :---: | :---: |
| A | $\mathbf{4 0}$ | $\mathbf{2 0}$ |
| B | $\mathbf{1 0 0}$ | $\mathbf{3 5}$ |
| C | $\mathbf{5 0}$ | $\mathbf{2 4}$ |
| D | $\mathbf{6 0}$ | $\mathbf{1 8}$ |

## Financial Management

## Risk \& Return

- Holding Period Return (HPR): $R_{t}=\frac{P_{t}-P_{t-1}}{P_{t-1}}=\frac{P_{t}}{P_{t-1}}-1$
- Holding Period Return (HPR) including dividend: $R_{t}=\frac{P_{t}-P_{t-1}+D_{t}}{P_{t-1}}=\frac{P_{t}+D_{t}}{P_{t-1}}-1$
- Continuously compounded return : $R_{t}=\ln \left(\frac{P_{t}}{P_{t-1}}\right)$
- Expected Return: This is the return that an investor expects a security to Qarn over the next period. Off course this is only an expectation, the actual return may beeither higher or lower. An investor's expectation may simply be the average return perpermod a security has earned over the past.

W ithout probability: $E(r)=\bar{r}=\frac{\sum r}{n}$
W ith probability: $E(r)=\bar{r}=\sum r p$
Here, $r=$ return and $p=$ probability corresponding return

- Risk: The uncertainty about the future return 60 m an investment is called the risk or volatility and can be measured using variancer standard deviation of the return.

$$
\begin{aligned}
& \text { Variance, } \sigma^{2}(\text { without probabilily })=\frac{\sum\left(r_{i}-\bar{r}\right)^{2}}{n-1}=\frac{1}{n-1}\left[\sum r_{i}^{2}-\frac{\left(\sum r_{i}\right)^{2}}{n}\right] \\
& \text { Variance, } \sigma^{2}\left(\text { with probe }{ }^{\prime} \text { ility }\right)=\sum(r-\bar{r})^{2} p=\sum r^{2} p-[E(r)]^{2}
\end{aligned}
$$



- Expected return:
- Security A: 6\%
- Security B: 5\%
- Security C: $-(0.25 \times 10 \%)+(0.25 \times 0)+(0.5 \times 20 \%)=7.5 \%$
- Security D: $-(0.25 \times 20 \%)+(0.5 \times 10 \%)+(0.25 \times 40 \%)=10 \%$

Example:

| Purchase price $=\$ 100$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| State of the Economy | Prob | Year-end price | Cash Dividends | Holding Period Return (HPR) |
| Boom | 0.3 | 129.5 | 4.5 | $?$ |
| Normal | 0.5 | 110.0 | 4.0 | $?$ |
| Recession | 0.2 | 80.5 | 3.5 | $?$ |
| Expected Return $=?$ |  |  |  |  |

## Sol:

| Purchase price $=\$ 100$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| State of the Economy | Prob | Year-end price | Cash Dividends | Holding Period Return (HPR) |
| Boom | 0.3 | 129.5 | 4.5 | $(129.5+4.5-100) / 100=0.34$ |
| Normal | 0.5 | 110.0 | 4.0 | $(110+4-100) / 1080.14$ |
| Recession | 0.2 | 80.5 | 3.5 | $(80.5+3.5-100) 00=0.16$ |
| Expected Return $=(0.34 \times 0.3)+(0.14 \times 0.5)+(-0.16 \times 0.2)=014)$ |  |  |  |  |

Variance, $\sigma^{2}=\Sigma r^{2} p-[E(r)]^{2}$
$=\left[\left(0.34^{2} x 0.3\right)+\left(0.14^{2} x 0.5\right)+\left(-0.16^{2} x 0.2\right)\right]-0.14^{2}=0.03$
Standard Deviation, $\sigma=\sqrt{\sigma^{2}}=\sqrt{0.03}=0.1732$

## Portfolio Returns and Risk:

- Portfolio Expected Return: When we investion more than one asset, the portfolio expected return is the weighted average of the expected returns of all asset held in your portfolio.
- $w_{i}$ are weights, i.e. the investment proportions, and $E\left(R_{i}\right)$ expected return on individual asset.
- Expected return on a portfoli申. $E\left(R_{p}\right)=\sum_{i=1} w_{i} E\left(R_{i}\right)$
- Expected return on a portfolio (2 assets): $\mathrm{E}\left(\mathrm{R}_{p}\right)=w_{A} x R_{A}+w_{B} x R_{B}$
- Expected returnda portfolio (3 assets): $\mathrm{E}\left(\mathrm{R}_{p}\right)=w_{A} x R_{A}+w_{B} x R_{B}+w_{C} x R_{C}$
- Variance of artfolio (2 assets):

$$
\sigma_{p}^{2}=w^{2} \sigma_{A}^{2}+w_{B}^{2} \sigma_{B}^{2}+2 w_{A} w_{B} \sigma_{A B}=w_{A}^{2} \sigma_{A}^{2}+w_{B}^{2} \sigma_{B}^{2}+2 w_{A} w_{B} \sigma_{A} \sigma_{B} \rho_{A, B}
$$

$$
=\text { Portfolio Variance } ; \sigma_{A}^{2}=\mathrm{V} \text { ariance of Security } \mathrm{A}
$$

$=$ Variance of Security B; $w_{A}=$ Proportion of investment on A
$w_{B}=$ Proportion of investment on $B ; \rho_{A, B}=$ Correlation coefficient between A \& B

- Variance of a portfolio (3 assets):

$$
\begin{aligned}
& \sigma_{p}^{2}=w_{A}^{2} \sigma_{A}^{2}+w_{B}^{2} \sigma_{B}^{2}+w_{C}^{2} \sigma_{C}^{2}+2 w_{A} w_{B} \sigma_{A B}+2 w_{A} w_{C} \sigma_{A C}+2 w_{B} w_{C} \sigma_{B C} \\
& =w_{A}^{2} \sigma_{A}^{2}+w_{B}^{2} \sigma_{B}^{2}+w_{C}^{2} \sigma_{C}^{2}+2 w_{A} w_{B} \sigma_{A} \sigma_{B} \rho_{A, B}+2 w_{A} w_{C} \sigma_{A} \sigma_{C} \rho_{A, C}+2 w_{B} w_{C} \sigma_{B} \sigma_{C} \rho_{B, C}
\end{aligned}
$$

Example: Calculate the expected return on a portfolio that has one third of your wealth invested in GM and two thirds in BP.

| Year | Prob. | GM | BP | Portfolio |
| :---: | :---: | :---: | :---: | :---: |
| 2 | $1 / 4$ | 0.10 | 0.15 | 0.1333 |
|  | $1 / 4$ | -0.05 | 0.10 | 0.05 |
|  | $1 / 4$ | 0.08 | 0 | 0.0267 |
|  | $1 / 4$ | 0.15 | -0.01 | 0.0433 |
| Expected Rate of return |  |  |  |  |

- Covariance and Correlations: Returns on individual securities are related one another. Covariance is a statistic measuring the inter-relationship between two securities. Alternatively, this relationship can be restated in terms of the correlaoh between the two securities.

A sample covariance is


Correlation Coefficient $(x, y)=r_{x y}=\frac{\sigma_{x y}}{\delta^{2}}{ }^{9}$

$$
\sum x_{i} y_{i}-\frac{\left(\sum x_{i}\right)\left(\sum y_{i}\right)}{n}
$$



Example: Out of $\mathbf{\$ 1 0 0}$, if andinvestor invests $\mathbf{\$ 6 0}$ in Security A and remaining $\$ 40$ in security $B$, then find theepected return and variance of the portfolio.

| State of the Economy | Rate of Return from Security A | Rate of Return from Security B |
| :---: | :---: | :---: |
| Borr | $-20 \%$ | $5 \%$ |
| Rormal | $10 \%$ | $20 \%$ |
| Recession | $30 \%$ | $-12 \%$ |
| Depression | $50 \%$ | $9 \%$ |

$\sum x^{2}=0.39 ; \quad \sum y^{2}=0.065 ; \quad \sum x=0.7 ; \quad \sum y=0.22 ; \quad \sum x y=0.019$;
$\bar{x}=0.175 ; \quad \bar{y}=0.055 ; \quad \sigma_{x}=0.2586 ; \quad \sigma_{y}=0.115$

$$
\sum x_{i} y_{i}-\frac{\left(\sum x_{i}\right)\left(\sum y_{i}\right)}{n}
$$

Correlation Coefficient $(x, y)=r_{x y}=\frac{\sum x_{i} y_{i}-\frac{\left(\sum x_{i}\right)\left(\sum y_{i}\right)}{n}}{\sqrt{\left\{\sum x_{i}^{2}-\frac{\left(\sum x_{i}\right)^{2}}{n}\right\}\left\{\sum y_{i}^{2}-\frac{\left(\sum y_{i}\right)^{2}}{n}\right\}}}$

$$
=\frac{0.019-\frac{0.7 x 0.22}{4}}{\sqrt{\left(0.39-\frac{0.7^{2}}{4}\right)\left(0.065-\frac{0.22^{2}}{4}\right)}}=-0.1639
$$

Expected Return, $\mathrm{E}\left(\mathrm{R}_{i}\right)=w_{A} x R_{A}+w_{B} x R_{B}=0.6 x 0.175+0.4 x 0.055=0.127$
$\sigma_{p}^{2}=w_{A}^{2} \sigma_{A}^{2}+w_{B}^{2} \sigma_{B}^{2}+2 w_{A} w_{B} \sigma_{A} \sigma_{B} \rho_{A, B}$
$=0.6^{2} \times 0.2586^{2}+0.4^{2} \times 0.115^{2}+2 \times 0.6 \times 0.4 \times 0.2586 \times 0.115 \times(-0.1639)=0$ (82 385
$\sigma_{P}=\sqrt{0.02385}=0.1544=15.44 \%$

## Systematic and Non-systematic Risk:

- When an investor diversifies across assets that are not perfectly'sorfelated, the portfolio's risk is less than the weighted average of the risks of the bedividual securities in the portfolio. The risk that is eliminated by diversification is called unsystematic risk (also called unique, diversifiable, or firm specific risk). Since te market portfolio contains all the risky assets, it must be well a diversified portfolio $\boldsymbol{\text { the }}$ risk that remains and can not be diversified away, is called the systematic risk (alsocalled non-diversifiable or market risk).
- The concept of systematic risk applies to individual securities as well as to portfolios. Some securities' returns are highly correlated with@verall market returns. Examples of firms that are highly correlated with market returns are luxury goods manufacturers such as Ferrari Automobiles and Harley Davidson Motorcycles. These firms have high systematic risk (ie they are very responsive to marker
- Other firms, such as utility companies, respond very little to changes in the systematic risk factors. These firms have very Tittle systematic risk. Hence total risk (measured by standard deviation) can be broken om into its component parts: unsystematic risk and systematic risk.

Figure 4: Risk vs. Number of Portfolio Assets


## Relationship between Risk and Expected Return (CAPM: Capital Asset Pricing Model):

- Capital Asset Pricing Model (CAPM) implies that the expected return on a security is linearly related to its beta. Because the average return on the market has been higher than the average risk free rate over long periods of time, market premium is presumably positive. Thus the formula implies that the expected return on a security is positively related to its beta.

Capital Asset Pricing Model (CAPM):

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{i}\right)= \mathrm{R}_{f} \\
&+\quad \beta_{i} \quad \mathrm{x}\left[\mathrm{E}\left(\mathrm{R}_{m}\right)-\mathrm{R}_{f}\right] \\
& \mathrm{R} \text { isk-free rate }+ \text { Beta of the security } \\
& \times \mathrm{M} \text { arket } \mathrm{R} \text { isk Premium }
\end{aligned}
$$

- If $\beta=0$, that is, the expected return on the security is equal to the risk free rate. Because a security with zero beta has no relevant risk, its expected return should equal reisk-free rate.
- If $\beta=1$, that is, the expected return on the security is equal to the expegted return on the market. This makes sense because the beta of the market portfolio is (alsb) 1 .
- Security Market Line (SML) is the graphical depiction of the gaset pricing model (CAPM).
- 




- The CML uses 据 ${ }^{2}$ risk $\sigma$ on the X-axis. Hence only efficient portfolios will plot on the CML. On the gther hand, the SML uses beta (systematic risk) on the X-axis. So in a CAPM world, all preperly priced securities and portfolios of securities will plot on the SML.


## Calculateand Interpret Beta:

- T@U sensitivity of an asset's return to the return on the market index in the context of the market model is referred to as its beta. Beta measures the responsiveness of a security to movements in the market portfolio.
- The contribution of a security to the variance of a diversified portfolio is best measured by beta. Therefore, beta is the proper measure of the risk of an individual security for a diversified investor.
- Beta measures the systematic risk of a security. Thus, diversified investors pay attention to the systematic risk of each security. However, they ignore the unsystematic risk of individual securities, since unsystematic risks are diversified away in a large portfolio.

$$
\beta_{i}=\frac{\operatorname{Cov}(i, m)}{\sigma_{m}^{2}}=\frac{\rho(i, m) \sigma_{i} \sigma_{m}}{\sigma_{m}^{2}}=\rho(i, m) \frac{\sigma_{i}}{\sigma_{m}}
$$

- One useful property is that the average beta across all securities, when weighted by the proportion of each security's market value to that of the market portfolio, is 1 . That is, the beta of the market portfolio is 1 . For aggressive securities beta $>1$; for defensive securities beta $<1$ and for neutral securities beta=1
Example: The expected return on the market is $15 \%$, the risk free rate is $8 \%$, and the beta for the stock $A$ is 1.2. Compute the rate of return that would be expected (required) on this stock.

$$
E\left(R_{A}\right)=0.08+1.2(0.15-0.08)=0.164=16.4 \% ; \text { Here, } \beta_{A}>1 ; E\left(R_{A}\right)>E\left(R_{M}\right)
$$

Example: The expected return on the market is $15 \%$, the risk free rate is $8 \%$, and the beta for the stock $B$ is 0.8 . Compute the rate of return that would be expected (required) on this stock.

$$
E\left(R_{B}\right)=0.08+0.8(0.15-0.08)=0.136=13.6 \% ; \text { Here, } \beta_{B}<1 ; E\left(R_{B}\right)<E\left(R_{M}\right)
$$

Example: Acme, Inc., has a capital structure that is $\mathbf{4 0 \%}$ debt and $\mathbf{6 0 \%}$ equity. The expected return on the market is $12 \%$, and the risk free rate is $4 \%$. What ©iscount rate should an analyst use to calculate the NPV of a project with an equity bgeta of 0.9 if the firm's after tax cost of debt is $\mathbf{5 \%}$

- Sol: The required return on equity for this project is, $0.04+0.9(0.12-0 .(04)=0.112=11.2 \%$
- The appropriate discount rate is a weighted average of the costs cos lebt and equity for this project, $0.40 \mathrm{x} 0.05+0.6 \mathrm{x} 0.112=0.0872=8.72 \%$

Example: Suppose risk free rate and market return Ace/7\% and $15 \%$ respectively. Compute the expected and required return on each stock, determine whether each stock is undervalued, overvalued or properly valued and outline an appropriate trading strategy.
Forecast Data

| Stock | Price Today | E(Price) in 1 Year | E(Dividend) in 1 Year | Beta |
| :---: | :---: | :---: | :---: | :---: |
| A | $\$ 25$ | $\$ 27$ | $\$ 1.00$ | 1.0 |
| B | 40 | 45 | 2.00 | 0.8 |
| C | 15 | 17 | 0.50 | 1.2 |
|  | Forecast Return |  |  |  |
| Stock | $(\$ 27-\$ 25+\$ 1) / \$ 25=12.0 \%$ | $0.07+(1.0)(0.15-0.07)=15.0 \%$ |  |  |
| A | $(\$ 45-\$ 40+\$ 2) / \$ 40=17.5 \%$ | $0.07+(0.8)(0.15-0.07)=13.4 \%$ |  |  |
| B | $(\$ 17-\$ 15+\$ 0.5) / \$ 15=16.6 \%$ | $0.07+(1.2)(0.15-0.07)=16.6 \%$ |  |  |
| C |  |  |  |  |

Stock $A$ is overvalued. It is expected to earn $12 \%$, but based on its systematic risk it should earn $15 \%$. It plots below the SML.
Stock B is undervalued. It is expected to earn $17.5 \%$, but based on its systematic risk it should earn $13.4 \%$. It plots above the SML.
Stock C is properly valued. It is expected to earn $16.6 \%$, and based on its systematic risk it should earn $16.6 \%$. It plots on the SML.

## The appropriate trading strategy is:

- Short sell Stock A.
- Buy Stock B.
- Buy, sell, or ignore Stock C.


## Assumptions of CAPM:

- Investors make their investment decisions according to mean-variance rule
- Investors can borrow or lend at the risk-free rate.
- No transactions cost for diversification

- Investors are price takers and have homogeneous expectations or beliefs regalaing future expected returns, variances and covariance.
- All assets are marketable and perfectly divisible.
- There are no market imperfections such as taxes, regulations, or delestrictions on short selling.


## Arbitrage Pricing Theory (APT) or Multi-factor model vgSingle Factor or Single Index Model or CAPM:

- APT has three major assumptions: 1) Capital marlets are perfectly competitive 2) Investors always prefer more wealth to less wealth with certainty 3) The stochastic process generating asset returns can be represented a K factors model.
- Multi-factor models to predict expected returns commonly use macro economic factors such as GDP growth, inflation, consamer confidence, along with fundamental factors such as earnings, earnings growth, firg size and research expenditures. The general form of a multi factor model with k factof is as follows:
- Expected Return, $\mathrm{E}\left(\mathrm{r}_{i}\right)=\mathbb{R}+\beta_{1} \mathrm{E}\left(\right.$ factor $\left._{1}\right)+\beta_{2} \mathrm{E}\left(\mathrm{factor}_{2}\right)+\ldots \ldots .+\beta_{k} \mathrm{E}\left(\right.$ factor $\left._{k}\right)$
- Single index model in contrast is a single factor or market model. The only factor is the expected excess returo the market portfolio (market index). The form of the single index model is as follo
- ExpectedRetion, $\mathrm{E}\left(\mathrm{r}_{i}\right)=R_{f}+\beta_{i}\left[\mathrm{E}\left(\mathrm{r}_{m}\right)-R_{f}\right]$ or, $\mathrm{E}\left(\mathrm{r}_{i}\right)-R_{f}=\beta_{i}\left[\mathrm{E}\left(\mathrm{r}_{m}\right)-R_{f}\right]$
- In this case the beta for asset i is a measure of how sensitive the excess return on asset i is to the ess return on the overall market portfolio.


## Risk \& Return Problems

Question 1: Suppose you have invested $\mathbf{\$ 3 0 , 0 0 0}$ in the following four stocks:

| Security | Amount Invested (\$) | Beta |
| :---: | :---: | :---: |
| Stock A | $\mathbf{5 , 0 0 0}$ | $\mathbf{0 . 7 5}$ |
| Stock B | $\mathbf{1 0 , 0 0 0}$ | $\mathbf{1 . 1 0}$ |
| Stock C | $\mathbf{8 , 0 0 0}$ | $\mathbf{1 . 3 6}$ |
| Stock D | $\mathbf{7 , 0 0 0}$ | $\mathbf{1 . 8 8}$ |

The risk-free rate is $4 \%$ and the expected return on the market portfolio is $15 \%$. Based on the CAPM, what is the expected return on the above portfolio?

Question 2: You have been provided the following data on the securities of three firms and the market:

| Security | $\bar{R}$ | $\sigma_{i}$ | $\rho_{i, m}$ | $\boldsymbol{o}_{i}^{\rho_{i}^{\circ}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Stock A | $\mathbf{0 . 1 3}$ | $\mathbf{0 . 1 2}$ | (i) | $\mathbf{0 . 9 0}$ |
| Stock B | $\mathbf{0 . 1 6}$ | (ii) | $\mathbf{0 . 4}$ | $\mathbf{1 . 1 0}$ |
| Stock C | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 2 4}$ | $\mathbf{0 . 7 5}$ | (iii) |
| Market | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 1}$ | (iv) | (v) |
| Risk-free Asset $\left(r_{f}\right)$ | $\mathbf{0 . 0 5}$ | (vi) | (qui) | (viii) |

Assume the CAPM holds true:
a) Fill in the missing values in the table.
b) Provide an evaluation of the investment perfoymance of the three firms
c) What is your investment recommendationzWhy?

Question 3:
a) What is the expected return and vakiance on an equally weighted portfolio of the following three stocks?
$\left.\begin{array}{|c|c|c|c|c|}\hline \text { State of } \\ \text { Economy } & \text { Probability of © } \\ \text { of Economy }\end{array}\right)$
b) What is the expected return and variance of a portfolio invested $20 \%$ each in $A$ and $B$, and the remaining $\mathrm{C} C$ ?

Question 4: Xour portfolio is invested $\mathbf{3 0 \%}$ each in stock $A$ and $C$ and the remaining in $B$ ? What isthe expected return, variance and standard deviation of the portfolio?

| State of <br> Economy | Prob. | Expected Return |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | B | C |  |
| Boom | $\mathbf{0 . 2 0}$ | $\mathbf{0 . 3 0}$ | $\mathbf{0 . 4 5}$ | $\mathbf{0 . 3 3}$ |
| Good | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 1 2}$ | $\mathbf{0 . 1 0}$ | $\mathbf{0 . 1 5}$ |
| Poor |  | $\mathbf{0 . 0 1}$ | $\mathbf{- 0 . 1 5}$ | $\mathbf{- 0 . 0 5}$ |
| Bust | $\mathbf{0 . 1 5}$ | $\mathbf{- 0 . 0 6}$ | $\mathbf{- 0 . 3 0}$ | $\mathbf{- 0 . 0 9}$ |

Question 5: You own a stock portfolio invested $\mathbf{2 5 \%}$ in stock $\mathrm{Q}, \mathbf{2 0 \%}$ in stock R, $\mathbf{1 5 \%}$ in stock $S$, and the remaining in stock $T$ ? The betas for these four stocks are $0.75,1.90,1.38$ and 1.16 respectively. What is the portfolio beta?
Question 6: You own a portfolio equally invested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.85 and the total portfolio is equally as risky as the market, what must the beta be for the other stock in your portfolio?

Question 7: A stock has a beta of $\mathbf{1 . 2 5}$, the expected return on the market is $\mathbf{1 2 \%}$ and the risk free rate is $\mathbf{5 \%}$. According to CAPM, what must the expected return on this stock be?

Question 8: A stock has an expected return of $14.2 \%$, the risk free rate is $4 \%$ and the market risk premium is $\mathbf{7 \%}$. According to CAPM, what must the beta of this stock be?

Question 9: A stock has an expected return of $10.5 \%$, its beta is 0.73 , and the risk free rate is $5.5 \%$. According to CAPM, what must the expected return on market be?

Question 10: A stock has an expected return of $\mathbf{1 6 . 2 \%}$, a beta of 1.75 , and the expecter return on the market is $11 \%$. According to CAPM, what must the risk free rate be?

Question 11: A stock has a beta of 0.92 , and an expected return of $10.3 \%$. A risk free asset currently earns $5 \%$.
a) Based on CAPM, what is the expected return on a portfolio thatisfequally invested in the two assets?
b) If a portfolio of the two assets has a beta of 0.50 , what arecthe portfolio weights?
c) If a portfolio of the two assets has an expected return of $\mathscr{O}$, what is its beta?
d) If a portfolio of the two assets has a beta of 1.84, whatgre the portfolio weights? How do you interpret the weights for the two assetsin this case? Explain.

Question 12: Stock $Y$ has a beta of 1.35 and an expected return of $14 \%$. Stock $Z$ has a beta of $\mathbf{0 . 8 5}$ and an expected return of $\mathbf{1 1 . 5 \%}$. The market risk premium is $\mathbf{6 . 8 \%}$.
a) If the risk free rate is $5.5 \%$, are these stocks correctly priced?
b) What would the risk free rate have tobe for the two stocks to be correctly priced?

Question 13:
a) If your portfolio is invested 40 each in $A$ and $B$ and the remaining in $C$, what is the portfolio expected retura, variance and the standard deviation?

| State of <br> Econdmy | Prob. | Expected Return |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | B | C |  |
| Boom | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 2 0}$ | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 6 0}$ |
| Normal | $\mathbf{0 . 4 0}$ | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 1 2}$ | $\mathbf{0 . 0 5}$ |
| Bust |  | $\mathbf{0 . 0 1}$ | $\mathbf{- 0 . 2 5}$ | $\mathbf{- 0 . 5 0}$ |

b) If the expegted T-bill rate is $3.80 \%$, what is the expected risk premium on the portfolise
c) If the expected inflation rate is $3.50 \%$, what are the approximate and exact expected real returns on the portfolio? What are the approximate and exact expected real risk premiums on the portfolio?
Question 14: You want to create a portfolio equally as risky as the market, and you have $\$ 1,000,000$ to invest. Given this information, fill in the rest of the following table:

| Asset | Investment (\$) | Beta |
| :--- | :---: | :---: |
| Stock A | $\mathbf{1 8 0 , 0 0 0}$ | $\mathbf{0 . 7 5}$ |
| Stock B | $\mathbf{2 9 0 , 0 0 0}$ | $\mathbf{1 . 3 0}$ |
| Stock C |  | $\mathbf{1 . 4 5}$ |
| Risk Free Asset |  |  |

Question 15: You have $\mathbf{\$ 1 0 0 , 0 0 0}$ to invest in a portfolio containing stock $X$, stock $Y$, and a risk-free asset. You must invest all of your money. Your goal is to create a portfolio that has an expected return of $10.7 \%$ and that has only $\mathbf{8 0 \%}$ of the risk of the overall market.

If $X$ has an expected return of $17.2 \%$ and a beta of 1.8 , $Y$ has an expected return of $8.75 \%$ and a beta of 0.5 , and the risk free rate is $7 \%$, how much money will you invest in stock X?

Question 16: You have invested $40 \%$ of your capital in stock $A$ and the remaining in stock B. Historical return over last 5 years shows the following pattern:

| Year | Stock A <br> $(\mathbf{x})$ | Stock B <br> $(\mathbf{y})$ |
| :---: | :---: | :---: |
| 2014 | $\mathbf{2 5 \%}$ | $\mathbf{1 5 \%}$ |
| 2013 | $\mathbf{3 2 \%}$ | $\mathbf{1 0 \%}$ |
| 2012 | $\mathbf{1 5 \%}$ | $\mathbf{3 0 \%}$ |
| 2011 | $\mathbf{2 0 \%}$ | $\mathbf{- 5 \%}$ |

Find the expected portfolio return and variance.
Question 17: Security $\mathbf{F}$ has an expected return of $\mathbf{1 0 \%}$ and a standard deriation of $\mathbf{2 6 \%}$ per year. Security $G$ has an expected return of $17 \%$ and a standard deviation of $58 \%$ per year.
a) What is the expected return on a portfolio composed of 30 of security $F$ and $70 \%$ of security G?
b) If the correlation between the returns of security $F$ and Security $G$ is 0.25 , what is the standard deviation of the portfolio described in patt (a)?

Question 18: Suppose the expected returns of stod $A$ and $B$ are $13 \%$ and $19 \%$ respectively whereas standard deviations are 38 and $62 \%$ respectively. Calculate expected return and standard deviation of a portholio that is composed of $45 \%$ of $A$ and $55 \%$ of $B$ when the correlation between the retirns on $A$ and $B$ is 0.5 .

Question 19: You have been provided the following data on the securities of three firms and the market:

| Security | $\bar{R} \cdot Q^{8}$ | $\sigma_{i}$ | $\rho_{i, m}$ | $\beta{ }_{i}$ |
| :---: | :---: | :---: | :---: | :---: |
| Stock A | 0.10 | 0.27 | (i) | 0.85 |
| Stock B | -10. 14 | (ii) | 0.5 | 1.50 |
| Stock C | 0.17 | 0.7 | 0.35 | (iii) |
| Market $1 \sim$ | 0.12 | 0.2 | (iv) | (v) |
| Risk-free Asset ( ${ }_{f}$ ) | 0.05 | (vi) | (vii) | (viii) |

Assume the CAPM holds true:
a) Fill int the missing values in the table.
b) Ane these stocks A, B and C correctly priced?
c) What is your investment recommendation? Why?

Question 20: The market portfolio has an expected return of $\mathbf{1 2 \%}$ and a standard deviation of $\mathbf{1 9 \%}$. The risk free rate is $5 \%$.
a) What is the expected return on a well diversified portfolio with a standard deviation of 7\%?
b) What is the standard deviation of a well diversified portfolio with an expected return of $20 \%$ ?

Question 21: A portfolio that combines the risk free asset and the market portfolio has an expected return of $\mathbf{9 \%}$ and a standard deviation of $13 \%$. The risk free rate is $5 \%$, and the expected return on the market portfolio is $\mathbf{1 2 \%}$. Assume the CAPM holds. What expected
rate of return would a security earn if it had a 0.45 correlation with the market portfolio and a standard deviation of $40 \%$ ?

Question 22: Suppose the risk free rate is $\mathbf{4 . 8 \%}$ and the market portfolio has an expected return of $\mathbf{1 1 . 4 \%}$. The market portfolio has a variance of $\mathbf{0 . 0 4 2 9}$. Portfolio $Z$ has a correlation coefficient with the market of 0.39 and a variance of 0.1783 . According to the CAPM, what is the expected return on portfolio $Z$ ?

Question 23: Suppose market risk premium is $\mathbf{7 . 5 \%}$ and the risk free rate is 4\%. Which of the following stock has the most systematic risk? Which one has the most unsystematic risk? Which stock is riskier? Explain.

| State of Economy | Probability | Return from Stock A | Return from Stock P $\boldsymbol{7}$ |
| :---: | :---: | :---: | :---: |
| Recession | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 0 9}$ | $\mathbf{- 0 . 3 0} \quad 0^{\mathbf{0}}$ |
| Normal | $\mathbf{0 . 5 5}$ | $\mathbf{0 . 4 2}$ | $\mathbf{0 . 1 2} \quad \mathbf{0}^{\circ}$ |
| Good | $\mathbf{0 . 3 0}$ | $\mathbf{0 . 2 6}$ | $\mathbf{0 . 4 4 0}$ |

Question 24: Suppose you observe the following situation:

| Security | Beta | Expected Retuch |
| :---: | :---: | :---: |
| A | 1.4 | 0.156 |
| B | 0.9 | 0.18 |

Assume these securities are correctly priced. Based on the EAPM, what is the expected return on the market? What is the risk free rate?

# Financial Management <br> Cost of Capital 

## Cost of Capital:

- If a firm's capital is financed with equity, then the cost of capital would be the required rate of return on equity. However, most firms use different types of capital, and due to differences in risk, these different securities have different required rates of return.
- The cost of capital used to analyze capital budgeting decisions should be a weighted average cost of the various capital sources (WACC: Weighted Average Cost of Capital)

$$
W A C C=w_{d} r_{d}(1-T)+w_{p} r_{p}+w_{e} r_{e}
$$

- Cost of debt $=r_{d}(1-T)$
- Cost of Preferred stock, $\mathbf{r}_{\mathbf{p}}=$ Preferred Dividend / Price
- Cost of Common Equity, $\mathbf{r}_{\mathrm{e}}$ : Cost of equity can be estimated using the following methods: Capital Asset Pricing Model (CAPM): $\mathrm{E}\left(\mathrm{R}_{i}\right)=\mathrm{R}_{f}+\beta_{i}\left[\mathrm{E}\left(\mathrm{R}_{m}\right)-\mathrm{R}_{f}\right]$
Discounted cash flow method:

$$
P_{0}=\frac{D_{1}}{r_{e}-g} \text {; i.e. } r_{e}=\frac{D_{1}}{P_{0}}+g=\text { Div yield }+ \text { Grow th or Capital gain }
$$

## Cost of Capital Problems and Salutions

Question 1: The Dybvig Corporation's common stockyas a beta of 1.15 . If the risk free rate is $4.5 \%$ and the expected return on the marret is $11 \%$, what is Dybbig's cost of equity capital?

Question 2: The Devon Co. just issued a dividend of \$2.4 per share on its common stock. The company is expected to maintain a constant $5.5 \%$ growth rate in its dividends indefinitely. If the stock sells for $\$ 52$ hare, what is the company's cost of equity?

Question 3: Stock in Country Road Industries has a beta of 0.85 . The market risk premium is $8 \%$, and T-bils are currently yielding $5 \%$. The company's most recent dividend was $\$ 1.6$ per shate, and dividends are expected to grow at a $6 \%$ annual rate indefinitely. If the stac sells for $\$ 37$ per share, what is your best estimate of the company's cost of equity?

Question 4: Adyfrice Inc., is trying to determine its cost of debt. The firm has a debt issue outstanding (x)th 12 years to maturity that is quoted at $95 \%$ of face value. The issue makes semimnual payments and has a coupon rate of $\mathbf{8 \%}$ annually. What is Advance's pretax cost of debt? If the tax rate is $35 \%$, what is after tax cost of debt?

Question 5: Shanken Corp. issued a 30 years, $7 \%$ semiannual bond 7 years ago. The bond currently sells for 108 percent of its face value. The company's tax rate is $\mathbf{3 5 \%}$.
a) What is the pretax cost of debt?
b) What is the after tax cost of debt?
c) Which is more relevant, the pretax or the after tax cost of debt? Why?
d) Suppose the book value of the debt issue is $\mathbf{\$ 6 0}$ million. In addition, the company has a second debt issue on the market, a zero coupon bond with seven years left to maturity; the book value of this issue is $\$ \mathbf{8 0}$ million and the bonds sell for $\mathbf{7 3 \%}$ of par. What is the company's total book value of debt? The total market value? What is your best estimate of the after tax cost of debt now?

Question 6: Fama's LIamas has a weighted average cost of capital of $\mathbf{9 . 8 \%}$. The company's cost of equity is $15 \%$, and its cost of debt is $7.5 \%$. The tax rate is $\mathbf{3 5 \%}$. What is Fama's debt-equity ratio?

Question 7: Filer manufacturing has 7.5 million shares of common stock outstanding. The current share price is $\$ 49$, and the book value per share is $\$ 4$. Filer Manufacturing also has two bond issues outstanding. The first bond issue has a face value of $\$ 60$ million, pays $\mathbf{7 \%}$ semi-annual coupon and sells for $\mathbf{9 3 \%}$ of par. The second issue has a face value of $\mathbf{\$ 5 0}$ million, pays $\mathbf{6 . 5 \%}$ semiannual coupon, and sells for $\mathbf{9 6 . 5 \%}$ of par. The first issue matures in 10 years, the second in 6 years.
a) What are Filer's capital structure weights on a book value basis?
b) What are Filer's capital structure weights on a market value basis?
c) Which are more relevant, the book value or market value weights? Why?
d) Suppose the company's stock has a beta of 1.2 , the risk free rate is $5.2 \%$, and the market risk premium is $\mathbf{7 \%}$. Assume that the overall cost of debt is $\boldsymbol{G}$ e weighted average of the two outstanding debt issues. The tax rate is 35 g . What is the company's WACC?

Question 8: Kose Inc., has a target debt equity ratio of 0.65 . Its tax rate is $\mathbf{3 5 \%}$.
a) If Kose's cost of equity is $15 \%$, what is its pretax cost debt?
b) If instead you know that the after tax cost of dêth is $6.4 \%$, what is the cost of equity?

Question 9: Given the following information for forntington Power Co. find the WACC. Assume the company's tax rate is $35 \%$, marketisk premium 7\% and risk free rate $6 \%$.

| Debt | $5,0008 \%$ <br> remaining, selling for 103\% of par |
| :--- | :--- |
| Common stock | $\mathbf{1 6 0 , 0 0 0}$ shares outstanding, selling for \$57 per share; the beta is 1.10. |

Question 10: Titan Mining Corporation has 8.5 million shares of common stock outstanding and $200,0007.5 \%$ semiannual bonds outstanding with par value of $\$ 1,000$ each. The common stock cuyrently sells for $\$ 34$ per share and has a beta of 1.2 and the bonds have 15 years to phaturity and sell for $93 \%$ of par. The market risk premium is $\mathbf{7 \%}$, T-bills are yielding $6 \%$, and company's tax rate is $35 \%$.
a) What is the finm's market value capital structure?
b) If Titan fining is evaluating a new investment project that has the same risk as the firnt typical project, what rate should the firm use to discount the project's cash 40 ws ?

Question 11: Suppose your company needs $\$ 20$ million to build a new assembly line. Your target debt-equity ratio is 0.75 . The floatation cost for issuing new equity is $8 \%$, but the floatation cost for debt is only $5 \%$. Your boss has decided to fund the project by borrowing money because the floatation costs are lower and the needed funds are relatively small.
a) What do you think about the rationale behind borrowing the entire amount?
b) What is your company's weighted average floatation cost, assuming all equity is raised externally?
c) What is the true cost of building the new assembly line after taking floatation costs into account?

Question 12: Southern Alliance Company needs to raise $\$ 45$ million to start a new project and will raise the money by selling new bonds. The company will generate no internal equity for the foreseeable future. The company has a target capital structure of $\mathbf{6 5 \%}$ common stock, $\mathbf{5 \%}$ preferred stock, and $\mathbf{3 0 \%}$ debt. Floatation costs for issuing common stock are $9 \%$, for new preferred stock $6 \%$, and for new debt $3 \%$. What is the true initial cost figure Southern should use when evaluating its project?

Question 13: Och, Inc. is considering a project that will result in initial after tax cash savings of $\$ 3.5$ million at the end of the first year, and these savings will grow at a rate of $5 \%$ per year indefinitely. The firm has a target debt-equity ratio of 0.65 , a cost of equity of $15 \%$, and an after tax cost of debt of $5.5 \%$. The cost saving proposal is somewhat riskier than the usual projects the firm undertakes; management uses the subjective approach and applies an adjustment factor of $+\mathbf{2 \%}$ to the cost of capital for sucPrisky projects. Under what circumstances should Och take on the project?

Question 14: Goodbye Inc. recently issued new securities to finance a newg show. The project cost $\$ 15$ million, and the company paid $\$ 850,000$ in floatation costs. In addition, the equity issued had a floatation cost of $\mathbf{7 \%}$ of the amount raised, whereas the debt issued had a floatation cost of $3 \%$ of the amount raised. If Good矿e issued new securities in the same proportion as its target capital structure, what is tre company's target debtequity ratio?

Question 15: Photochronograph Corporation ( $\mathrm{P} \mathrm{O}^{\circ}$, manufactures time series photographic equipment. It is currently at its target debt-equity ratio of 0.7 . It's considering building a new $\$ 45$ million manufactuping facility. This new plant is expected to generate after tax cash flows of $\$ 6.2$ million a year in perpetuity. The company raises all equity from outside financing. There are three financing options:

1) A new issue of common stock: The floatation costs of the new common stock would be $8 \%$ of the amount raised. Theorequired return on the company's new equity is $14 \%$.
2) A new issue of 20 year bonds: The floatation costs of the new bonds would be $4 \%$ of the proceeds. If the company issues these new bonds at an annual coupon rate of $8 \%$, they will sell at par.
3) Increased use of accounts payable financing: Because this financing is part of the company's ongoing daily business, it has no floatation costs, and the company assigns it areost that is the same as the overall firm WACC. Management has a target rafie of accounts payable to long term debt of 0.20 (assume there is no differepe between the pretax and after tax accounts payable cost.)
What is the 1 PV of the new plant? Assume that PC has a $35 \%$ tax rate.
Question 16: Trower Corp. has a debt equity ratio of 1.2. The company is considering a new Plant that will cost $\$ 145$ million to build. When the company issues new equity, it incurs a floatation cost of $8 \%$. The floatation cost on new debt is $\mathbf{3 . 5 \%}$.
a) What is the initial cost of the plant if the company raises all equity externally?
b) What if it typically uses $\mathbf{6 0 \%}$ retained earnings?
c) What if all equity investments are financed through retained earnings?

Question 17: Suppose a firm has both a current and a target debt-equity ratio of $\mathbf{0 . 6}$, a cost of debt of $5.15 \%$, and a cost of equity of $10 \%$. The corporate tax rate is $34 \%$. The firm is considering taking on a warehouse renovation costing $\mathbf{\$ 6 0}$ million that is expected to yield cost savings of $\mathbf{\$ 1 2}$ million a year for six years. Should the firm take on the warehouse renovation?

Question 18: Consider a firm whose debt has a market value of $\$ 40$ million and whose stock has a market value of $\$ 60$ million (3million outstanding shares of stock, each selling for $\$ 20$ per share). The firm pays a $5 \%$ rate of interest on its new debt and has a beta of 1.41. The corporate tax rate is $34 \%$. The risk premium on the market is $\mathbf{9 . 5 \%}$ and that the current Treasury bill rate is $1 \%$. What cost of capital the firm should use to discount its future cash flows?

Question 19: The Weinstein Corporation has a target capital structure of $\mathbf{8 0 \%}$ equity and $20 \%$ debt. The floatation costs for equity issues are $20 \%$ of the amount raised; the floatation costs for debt issues are $6 \%$. If Weinstein needs $\$ 65$ million for a new manufacturing facility, how much it will have to raise?

Question 20: Tripleday Printing Company is currently at its target debt-equity ditio of $^{\text {of }}$ $\mathbf{1 0 0 \%}$. It is considering building a new $\mathbf{\$ 5 0 0 , 0 0 0}$ printing plant in Kansas. This-new plant is expected to generate after tax cash flows of $\$ 73,150$ per year forever. The tax rate is $34 \%$. To finance the project, the firm is considering all debt or all equity option. The issuance costs of the new common stock and new debt would be about $\mathbf{1 0 \%}$ and $\mathbf{2 \%}$ respectively. The required return on the company's new equity is $20 \%$ and cost of new debt is $10 \%$. Should the firm accept the project?

Financial Management Sources of Finance


Criteria for choosing between sources of finance:

- A firm must consider the following factors:

| Factors | Issue to Consider |
| :--- | :--- |
| Cost | Debt usually cheaper thanequity |
| Duration | Long-term finance qually more expensive but secure <br> Firms should mat duration of asset purchased with duration of liability. <br> Term structure <br> of interest rates |
| Gearing | Usually short term loan is cheaper but not always. <br> Accessibility more debt is cheaper but high gearing is risky |

## Short Tem Sources of Finance:

1) Banc overdrafts 2) Bank loans 3) Better management of working capital
2) Leasing 4) Sale and leaseback

## Leasing:

- A lease is a contract between a lessor and a lessee for the hire of a particular asset.
- Lessor retains ownership of the asset
- Lessor conveys the right of the use of the asset to the lessee for an agreed period and in return lessor receives specified rentals
- Operating lease: The lease period is less than the useful life of the asset. The lessor relies on subsequent leasing or eventual sale of the asset to cover his capital outlay and show a profit. In this type of lease, the lessor is normally responsible for repairs and maintenance.
- Finance Lease- Long Term Source of Finance: The key difference between an operating lease (short term) and a finance lease (medium to long term) is that the former equates to renting an asset whereas the latter equates to borrowing money in order to purchase the asset. Long term lease agreements are likely to be a finance lease.
- One lease exists for the whole useful life of the asset
- The lessor does not retain the risks or rewards of ownership. Lessee is responsible-for repairs and maintenance. The lease agreement can not be cancelled; the lessee has a liability for all payments.


## Sale and Leaseback: Short Term Source of Finance:

- A company that owns its own premises can obtain finance by selling ©ne property for cash and rent it back from the buyer under sale and leaseback agrénent. Although this can provide an immediate source of cash, often more than that Could be obtained from a mortgage, there are a number of disadvantages including:
- The company loses ownership of the property and will therefore miss out on any appreciation in the property's future value.


## Raising Equity Finance:

- There are three main sources of equity innance:

1. Internally generated funds- retained earnings
2. Private Placement
3. Public Issue: a) Initial Putic Offering (IPO) or unseasoned issue b) Seasoned issue or right issues

- Internally generatedrunds: Internally generated funds are earnings retained in the business, i.e. undistubuted profits attributable to ordinary shareholders.
- Such finance isocheap and quick to raise, require no transaction costs, professional assistance, or dime delay.


## Private Placenent of Equity:

- Anget: For most startups, the first round of external financing comes through a private (acement of equity to one or two individual investors, called angels. In return for a typical
investment in the range of $\$ 50,000$ to $\$ 400,000$, the angels receive stock and perhaps also a seat on the board of directors. Because angels can influence the strategic decision of the company, it is best that they bring experience and industry contacts to the table, not just cash.
- The Securities Act restrict the number and type of investors who may participate in private placements that are not registered with the SEC. Accredited investors include the officers and directors of the company, high-wealth individuals, and institutional investors.
- In a non-registered private placement, the company may issue securities to an unlimited number of accredited investors, but only to 35 non-accredited investors. In addition, none of the investors can sell their securities in the secondary market to the general public.
- Venture Capital Fund: As the company grows, its financing requirements may exceed the resources of individual investors, in which case it is likely turn to a venture capital fund. A venture capital fund is a private limited partnership, which raises fund from a relatively small group of primarily institutional investors, including pension funds, college endowments, and corporations. The managers of a venture capital fund, called venture capitalists or VCs, are usually very knowledgeable and experience in a particular industry.
- The venture capital fund buys shares of the portfolio of companies, and the VCs sit on the companies' board of directors. The venture capital fund usually has a pre-specified life of seven to ten years, after which it is dissolved, either by selling the portfolio companies stock and distributing the proceeds to the funds' investors or by directly distributing-the stock to the investors.


## The Public Issue of Equity:

- The Basic Procedure for a IPO or unseasoned issue:

1. Management's first step in any issue of securities to the publio to obtain approval from the board of directors.
2. Next, the firm must prepare and file a registration staternent with the SEC. This statement contains a great deal of financial information, Including a financial history, details of the existing business, proposed financing, ad plans for the future.
3. The SEC studies the registration statement during a waiting period. During this time, the firm may distribute prospectus contains कnuch of the information put into the registration statement, and is given to potenit investors by the firm. The company can not sell the securities during the waitinOperiod; however, oral offers can be made. A registration statement will become effective on the 20th day after its filing unless the SEC sends a letter of comment suggesting changes.
4. The registration statement doef hot initially contain the price of the new issue. On the effective date of the registration statement, a price is determined and a full-fledged selling effort gets under way.

## Investment Bank in the process of IPO or unseasoned issue:

- Investment bankere involved in issuing IPO securities. In addition to aiding in the sale of securities, the may facilitate mergers, and other corporate restructuring, act as brokers to both individual and institutional clients.
- Investmen bankers perform following services in IPO issue:

1. Formulating the method used to issue the securities

Pricing the new securities
Selling the new securities
There are three basic methods of issuing securities for cash:

- Firm commitment: Under this method, the investment bank (or a group of investment banks-syndicate) buys the securities for less than the offering price and accepts the risk of not being able to sell them. Because this function involves risk, we say that the investment banker underwrites the securities in a firm commitment. To minimize the risk, investment bankers combine to form an underwriter group (syndicate) to share the risk and to help sell the issue.
- The difference between the underwriter's buying price and the offering price is called the spread or discount. It is the basic compensation received by the underwriter. The issuer receives the full amount of the proceeds less the spread, and all the risk is transferred to the underwriter.
- Best Efforts: The underwriter bears risk with a firm commitment method by buying the entire issue. Conversely, the syndicate avoids this risk under a best efforts offering method because it does not purchase the shares, instead it merely acts as an agent, receiving commission for each share sold. The syndicate is legally bound to use its best efforts to sell the securities at the agreed upon offering price. If the issue can not be sold at the offering price, it is usually withdrawn. This form of underwriting has become relatively rare.
- Dutch Auction Underwriting: In this method, the underwriter does not set a fixed arice for the shares to be sold. Instead, the underwriter conducts an auction in which infestors bid for shares. The offer price is determined based on the submitted bids. Thiomethod is comparatively new in IPO market while more common to bond market.


## Green Shoe Provision:

Many underwriting contracts contain a Green Shoe Provision, wheh gives the members of the underwriting syndicate the option to purchase additional ares at the offering price. This provision usually last for about 30 days and involve no than $15 \%$ of the newly issued shares. This option is a benefit to the underwriting sxelicate and a cost to the issuer. If the market price of the new issue goes above the offeriog price within 30 days, the underwriters can buy shares from the issuer and immediately reell the shares to the public.

## Right or Seasoned Issue:

- A right issue is an offer to existinestareholders to subscribe for new shares, at a discount to the current market price, ineproportion to their existing holdings. Shareholders no wishing to take up their rightsean sell them on the stock market
- Advantages of right issuce (1) it is cheaper that a public share issue 2) No need to place shares with the help of idevestment banker
- Theoretical Ex-righe price (TERP): The new share price after the issue is known as the theoretical ex-rights price and is calculated by finding the weighted average of the old price and the rights price, weighted by the number of shares.


Exárple: $X$ company, which has an issued capital of 2 million shares, having a current market price of $\$ 2.70$ each, makes a rights issue of one new share for every two existing shares at a price of $\mathbf{\$ 2 . 1 0}$. Find theoretical ex right price.

$$
\begin{aligned}
& \text { TERP }=\frac{\text { Market value of shares already in issue }+ \text { Proceeds from new share issue }}{\text { Number of shares in issue after the rights issue }} \\
& =\frac{\$ 2.70 \times 2 \mathrm{million}+\$ 2.10 \times 1 \mathrm{million}}{2 \mathrm{million}+1 \mathrm{million}}=\$ 2.5 \text { per share }
\end{aligned}
$$

## Hybrid Financing:

- There are three types of long term hybrid financing: Preferred stock, warrants, and convertibles.
- Preferred Stock: Although preferred stock is considered to be equity, it is often included in the fixed income securities like bond. This is because, like bonds, preferred stock promises to pay a fixed stream of dividends. However, unlike bonds, the failure to pay the promised dividend does not result in corporate bankruptcy. Instead, the dividends owed simply accumulate, and the common stockholder may not receive any dividends until the preferred stockholders have been paid in full.
- In the event of bankruptcy, preferred stockholders' claim to the firm's assets has lower priority than those of bondholders, but higher priority than those of common stockholders.
- Warrants: A warrant is a derivative certificate issued by a company that gives the holder the right to buy a stated number of shares of the company' stock at a speciఱed price for some specified length of time. Generally, warrants are issued along with debt, and they are used to induce investors to buy long term debt with a lower conpon rate that would otherwise be required.
- Warrants are long term call options that have value because hiluers can buy the firm's common stock at the exercise price regardless of how high market price climbs. This option offers the low interest rate on the bonds and makes the package of low yield bonds plus warrants attractive to investors.
- Warrants generally are used by small, rapidly growing firms as sweeteners when they sell debt or preferred stock
- Warrant Example: ABC company wants aise $\$ 50$ million of debt finance by issuing 30 years bonds in the market. Counpany's CFO and investment banker found that the bonds would be difficult to stll and that a coupon rate of $10 \%$ would be required. However, as an alternatiet they also suggested that investors might be willing to purchase the bonds at $8 \%$ ertpon rate if the company would offer 20 warrants with $\$ 1000$ bond, each warran entitling the holder to purchase one share of common stock at an exercise price of \$2 per share. The stock was selling for \$20 per share and the warrant would expieafter 10 years.
- Convertibles: Contertible securities are bonds or preferred stocks that under specified terms and conditions, can be exchanged for (that is converted into) common stock at the option of the iolder. Unlike the exercise of warrants, which brings in additional finds to the firm, coniersion does not provide new capital, debt or preferred stock is simply replaced by common stock in the balance sheet. Exercise of this option will reduce the debt or preferred steck and will improve the firm's financial strength and make it easier to raise additional acapital.
- Conversion Ratio and Conversion Price: One of the most important provisions of a convertible security is the conversion ratio, CR , defined as the number of shares of stock a bondholder will receive upon conversion. Related to the conversion ration is the conversion price, $\mathrm{P}_{\mathrm{c}}$, which is the effective price investors pay for the common stock when conversion occurs. Once $C R$ is set, the value of $P_{c}$ is established.

$$
\text { Conversion Price, } \mathrm{Pc}=\frac{\text { Par value of bond given up }}{\text { Shares received or CR }}
$$

## Financial Management <br> Dividend Payout Policy

## Distribution to Shareholders:

- A profitable company regularly faces three important questions:

1. How much of its free cash flow should it pass on to shareholders?
2. Should it provide this cash to shareholder by raising the dividend or by repurchasing stock?
3. Should it maintain a stable, consistent payment policy, or should it let the paynents vary as conditions change?

- There are only six good uses of free cash flow:

1. Re-invest in the business by issuing stock dividends
2. Pay interest expenses
3. Pay debt principal
4. Pay dividends
5. Repurchase stock
6. Buy non-operating assets such as treasury bills or other mazaketable securities.

## The Level of Distributions and Firm Value:

- Can a company increase its value through its chice of distribution policy, defined as the level of distributions, the form of distributions (cash dividends vs stock repurchases) and the stability of distributions?
- The answer to above question depends on investor' preferences for returns as dividend versus capital gains. A high distribution ratio and a high payout ratio mean that a company pays large dividends and has surall or zero stock repurchases. In this situation, dividend yield is relatively high and the expected capital gain is low. Conversely, if a company has large distribution ratio boull payout ratio, then it pays low dividends but regularly repurchases stock, resôing in a low dividend yield but a relatively high expected capital gain.


## Investor' Preferences for Dividend Yield versus Capital Gains:

- Dividerd Irrelevance Theory: Merton Miller and Franco Modigliani (MM) theory states that dividend policy has no effect on either the price of a firm's stock or its cost of capital. They argued that the firm's value is determined only by its basic earning power and its business risk. In other words, MM argued that the value of the firm depends only on the income produced by its assets, not on how this income is split between dividends and retained earnings. The principal conclusion of MM theory is that dividend policy does not affect the required return on equity. MM proposition is based on the assumption of no tax and brokerage costs. However, in real world tax and brokerage cost do exist, so the MM irrelevance theory may not be true.
- Bird-in-the-Hand Theory: Dividends Are Preferred: Myron Gordon and John Linter opposed the MM theory and state that required return on equity decreases as the dividend payout is increased because investors are less certain of receiving the capital gains than
they are of receiving dividend payments. Gordon and Linter said, investors value a dollar of expected dividends more than a dollar of expected capital gains because the dividend yield component is less risky than the expected capital gain.
- Tax Preference Theory: Capital Gains are Preferred: Due to time value effects, a dollar of taxes paid in the future has a lower effective cost than a dollar paid today. So even if dividends and gains are taxed equally, capital gains are taxed only when the investor sell shares. If an investor holds his shares for long term, then he will be concerned only on taxes on dividend income and thereby prefer to have companies minimize dividends. If so, investors would be willing to pay more for low payout companies than for otherwise similar high payout companies.


## Stock Dividends and Repurchase of Stock:

- Stock Dividend: Stock dividend is paid out in shares of stock. It is ngtoa true dividend because no cash leaves the firm. Rather, a stock dividend increases the number of shares outstanding, thereby reducing the value of each share. A stock rividend is commonly expressed as a ratio; for instance, with a $5 \%$ stock dividend a $\&$ freholder receives 1 new share for every 20 currently owned.
- Stock Repurchase: Instead of paying dividends, a firm mayuse cash to repurchase shares of its own stock. Share repurchases are typically accomprished in one of three ways:

1. Companies may simply purchase their own sto just as anyone would buy shares of a particular stock. In these open market purchaies, the firm does not reveal itself as the buyer. Thus, the seller does not know whather the shares were sold back to the firm or to just another investor.
2. Companies could institute a tender offer. Here, the firm announces to all of its stockholders that it is willing douty a fixed number of shares at a specific price. The firm can also purchase sharesfrom an auction.
3. Finally, firms may repurchase shares from specific individual stockholders, a procedure called a targeted repurdase. Companies engage in targeted repurchases for a variety of reasons. A single lare stockholder can be bought out at a price lower than that in tender offer. The legalfees in a targeted repurchase may also be lower than those in more typical buybaek.

## Financial Management <br> Working Capital Management

## The Elements of Working Capital:

- Working capital is the capital available for conducting the day-to-day operations of an organization; normally the excess of current assets over current liabilities.
- Working capital management is the management of all aspects of both current assets and current liabilities, to minimize the risk of insolvency while maximising the return on assets.



## The Elements of Working Capital:

- Investing in working capital has a cost, which can be expressed either as:

1. The cost of funding it
2. The opportunity cost of lost mnestment opportunities because cash is tied up and unavailable for other usesp

- Working capital is an invent which affects cash flows in the following ways:

1. When inventory is p chased, cash is paid to acquire it.
2. Receivables represent the cost of selling goods or services to customers, including the costs of the raterials and the labor incurred.
3. The cash the up in working capital is reduced to the extent that inventory is financed by traderayables. If suppliers supply goods to the firm on credit, the firm's cash flows are inproved and working capital is reduced.

## Objectives of Working Capital Management:

- The main objective of working capital management is to get the balance of current assets and current liabilities right.

1. Ensuring current assets are sufficiently liquid to minimize the risk of insolvency
2. Investing in less liquid assets in order to maximize return.

## Trade-off between Liquidity versus Profitability:

- Liquidity versus Profitability: The decision regarding the level of overall management in working capital involves a trade-off between liquidity versus profitability. Liquidity is as important as profitability. Unprofitable companies can survive if they have liquidity whereas profitable companies can fail if they run out of cash to pay their liabilities (wages, amounts due to suppliers, overdraft interest etc). Liquidity in the context of working capital management means having enough cash or ready access to cash to meet all payment obligations when these fall due. The main sources of liquidity are: cash in the bank, short term investments that can be cashed in easily and quickly, cash inflows from normal trading operations, an overdraft facility or other ready source of extra borrowing.
- Some examples of transactions that have this trade-off effect on liquidity and on profts are as follows:
- Purchase of non-current assets for cash. The cash will be paid in full to thertipplier when the asset is delivered; however profits will be charged gradually over the $1 \mathbb{1 T E}$ of the asset in the form of depreciation.
- Receiving a bulk purchase discount for buying more inventory tha is currently required improves profitability but reduces liquidity.
- Sale of goods on credit attracts more customers so improfles profitability but reduces liquidity. Profits will be credited in full once the sale has geep confirmed; however the cash may not be received for some considerable periods aftepuards.
- With some payments such as tax there may be a singlicant timing difference between the impact on reported profit and the cash flow.
- Offering an early settlement discount to customers will improve liquidity at the cost of profitability.


## Policies Regarding Working Capitat N九̂nagement

- Aggressive vs Conservative taproach: A firm choosing to have a lower level of working capital than rivals is said to have an aggressive approach, whereas a firm with a higher level of working capital has aconservative approach.
- An aggressive appoach will result in higher profitability with higher risk, while a conservative appach will result in lower profitability and lower risk.
- Over-capitalization in working capital: If there are excessive inventories, accounts receivable aud cash, and very few accounts payable, there will be an over-investment by the conaQ any in current assets. Working capital will be excessive and the company will be over-capitalized.
- ODertrading: Cash flow is the lifeblood of the thriving business. Effective and efficient
management of the working capital investment in essential to maintain a control of business cash flow. Management must have full awareness of the profitability versus liquidity tradeoff.
- For example, healthy trading growth typically produces: increased profitability, the need to increase investment in non-current assets and working capital. In contrast to overcapitalization, if the business does not have access to sufficient capital to fund the increase, it is said to be "overtrading." this can cause serious trouble for the business as it is unable to pay its business creditors.


## Working Capital Ratios-Liquidity:

- Two key measures, the current ratio and the quick ratio, are used to assess short-term liquidity. Generally a higher ratio indicates better liquidity.
- Current Ratio: Current ratio measures how much of the total current assets are financed by current liabilities.

$$
\text { Current Ratio }=\frac{\text { Current Assets }}{\text { Current Liability }}
$$

- A current ration of 2:1 means that current liabilities can be paid twice over out of existing current assets.
- Quick (Acid Test) Ratio: The quick or acid test ratio measures how well current liablyties are covered by liquid assets. This is particularly useful where inventory holding pefiods are long.

$$
\text { Quick Ratio }=\frac{\text { Current Assets - Inventory }}{\text { Current Liability }}
$$

- A quick ratio of $1: 1$ means that the company is able to meet existing liabilities if they all fall due at once.
- These liquidity ratios are a guide to the risk of cash flow prems and insolvency. If a company suddenly finds that it is unable to renew its short term liabilities (for instance if the bank suspends its overdraft facilities) there will bedanger of insolvency unless the company is able to turn enough of its current assets into cash quickly.
- In general, high current and quick ratios are considered 'good' because it indicates that the firm has the resources to meet its commitments as they fall due. However, it may also indicate that working capital is not being used efficiently, or the company have too much idle cash tied up in the business that could be used to earn profit.
- Ideal current and quick ratio varies depending on the nature of the business. For example, a supermarket business operating a system will have little inventory and since most of its sales are for cash they will have receivables. In addition, the ability to negotiate long credit periods with supplierscan result in a large payables figure. This can result in net current liabilities and a ecurent ratio below 1 but does not mean the business has a liquidity problem.


## Cash Operating Cycle:

- The cash porating cycle is the length of time between the company's outlay on raw materials wages and other expenditures and the inflow of cash from the sale of goods.
- The fer a firm can push items around the cycle the lower its investment in working ca) il .

| Purchases | Sales |  | Receipts from receivables |
| :---: | :---: | :---: | :---: |
| Inventory Days |  | Receivable Days |  |
| Payable Days |  | Cash operating cycle |  |
| Pay Payables |  |  |  |

## Calculation of the Cash Operating Cycle:

## Type of Business: Manufacturing

Raw materials holding period X
WIP holding period X
Finished goods holding period X
Inventory days X
Receivables' collection period X
Less: Payables' payment period
Cash operating cycle

Type of Business: Wholesale or Retail
Inventory holding period
Receivables' collection period

Operating Cycle Ratios:




Trade Receivable Days $=\frac{\text { Receivables }}{\text { Credit Sales }} x 365$

Trade Payable Days $=\frac{\text { Trade Payables }}{\text { Credit Purchases }} \times 365$

## Example: Evaluate working capital management of the following company:

|  | 2006 |  | 2007 |  |
| :--- | :---: | :---: | :---: | :---: |
| Current Assets | $\$ 000$ | $\$ 000$ | $\$ 000$ | $\$ 000$ |
| Inventory | 37 | 42 |  |  |
| Receivables | 23 | 29 |  |  |
| Bank | 5 | $\frac{3}{74}$ |  |  |
|  |  | 65 |  |  |

Current
Liabilities

| Trade Payables | 55 |
| :--- | ---: |
| Tax Payables | 10 |
|  | 65 |

196

37
162
(42)

157

52
a) Calculate liquidity ratios in 200 and 2007
b) Calculate the length of operafing cycle in 2006 and 2007
c) Comment on your resulfernd appraise how effectively the working capital is being managed.
Sol:

| Liquidity Ratios $\sim^{*}$ |  | 2006 | 2007 |
| :---: | :---: | :---: | :---: |
| Current Ratio Current Assets |  | $\frac{65}{65}=1$ | $\frac{74}{46}=1.6$ |
| $\text { Quick RAMo }=\frac{\text { Current Assets - Inventory }}{\text { Current Liability }}$ |  | $\frac{65-37}{65}=0.43$ | $\frac{74-42}{46}=0.7$ |
| $\text { Inventory Holding Period }=\frac{\text { Inventory }}{\text { Cost of sales }} \times 365$ | 2006 |  | 2007 |
|  | $\frac{(29+37) / 2}{151} \times 365=80 \text { days }$ |  | $\frac{(37+42) / 2}{157} \times 365=92 \text { days }$ |
| $\text { Trade Receivable Days }=\frac{\text { Receivables }}{\text { Credit Sales }} \times 365$ | $\frac{23}{196} \times 365=43 \text { days }$ |  | $\frac{29}{209} \times 365=51 \mathrm{days}$ |
| Trade Payable Days $=\frac{\text { Trade Payables }}{\text { Credit Purchases }} \times 365$ <br> Cash Operating Cycle | $\frac{55}{159} \times 365=126.3 \mathrm{days}$ |  | $\frac{36}{162} \times 365=81.1 \mathrm{~d}$ a s |
|  | 80+4 | -126.3=(3.3days) | 92+51-81.1=61.9days |

## Comments:

- Liquidity Ratio: The company improved in its liquidity condition in 2007 from 2006. Liabilities payables are $70 \%$ covered by cash and receivables (liquid assets, close to cash) in 2007.
- The inventory holding period has lengthened. In general, the shorter the stock holding period the better. It is expensive to hold stock for longer period; the company should focus on its sells effort by providing training to sales staff and effective promotion strategy.
- The receivables collection period scenario has worsened in 2007 compared to 2006. The company need to focus on its credit policy and try to reduce it to less than 30 days.
- The payables' payment period has reduced substantially from last year. It indicates that the company has improved relationship with its suppliers by settling payment early.
- Cash Operating Cycle: In 2007, there is approximately a 62 day gap between poying cash to suppliers and cash received from customers. However, in 2006, there somewhat unusual situation where cash was received from customers more than 3 days before the payment to suppliers was needed.


## Financial Management <br> Microsoft Excel Applications

Example: What is the present value of $\$ 2,500$ payable 4 years from now at $\mathbf{8 \%}$ compounded quarterly?

Excel formula: $=\mathrm{PV}($ rate, neper, amt, [fy], [type] $)==\mathrm{PV}(8 \% / 4,16,2500)=(\$ 1,821.11)$
Example: If $\$ 800$ is invested at $6 \%$ compounded semiannually(every 6 months), what will be the amount in 5 years?

Excel formula: $=\mathrm{FV}($ rate, nper,pmt,[pv],[type] $)==\mathrm{FV}(6 \% / 2,10,, 800)=(\$ 1,075.13)$
Example: If $\$ 100$ is deposited in an account each month for 10years and the account earns $\mathbf{7 \%}$ compounded monthly, how much will be in the account after the last depositisthade.

Excel formula: $=\mathrm{FV}($ rate,nper,pmt, $[\mathrm{pv}],[$ type $])=\mathrm{FV}(7 \% / 12,120,100,, 0)=(\$ 12,308.48)$
Example: How much should be deposited in a sinking fund at the end of each quarter for 5 years to accumulate $\mathbf{\$ 1 0 , 0 0 0}$ if the fund earns $\mathbf{8 \%}$ compounded quarterly.

Excel formula: PMT(rate, neper, jv, [fv], [type] $)=\operatorname{PMT}(8 \% / 4,2 Q, 10000,0)=(\$ 411.57)$
Example: What sum deposited now in an account earning $8 \%$ interest compounded quarterly will provide quarterly payments of $\$ 1,000$ for 10 years die first payment to be made 3 months from now?

Excel formula: $\mathrm{PV}\left(\right.$ rate, neper, pms, $[f v],[t y p e)_{9}=\mathrm{PV}(8 \% / 4,40,1000,0)=(\$ 27,355.48)$
Example: Sam borrowed $\$ 5,000$ to bu ar car. He will amortize the loan by monthly payments over 3 years. Find the monthly payment amount if interest is $\mathbf{1 2 \%}$ compounded monthly
Excel formula: $=\operatorname{PMT}($ rate, npexpv, $[\mathrm{fv}],[$ type $])=\operatorname{PMT}(12 \% / 12,36,5000,, 0)=(\$ 166.07)$

## Goal Seek Examples

Example: Suppose $y$ you want to borrow money from a bank to construct a building. Bank will give 30 years loan at the rate of $9 \%$ compounded monthly. How much you will be able to borrow depends on how much you can pay per month as installment. If you can pay $\$ 9,000$ per month then howgruch you can borrow from the bank?

And. We can solve this using present value formula:
Excel formula: $\mathrm{PV}($ rate, neper, mt, [fy], [type] $)==\mathrm{PV}(9 \% / 12,360,9000)=(\$ 1,118,536.79)$

Thus, you can borrow up to $\$ 1,118,536.79$ if you are able to pay $\$ 9,000$ per month. However, if you want to find how much you can borrow if you can pay suppose $\$ 6,000$ or $\$ 8,000$ or different amount, then you can use goal seek argument option to find out this very easily.

Suppose you can borrow $\$ 900,000$ if you can pay $\$ 7,241.60$ per month.

| A | B |  |
| :--- | :--- | ---: |
| 1 | Loan Amount | $\$ 900,000$ |
| 2 | Interest rate | $9 \%$ |
| 3 | Years | 30 |
| 4 |  |  |
| 5 | Payment | $(\$ 7,241.60)$ |


| Goal Seek |  | 8 | 83 |
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| A | B |  |
| :--- | :--- | ---: |
| 1 | Loan Amount | $(\$ 1,118,537)$ |
| 2 | Interest rate | $9 \%$ |
| 3 | Years | 30 |
| 4 |  |  |
| 5 | Payment | $\$ 9,000.00$ |

Example:At $8 \%$ compounded annually, how many years will it take for $\$ 2,000$ to grow to $\mathbf{\$ 3 , 0 0 0}$ ? Ans: Using goal seek we can solve this,

Future value formula: $=\mathrm{B} 1^{*}(1+\mathrm{B} 2)^{\wedge} \mathrm{B} 3$

| A | B |  |
| :--- | :--- | ---: |
| 1 | Present value | $\$ 2,000$ |
| 2 | Interest rate | $8 \%$ |
| 3 | Years |  |
| 4 | Future value | $\$ 2,000$ |



| A |  | B |
| :--- | :--- | ---: |
| $\mathbf{1}$ | $\$$ |  |
| 1 | Present value | $\$ 2,000$ |
| 2 | Interest rate | $8 \%$ |
| 3 | Years | 5.268446652 |
| 4 | Future value | $\$ 3,000$ |

Ans: 5.268 years
Example: At $\mathbf{2 \%}$ compounded monthly, how many years will it take for $\$ 500$ to grow to $\mathbf{\$ 2 , 2 0 0}$ ?
Ans: Future value formula: $=\mathrm{B} 1 *(1+\mathrm{B} 2 / 12)^{\wedge}(\mathrm{B} 3 * 12)$

| 4 | A | B |
| :---: | :--- | :---: |
| 1 | Present value | $\$ 500$ |
| 2 | Interest rate | $12 \%$ |
| 3 | Years |  |
| 4 | Future value | $\$ 500$ |



Ans: 12.4 years

## Example: At what interest rate compounded annually will a sum of money double in $\mathbf{1 0}$ years $\boldsymbol{\sim}$ )



Ans: 7.177\%

## Example: At what interest rate compounded monthly will a sum of money triple in 15 years?



| Goal Seek |  | 9 $x$ |
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| 4 | A | B |
| :--- | :--- | ---: |
| 1 | Present value | $\$ 500$ |
| 2 | Interest rate | $7.346 \%$ |
| 3 | Compounding | 12 |
| 4 | Years | 15 |
| 5 | Future value | $\$ 1,500$ |

Ans: 7.346\%
Example: Find the effective rate (or equivalent annual rate) of $\mathbf{1 2 \%}$ compounded monthly.
$\operatorname{EFFECT}($ nominal_rate, npery) $=\operatorname{EFFECT}(12 \%, 12)=12.683 \%$

Example: If the effective rate (or equivalent annual rate) is $\mathbf{1 6 \%}$ then find the nominal rate for monthly compounding.

NOMINAL(effect_rate, npery) $=\operatorname{NOMINAL}(16 \%, 12)=14.934 \%$
Example: Suppose you want to borrow money for purchasing a car. A bank is offering you EMI option for the following packages. Find the EMI amount for all the packages.

| Packages | Package 1 | Package 2 | Package 3 | Package 4 |
| :--- | ---: | ---: | ---: | ---: |
| Loan | $\$ 50,000$ | $\$ 90,000$ | $\$ 100,000$ | $\$ 100,000$ |
| Interest rate | $7.000 \%$ | $9.000 \%$ | $5.000 \%$ | $10.000 \%$ |
| Compounding | 12 | 12 | 12 | 12 |
| Years | 6 | 8 | 2 | 10 |
| Payment | $\mathbf{( \$ 8 5 2 . 4 5 )}$ | $\mathbf{( \$ 1 , 3 1 8 . 5 2 )}$ | $\mathbf{( \$ 4 , 3 8 7 . 1 4 )}$ | $\mathbf{( \$ 1 , 3 2 1 . 5 1 )}$ |

Using Scenario Analysis we can change the package conditions and find the paynent amounts.

| Scenario Summary |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Current Values: | Package 1 | Package 2 | Package 3 | Package 4 |
| Changing Cells: |  |  |  |  |  |
| \$B\$3 | \$50,000 | \$50,000 | \$90,000 | \$100,000 | \$100,000 |
| \$B\$4 | 7.000\% | 7.000\% | 9.000\% | 5.000\% | 10.000\% |
| \$B\$5 | 12 | 12 | 12 | 12 | 12 |
| \$B\$6 | 6 | 6 | 8 | 2 | 10 |
| Result Cells: |  |  |  |  |  |
| \$B\$7 | (\$852.45) | (\$822.45) | (\$1,318.52) | (\$4,387.14) | (\$1,321.51) |

We can design any packages according to customer demands by changing loan amount, interest rate, no. of compounding per year'and no of years.

Example: Find NPV and HRK of the following project.

| Year | Cash Flow |
| :--- | :--- |
| $\mathbf{0}$ | $(240,000)$ |
| 1 | $\mathbf{8 0 , 0 0 0}$ |
| 2 | 120,000 |
| 3 | 70,000 |
| 4 | 40,000 |
| 5 | 20,000 |
| 6 | 15,000 |
| 7 | $\mathbf{1 2 , 0 0 0}$ |

$=\mathbf{N P V}($ rate,future cash flows)+Initial Investment
$=\operatorname{IRR}($ values, $[$ guess $])=\mathbf{1 7 . 0 1 \%}$

## Financial Management

## Time Value of Money Problems and Solutions

Question 1-2: In problems 1 through 2, find the future value at the stated nominal interest rate compounded annually.

1) $\$ 200 ; 20$ years; 5 percent

Sol: Here, Present value, $\mathrm{P}=\$ 200, \mathrm{t}=20$ years, $\mathrm{r}=5 \%=0.05$
Future value, $F_{t}=P(1+r)^{t}=200(1+0.05)^{20}=\$ 530.66$
2) $\$ 300$; 10 years; 6 percent

Sol: Here, Present value, $\mathrm{P}=\$ 300, \mathrm{t}=10$ years, $\mathrm{r}=6 \%=0.06$
Future value, $F_{t}=P(1+r)^{t}=300(1+0.06)^{10}=\$ 537.25$
Question 3-4: In problems 3 through 4, find the future value using the approprate interest and number of periods.
3) $\$ 150 ; 8$ years; $\mathbf{8}$ percent compounded quarterly

Sol: Here, Present value, $\mathrm{P}=\$ 150, \mathrm{t}=8$ years x $4=32, \mathrm{r}=8 / 4 \%=2 \%=0.02$
Future value, $F_{t}=P(1+r)^{t}=150(1+0.02)^{32}=\$ 282.68$
4) $\$ 600 ; 20$ years; 8 percent compounded semi-annually

Sol: Here, Present value, $P=\$ 600, t=20$ years x $2=40, r=8 / 2 \%=4 \%=0$ e4
Future value, $F_{t}=P(1+r)^{t}=600(1+0.04)^{40}=\$ 2,880.61$
Question 5: How many years will it take for $\mathbf{\$ 5 , 0 0 0}$ to annount to $\$ 20,000$ at $\mathbf{7 \%}$ compounded annually?
Sol: Here, present value, $P=\$ 5,000$, Future value, $F=\$ 20800$, time periods, $t=$ ?

$$
\mathrm{r}=7 \%=0.07
$$

Future value, $F_{t}=P(1+r)^{t}$
i.e. $\$ 20,000=\$ 5,000(1+0.07)^{t}$
i.e. $1.07^{t}=\frac{20,000}{5,000}=4$
i.e. $\ln \left(1.07^{t}\right)=\ln (4) \quad ;$ TTaking $\ln$ in bort sides]
i.e. $t \mathrm{x} \ln (1.07)=\ln (4)$
i.e. $t=\frac{\ln (4)}{\ln (1.07)}=20.489 \approx 20 \%$ years

Question 6: How many years will it take for a sum of money to double at $\mathbf{1 0 \%}$ compounded semi,ánurually?
Sol: Here, $\mathrm{P}=\$ 1 \mathrm{Q}, \mathrm{F}=\$ 200$, time periods, $\mathrm{t}=\mathrm{xyears}(2)=2 \mathrm{x} \mathrm{r}=10 / 2 \%=0.05$

```
Future value, r' 
i.e.$200=$100(1+0.05)
i.e. 1.05 =2
irem(1.05 2x})=\operatorname{ln}(2)\quad; [Taking ln in both sides
i.e. 2x ln(1.05)=\operatorname{ln}(2)
i.e. 2x = 䇂(2)
i.e. }x=7.1\mathrm{ years
```

Question 7: Find the rate of interest compounded annually at which a sum of money will double in 20 years.
Sol: Here, present value, $\mathrm{P}=\$ 100$, Future value, $\mathrm{F}=\$ 200$, time periods, $\mathrm{t}=20$ years, $\mathrm{r}=$ ?
Future value, $F=P(1+r)^{t}$
i.e. $200=100(1+r)^{20}$
i.e. $(1+r)^{20}=2$
i.e. $\ln (1+r)^{20}=\ln (2) \quad ;[\mathrm{T}$ aking $\ln$ in both sides]
i.e. $20 \ln (1+r)=\ln (2)$
i.e. $\ln (1+r)=\frac{\ln (2)}{20}$
i.e. $(1+r)=e^{\frac{\ln (2)}{20}}$
$i . e . r=e^{\frac{\ln (2)}{20}}-1=0.0352=3.52 \%$

$$
\begin{aligned}
& (1+r)^{20}=2 \\
& \text { i.e. }\left[(1+r)^{20}\right]^{\frac{1}{20}}=(2)^{\frac{1}{20}} \\
& \text { i.e. }(1+r)=(2)^{\frac{1}{20}} \\
& \text { i.e. } r=(2)^{\frac{1}{20}}-1=0.0352=3.52 \%
\end{aligned}
$$

Question 8: Find the rate of interest compounded semi-annually at which $\$ \mathbf{5 , 0 0 0}$ will grow to $\$ 12,000$ in 8 years.
Sol: Here, present value, $\mathrm{P}=\$ 5,000$, Future value, $\mathrm{F}=\$ 12,000, \mathrm{t}=\mathrm{t}$ ars x $2=16, \mathrm{r}=$ ?
Future value, $F=P(1+r)^{t}$
i.e. $\$ 12,000=\$ 5,000\left(1+\frac{r}{2}\right)^{16}$
i.e. $\left(1+\frac{r}{2}\right)^{16}=\frac{\$ 12,000}{\$ 5,000}=2.4$
i.e. $\ln \left(1+\frac{r}{2}\right)^{16}=\ln (2.4) \quad ;[$ Taking $\ln$ in both sides $]$
i.e. $16 \ln \left(1+\frac{r}{2}\right)=\ln (2.4)$
i.e. $\ln \left(1+\frac{r}{2}\right)=\frac{\ln (2.4)}{16}$
i.e. $\left(1+\frac{r}{2}\right)=e^{\frac{\ln (2.4)}{16}}$
i.e. $\frac{r}{2}=e^{\frac{\ln (2.4)}{16}}-1=0.0,562$
i.e. $r=0.0562 \times 0.11248=11.248 \%$

Question $9 \times$ bank pays $5.25 \%$ compounded daily on savings accounts running for 6 years. Using 365 days per year, compute the future value of a deposit of $\$ 5,000$ for 6 years
Sol N Here, $\mathrm{r}=5.25 / 365 \%=0.0525 / 365, \mathrm{t}=6$ years x $365=2190$, present value, $\mathrm{P}=\$ 5,000$
Future value, $F=P(1+r)^{t}=\$ 5,000\left(1+\frac{0.0525}{365}\right)^{2190}=\$ 6,851.14$
Question 19-20: Compute the future value:
19) $\$ 4,000 ; 8$ percent compounded continuously; 5 years 8 months

Sol: Here, Present value, $\mathrm{P}=\$ 4,000, \mathrm{t}=5.67$ years, $\mathrm{r}=8 \%=0.08$
Future value, $F_{t}=P e^{r t}=\$ 4000 x e^{0.08 \times 5.67}=\$ 6,295.87$
20) $\mathbf{\$ 5 , 0 0 0 ; 1 2}$ percent compounded continuously; 6months

Sol: Here, Present value, $\mathrm{P}=\$ 5,000, \mathrm{t}=0.50$ years, $\mathrm{r}=12 \%=0.12$
Future value, $F_{t}=P e^{r t}=\$ 5000 x e^{0.12 \times 0.5}=\$ 5,309.18$

Question 21-22: Compute the present value:
21) $\$ 2,000$; 7 percent compounded continuously; $\mathbf{3}$ years 2 months

Sol: Here, Future value, $F=\$ 2,000, t=3.17$ years, $r=7 \%=0.07$
Present value, $P_{t}=F e^{-r t}=\$ 2,000 x e^{-0.07 x 3.17}=\$ 1,602$
22) $\mathbf{\$ 2 5 0 ; 1 2}$ percent compounded continuously; 1year $\mathbf{6 m o n t h s}$

Sol: Here, Future value, $\mathrm{F}=\$ 250, \mathrm{t}=1.5$ years, $\mathrm{r}=12 \%=0.12$

$$
\text { Present value, } P_{t}=F e^{-r t}=\$ 250 x e^{-0.12 \times 1.5}=\$ 208.81
$$

Question 23-24: The rate in problems 5 through 6 are nominal rates, find the effective rate when the interest rate is compounded continuously:
23) $\mathbf{1 2 \%}$, Sol: Effective rate, $\mathrm{r}_{e}=e^{r}-1=e^{0.12}-1=0.12749=12.749 \%$
24) 8\%, Sol: Effective rate, $\mathrm{r}_{e}=e^{r}-1=e^{0.08}-1=0.08328=8.328 \%$

Question 25-26: The rate in problems 25 through 26 are effective interest rates, find the nominal rate when the interest rate is compounded continuously:
25) Effective rate $=\mathbf{5 \%}$

Here, Effective rate, $\mathrm{r}_{e}=5 \%=0.05$, find the nominal rate, $r$
$\mathrm{r}_{e}=0.05=e^{r}-1 ;$ i.e. $e^{r}=1.05$
or, $\ln e^{r}=\ln (1.05)$
or, $r \ln e=0.04879 \quad ;[\ln \mathrm{e}=1]$
$o r, r=4.879 \%$
26) Effective rate $=\mathbf{1 0 \%}$

Here, Effective rate, $r_{e}=10 \%=0.10$, find nom inal rate, $r$
$\mathrm{r}_{e}=0.10=e^{r}-1 ; \quad$ i.e. $e^{r}=1.10$
or, $\ln e^{r}=\ln (1.10)$
or, $r \ln e=0.09531 \quad ;[\ln \mathrm{e}=1]$
$o r, r=9.531 \%$
Question 27: How much will a deposit of $\$ 5000$ grow to in 20 years at $7.2 \%$ interest compounded continuously?
Sol: Here, Present value, $\mathbb{R}=\$ 5,000, \mathrm{t}=20$ years, $\mathrm{r}=7.2 \%=0.072$
Future value, $F_{t}=p^{\gamma^{0}}=\$ 5000 x e^{0.072 \times 20}=\$ 21,103.47$
Question 28: How much should be deposited now at $\mathbf{8 . 4 \%}$ compounded continuously if the amount in the account 10 years from now is to be $\$ 8000$ ?
Sol: Here, Futrre value, $\mathrm{F}=\$ 8,000, \mathrm{t}=10$ years, $\mathrm{r}=8.4 \%=0.084$
Presentralue, $P_{t}=F e^{-r t}=\$ 8,000 x e^{-0.084 \times 10}=\$ 3,453.68$
Question 29: Sam invests $\$ 15,000$ in a bank account paying $8.4 \%$ compounded contmuously for 15 years. How much will be in the account at the end of this time?
Sol: Here, Present value, $\mathrm{P}=\$ 15,000, \mathrm{t}=15$ years, $\mathrm{r}=8.4 \%=0.084$
Future value, $F_{t}=P e^{r t}=\$ 15,000 x e^{0.084 \times 15}=\$ 52,881.32$
Question 30: History tells us that Peter Minuit purchased Manhattan Island in New York from the Indians for $\$ 24$ about 370 years ago. If the $\$ 24$ had been invested at 5\% compounded continuously, what would be its amount after 370 years.
Sol: Here, Present value, $\mathrm{P}=\$ 24, \mathrm{t}=370$ years, $\mathrm{r}=5 \%=0.05$
Future value, $F_{t}=P e^{r t}=\$ 24 x e^{0.05 \times 370}=\$ 2,598,119,706$

Question 31: A company issues $\$ 1$ million of bonds and sets up a sinking fund at $\mathbf{8 \%}$ compounded quarterly to accumulate $\$ 1$ million by 10 years to redeem the bonds. Find the quarterly payment to the sinking fund.
Sol: Here, Future value, $\mathrm{FV}=\$ 1,000,000$, interest rate, $\mathrm{r}=8 / 4 \%=2 \%=0.02, \mathrm{t}=10 \times 4=40$
$F V=1,000,000=\frac{c}{r}\left[(1+r)^{T}-1\right]=\frac{C}{0.02}\left[(1+0.02)^{40}-1\right]$
or $, 1,000,000 \times 0.02=C\left(1.02^{40}-1\right)$
or, $C=\frac{20,000}{\left(1.02^{40}-1\right)}=\$ 16,555.74$
using annuity table:
$\mathrm{FV}=1,000,000=C x \mathrm{FV} \mathrm{IFA}_{40,2 \%}=C x 60.402$
i.e. $C=\frac{1,000,000}{60.402}=\$ 16,555.74$

Question 32: In order to accumulate $\mathbf{\$ 1 5 , 0 0 0}$ for a down payment on a home 8 years from now, the Jonses are going to deposit a sum of money at the end of each $\sigma$-month period in an account earning $8 \%$ compounded semi-annually. What should be the amount of each deposit?
Sol: Here, Future value, $\mathrm{FV}=\$ 15,000, \mathrm{t}=8 \times 2=16$, interest rate $R 8 / 2 \%=4 \%=0.04$
$F V=15,000=\frac{c}{r}\left[(1+r)^{T}-1\right]=\frac{C}{0.04}\left[(1+0.04)^{16}-1\right]$
or, $15,000 \times 0.04=C\left(1.04^{16}-1\right)$
or,$C=\frac{600}{\left(1.04^{16}-1\right)}=\$ 687.29$
using annuity table:
$\mathrm{FV}=15,000=C x \mathrm{FVIFA}_{16,4 \%}=C x 21.825$
i.e. $C=\frac{15,000}{21.825}=\$ 687.28$

Question 33: When Kathy yas 万orn, her parents decided to deposit $\$ 500$ every 6 months thereafter for 15 years in anaccount earning $6 \%$ compounded semi-annually. How much will be in the account after the last deposit is made?
Sol: Here, Future values $=$ ? Semi-annual deposit, $C=\$ 500, t=15 \times 2=30, r=6 / 2 \%=3 \%$ $=0.03$
$F V=\frac{c}{r}\left[(1+r)^{2}-1\right]=\frac{500}{0.03}\left[(1+0.03)^{30}-1\right]=\$ 23,787.7$
using annefy table: $\mathrm{FV}=\$ 500 x \mathrm{FVIFA}_{30,3 \%}=\$ 500 \times 47.575=\$ 23,787.5$
Queston 34: A sum of money invested now at $10 \%$ compounded semi-annually is to prgkde payments of $\$ 1,500$ every 6 months for 8 years, with the first payment due 6 months from now. How much should be invested? How much interest will the investment earn?
Sol: Here, Present value, $\mathrm{PV}=? \mathrm{r}=10 / 2 \%=5 \%=0.05$, Semi-annual payments, $\mathrm{C}=\$ 1,500$ $\mathrm{t}=8 \times 2=16$
$P V=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{1,500}{0.05}-\frac{1,500}{0.05}(1+0.05)^{-16}=\$ 16,256.65$
using annuity table: $\mathrm{PV}=\$ 1,500 x$ PVIFA $_{16,5 \%}=\$ 1,500 x 10.838=\$ 16257$
Interest earned on investment $=\$ 1,500 \times 16-\$ 16,257=\$ 7,743$

Question 35: The directors of a company have voted to establish a fund that will pay a retiring accountant or his estate $\$ 1,000$ per month for the next 3 years, with the first payment to be made a month from now. How much should be placed in the fund if it earns interest at $\mathbf{1 2 \%}$ compounded monthly? How much interest will the fund earn during its existence?
Sol: Here, monthly payments, $\mathrm{C}=\$ 1,000, \mathrm{t}=3 \times 12=36$, present value, $\mathrm{PV}=? \mathrm{r}=12 / 12 \%$ $=1 \%=0.01$
$P V=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{1,000}{0.01}-\frac{1,000}{0.01}(1+0.01)^{-36}=\$ 30,107.5$
using annuity table: $\mathrm{PV}=\$ 1,000 x$ PVIFA $_{36,1 \%}=\$ 1,000 \times 30.108=\$ 30,108$
Interest earned on investment $=\$ 1,000 \times 36-\$ 30,108=\$ 5,892$
Question 36: A company has borrowed $\$ 50,000$ at $8 \%$ compounded quarterly. The debt is to be amortized by equal payments each quarter over 10 years. a) find the quarterly payment b) how much interest will be paid?
Sol: Here, present value, $\mathrm{PV}=\$ 50,000, \mathrm{r}=8 / 4 \%=2 \%=0.02, \mathrm{t}=10 \times 4=40$, quarterly payments, $\mathrm{C}=$ ?
$P V=\$ 50,000=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{C}{0.02}\left[1-(1+0.02)^{-40}\right]$
or , $1,000=C\left(1-1.02^{-40}\right) ; \quad$ or, $C=\frac{\$ 1,000}{\left(1-1.02^{-40}\right)}=\$ 1,827.78$
using annuity table: $\mathrm{PV}=\$ C x \mathrm{PVIFA}_{40,2 \%}=\$ C x 27.355$
or $, C=\frac{P V}{27.355}=\frac{\$ 50,000}{27.355}=\$ 1,827.82$
Interest will be paid $=\$ 1,827.78 \times 40-\$ 50,000-123,111.2$
Question 37: A real estate developer borrews $\$ 100,000$ at $12 \%$ compounded monthly. The debt is to be discharged by monthly payments over the next 3 years. a) find the monthly payment b) how much interest will bepaid?
Sol: Here, present value, $\mathrm{PV}=\$ 10 \%, 000, \mathrm{r}=12 / 12 \%=1 \%=0.01, \mathrm{t}=3 \times 12=36$, monthly payments, $\mathrm{C}=$ ?
$P V=\$ 100,000=\frac{c}{r}-\frac{c}{r}\left(1+5=\frac{C}{0.01}\left[1-(1+0.01)^{-36}\right]\right.$
or $, 1,000=C\left(1-1.01-\frac{10}{2}\right) ; \quad$ or $, C=\frac{\$ 1,000}{\left(1-1.01^{-36}\right)}=\$ 3,321.43$
using annuity tale $\mathrm{PV}=\$ C x \mathrm{PV}^{2}$ IFA $_{36,1 \%}=\$ C x 30.108$
or $, C=\frac{P C}{6.108}=\frac{\$ 100,000}{30.108}=\$ 3,321.37$
Interest will be paid $=\$ 3,321.43 \times 36-\$ 100,000=\$ 19,571.48$
Question 38: A company borrows $\$ 100,000$ at $12 \%$ compounded semi-annually. The debt is amortized by making equal payments at the end of each 6 months for 7 years.
a) Find the amount of each payment
b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
c) How much of the second payment is for interest, and by how much does it reduce the balance owed?
Sol: Here, present value, $\mathrm{PV}=\$ 100,000, \mathrm{r}=12 / 2 \%=6 \%=0.06, \mathrm{t}=7 \times 2=14$
a) Semi-annual payments, $\mathrm{C}=$ ?
$P V=\$ 100,000=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{C}{0.06}\left[1-(1+0.06)^{-14}\right]$
or $, 6,000=C\left(1-1.06^{-14}\right) ; \quad$ or $, C=\frac{\$ 6,000}{\left(1-1.06^{-14}\right)}=\$ 10,758.49$
using annuity table: $\mathrm{PV}=\$ C x \mathrm{PV}^{\text {IFA }}{ }_{14,6 \%}=\$ C x 9.295$
or $, C=\frac{P V}{9.295}=\frac{\$ 100,000}{9.295}=\$ 10,758.47$
b) At the time the first payment is due, 6 months has passed and the interest due $=\$ 100,000 \mathrm{x}$ $6 \%=\$ 6,000$

| Ans. | Period | Semi-annual <br> Payment | Interest charged | Balance <br> Reduced | New Balance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| b) | 1 | $\$ 10,758.49$ | $\$ 100,000 \times 6 \%$ <br> $=\$ 6,000$ | $\$ 4,758.49$ | $\$ 100,00-\$ 4,758.49$ <br> $605,241.51$ |
| c) | 2 | $\$ 10,758.49$ | $\$ 95,241.51 \times 6 \%$ <br> $=\$ 5,714.49$ | $\$ 5,044$ | 0 |

Question 39: Mr. Smith have taken out a $\$ 35,000,30$ year mortgage on their home, with interest at $8.75 \%$ compounded monthly.
a) Find the amount of each payment
b) How much of the first payment is for interest, and by how much does it reduce the balance owed?
Sol: Here, present value, $\mathrm{PV}=\$ 35,000, \mathrm{t}=30 \times 12=360, \mathrm{r}=8.75 / 12 \%=0.0875 / 12=0.00729$, a) Monthly payments, $\mathrm{C}=$ ?
$P V=\$ 35,000=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{C}{0.000^{20}}\left[10-(1+0.00729)^{-360}\right]$
or, $255.15=C\left(1-1.00729^{-360}\right) ; \quad$ or,$C=\frac{\$ 255.15}{\left(1-1.00729^{-360}\right)}=\$ 275.29$
b) At the time the first paymeat is due, one month has passed and the interest due $=\$ 35,000 \mathrm{x}$ $0.00729=\$ 255.15$

| Ans. | Period | Semíannual <br> Payment | Interest charged | Balance <br> Reduced | New Balance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| b) | 1 | $\$ 275.29$ | $\$, 000 \times 0.00729$ <br> $=\$ 255.15$ | $\$ 20.14$ | $\$ 35,000-\$ 20.14$ <br> $=\$ 34,979.86$ |

Question 40: Mr. Fran borrowed $\mathbf{\$ 2 , 0 0 0}$ from Silverbank and signed a note promising to discharge the debt with interest at $12 \%$ compounded monthly at a maturity date 2 years from now. Six months later, Silverbank needed more cash and sold Fran's note to Goldbank. Goldbank computed the maturity amount of Fran's note and gave Silverbank the present value of this amount, computed at $8 \%$ compounded quarterly. How much did Silverbank receive?

Sol.: The future or maturity value, Mr. Fran supposed to pay to Silverbank, 2 years from now:

$$
\text { Future value, } F_{t}=P(1+r)^{t}=\$ 2,000(1+0.01)^{24}=\$ 2539.47
$$

$$
\begin{aligned}
& \text { Here, } \mathrm{p}=\$ 2,000, \mathrm{r}=12 / 12 \%=1 \% \\
& =0.01, \mathrm{t}=2 \times 12=24
\end{aligned}
$$

The present value of the note (maturity value $=\$ 2539.47$ ) paid to Silverbank by Goldbank, after 6 months,

Present value, $\mathrm{P}_{t}=F(1+r)^{-t}=\$ 2,539.47(1+0.02)^{-6}=\$ 2,255$

Here, $\mathrm{p}=$ ?, $\mathrm{r}=8 / 4 \%=2 \%=0.02$, $\mathrm{t}=1.5 \times 4=6$

Silverbank received \$2,255
Question 41: Sue borrowed $\$ 7,000$ at $12 \%$ compounded monthly for 3 years to buy a car. How much will she have to pay at the end of each month to discharge the debt?
Sol: Here, present value, $\mathrm{PV}=\$ 7,000, \mathrm{r}=12 / 12 \%=1 \%=0.01, \mathrm{t}=3 \times 12=36$, monthly payments, $\mathrm{C}=$ ?

$$
\begin{aligned}
& P V=\$ 7,000=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{C}{0.01}\left[1-(1+0.01)^{-36}\right] \\
& o r, 70=C\left(1-1.01^{-36}\right) ; \quad o r, C=\frac{\$ 70}{\left(1-1.01^{-36}\right)}=\$ 232.5 \\
& \text { using annuity table: } \mathrm{PV}=\$ C x \mathrm{PV}^{\mathrm{IFA}} \mathrm{IF}_{36,1 \%}=\$ C x 30.108 \\
& o r, C=\frac{P V}{30.108}=\frac{\$ 7,000}{30.108}=\$ 232.49
\end{aligned}
$$

Question 42: How much should be deposited now at 7\% compounded annually to provide an income of $\mathbf{\$ 2 0 , 0 0 0}$ at the end of each year for the next 22 ygars?
Sol: Here, present value, $\mathrm{PV}=?, \mathrm{r}=7 \%=0.07$, annual paymedt, $\ell=\$ 20,000, \mathrm{t}=22$

$$
P V=\frac{c}{r}-\frac{c}{r}(1+r)^{-T}=\frac{20,000}{0.07}-\frac{20,000}{0.07}(1+0.07)^{-22}=\$ 22{ }^{22} 24.81
$$

using annuity table: $\mathrm{PV}=\$ 20,000 \times \mathrm{PV}$ IFA ${ }_{22,7 \%}=\$ 2 \pi 000 \times 11.061=\$ 221,220$
Question 43: Mr. James will make 20 equal semi-annual deposits to an account earning $\mathbf{8 \%}$ compounded semi-annually. Then, aftey the last deposit, she will use the amount in the account to establish an ordinary anntuity earning $6 \%$ compounded annually which will provide her with $\$ 10,000$ at the end of each year for 5 years. How much should his semi-annual deposit be?


Sol: Here, ${ }^{\text {rresent }}$ value of annuity 2 at the end of 10 years, $\mathrm{PV}=$ ?, annual payments, $\mathrm{C}=$ $\$ 10,0 \mathrm{R} \mathrm{Q}^{\top}=6 \%=0.06, \mathrm{t}=5$

using annuity table: $\mathrm{PV}=\$ 10,000 x \mathrm{PVIFA}_{5,6 \%}=\$ 10,000 x 4.2124=\$ 42,124$
Now, semi-annual deposit, $\mathrm{C}=$ ?, $\mathrm{r}=8 / 2 \%=4 \%=0.04, \mathrm{t}=20$

$$
\begin{aligned}
& F V=\$ 42,124=\frac{c}{r}\left[(1+r)^{T}-1\right]=\frac{C}{0.04}\left[(1+0.04)^{20}-1\right] \\
& \text { or }, \$ 1,684.96=C\left(1.04^{20}-1\right) \\
& \text { or }, C=\frac{\$ 1,684.96}{\left(1.04^{20}-1\right)}=\$ 1,414.59
\end{aligned}
$$

using annuity table:
$\mathrm{FV}=42,124=C x \mathrm{FVIFA}_{20,4 \%}=C x 29.778$
i.e. $C=\frac{\$ 42,124}{29.778}=\$ 1,414.6$

# Financial Management <br> Capital Budgeting Problems and Solution 

## Question 1:

| Year | Cash Flow |
| :--- | :--- |
| 0 | $(240,000)$ |
| 1 | 80,000 |
| 2 | 120,000 |
| 3 | 70,000 |
| 4 | 40,000 |
| 5 | 20,000 |
| 6 | 15,000 |
| 7 | 12,000 |

a) Find out the payback period and discounted payback period of the aboveproject Sol:

| Year | Cash Flow | Cumulative Cash Flow |
| :---: | :---: | :---: |
| 0 | $(240,000)$ |  |
| 1 | 80,000 | $(160,000)$ |
| 2 | 120,000 | $(40,000)$ |
| 3 | 70,000 | 30,000 |
| 4 | 40,000 | 70,000 |
| 5 | 20,000 | 90,000 |
| 6 | 15,000 | 1-10,000 |
| 7 | 12,000 | O17,000 |

Payback period is between the year 2 and year 3. If we assume a constant rate of cash flow throughout the $3^{\text {rd }}$ year, we could estimate payback period will be two years plus $(40,000 / 70,000)$ of 3 th year, which is 2.5714 years or 2 years 6.85 months.
Discounted Payback Periodecalculation

| ${ }^{+}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Year | Cash Flow | Present Value@15\% | Cumulative PVs |
| 0 | $(240,000)$ | $(240,000)$ |  |
| 1 | 80.000 | $80,000(1+0.15)^{-1}=\$ 69,565.21$ | $(170,434.79)$ |
| 2 | d20,000 | $120,000(1+0.15)^{-2}=\$ 90,737.24$ | $(79,697.55)$ |
| 3 | 70,000 | $70,000(1+0.15)^{-3}=\$ 46,026.13$ | (33,671.42) |
| 2 | 40,000 | $40,000(1+0.15)^{-4}=\$ 22,870.13$ | $(10,801.29)$ |
| 5 | 20,000 | $20,000(1+0.15)^{-5}=\$ 9,943.53$ | (857.76) |
| 6 | 15,000 | $15,000(1+0.15)^{-6}=\$ 6,484.91$ | 5,627.15 |
| 7 | 12,000 | $12,000(1+0.15)^{-7}=\$ 4,511.24$ | 10,138.39=NPV |

Discounted Payback Period $=5$ years $+\frac{857.76}{6,484.91} \times 12 \mathrm{month} s=5$ years +1.58 months
b) Find NPV of the project at $\mathbf{1 5 \%}$ hurdle rate.
$\mathrm{NPV}=10,138.39$
Decision: Accept the project because it not only generates enough cash to fulfil required return but also achieve extra return of $\$ 10,138$.

## Question 2:

| Year | Cash Flow |
| :--- | :--- |
| 0 | $(\mathbf{2 5 , 0 0 0})$ |
| 1 | $\mathbf{6 , 0 0 0}$ |
| 2 | $\mathbf{1 0 , 0 0 0}$ |
| 3 | $\mathbf{8 , 0 0 0}$ |
| 4 | 7,000 |

a) Find out the payback period and discounted payback period of the above project Sol:

| Year | Cash Flow | Cumulative <br> Cash Flow |
| :--- | :--- | :--- |
| 0 | $(\mathbf{2 5 , 0 0 0})$ |  |
| 1 | $\mathbf{6 , 0 0 0}$ | $(\mathbf{1 9 , 0 0 0 )}$ |
| 2 | $\mathbf{1 0 , 0 0 0}$ | $\mathbf{( 9 , 0 0 0 )}$ |
| 3 | $\mathbf{8 , 0 0 0}$ | $(\mathbf{1 , 0 0 0})$ |
| 4 | $\mathbf{7 , 0 0 0}$ | $\mathbf{6 , 0 0 0}$ |

Payback period is between 3 and 4 years. If we assume a comstant rate of cash flow throughout the $4^{\text {rd }}$ year, we could estimate payback periog Will be three years plus $(1,000 / 6,000)$ of 4th year, which is 3.167 years or 3 years $\mathcal{C}$ months.

Discounted Payback Period Calculation

| Year | Cash Flow | Present Value@22\% | $\underset{\text { Pumulative }}{\text { Cus }}$ |
| :---: | :---: | :---: | :---: |
| 0 | $(25,000)$ | $(25,000) \sigma^{\circ}$ |  |
| 1 | 6,000 | $6,000(1+0.22) \mathrm{O}=\$ 4,918.03$ | (20,081.96) |
| 2 | 10,000 | $10,000\left(1+(0,2)^{-2}=\$ 6,718.62\right.$ | $(13,363.34)$ |
| 3 | 8,000 | $\left.8,000(1)^{0} .22\right)^{-3}=\$ 4,405.65$ | (8,957.68) |
| 4 | 7,000 | $7.800(1+0.22)^{-4}=\$ 3,159.79$ | (5,797.89)=NPV |

Discounted payback petiod is $4+$ years.
b) Find NPV of thefproject at $22 \%$ hurdle rate.
$\mathrm{NPV}=(\$ .897 .89)$
Decisin: Reject the project because it fails to generate enough cash to fulfil required retom.

## Question 3:

| Year | Cash Flow |
| :--- | :--- |
| 0 | $(50,000)$ |
| 1 | 18,000 |
| 2 | 25,000 |
| 3 | 20,000 |
| 4 | $\mathbf{1 0 , 0 0 0}$ |

a) Find out the payback period and discounted payback period of the above project
b) Find NPV of the project when required rate of return is $\mathbf{1 2 \%}$.

Question 4: Find IRR of the following project

| Discount Rate | NPV |
| :--- | :--- |
| $18 \%$ | $\mathbf{1 8 8 4 9 . 1 6 0 2}$ |
| $20 \%$ | $\mathbf{8 1 4 4 . 7 1 8 7 9}$ |
| $22 \%$ | $\mathbf{- 1 5 1 3 . 9 6 1 9 7}$ |
| $24 \%$ | -10236.373 |

Sol: Find two costs of capital at which the NPV is positive and negative respectively but close to zero.
$I R R=L+\left[\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)\right]$
$\mathrm{L}=$ lower rate of interest $=20 \% ; \mathrm{H}=$ higher rate of interest $=22 \%$
$N P V_{L}=\$ 8,144.71879, \quad N P V_{H}=-1513.96197$
$I R R=0.20+\left[\frac{8144.71879}{8144.71879-(-1513.96197)} x(0.22-0.20)\right]=0.20+\left[\frac{81448879}{9658.68076} x 0.02\right]$
$=0.21686=21.686 \%$
Question 5: Find IRR of the following project

| Discount Rate | NPV |
| :--- | :--- |
| $20 \%$ | $\mathbf{5 1 5 9 . 0 3 6 4}$ |
| $22 \%$ | 2747.010 |
| $24 \%$ | $\mathbf{5 6 3} .18611$ |
| $26 \%$ | -1455.7951 |
| $28 \%$ | -3210.5872 |
| $30 \%$ | -5141.7951 |

Sol: Find two costs of capital at which the NPV is positive and negative respectively but close to zero.
$I R R=L+\left[\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)\right.$
$\mathrm{L}=$ lower rate of interest $=24 \% ; \mathrm{H}=$ higher rate of interest $=26 \%$

$$
N P V_{L}=\$ 563.18611, \mathcal{N} \quad N P V_{H}=-1415.7951
$$



Question 6: AdCmpany is considering the following 5 investment projects and has $\$ 100,000$ akárable for investment. All this projects are divisible. Determine which projects fould be chosen to maximize the return to the business.

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ |
| :--- | :---: | :---: |
| A | $\mathbf{4 0}$ | $\mathbf{2 0}$ |
| B | $\mathbf{1 0 0}$ | $\mathbf{3 5}$ |
| C | $\mathbf{5 0}$ | $\mathbf{2 4}$ |
| D | $\mathbf{6 0}$ | $\mathbf{1 8}$ |
| E | $\mathbf{5 0}$ | $\mathbf{( 1 0 )}$ |

Sol:

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ | Profitability <br> Index (PI) | Ranks |
| :--- | :---: | :---: | :---: | :---: |
| A | 40 | 20 | $20 / 40=0.5$ | 1 |
| B | 100 | 35 | $35 / 100=0.35$ | 3 |
| C | 50 | 24 | $24 / 50=0.48$ | 2 |
| D | 60 | 18 | $18 / 60=0.3$ | 4 |
| E | 50 | $(10)$ | Rejected |  |

Capital Rationing:

| Projects | Ranks | Available Fund <br> $(\$ 000)$ | Fund Invested <br> $(\$ 000)$ | Fund Remaining <br> $(\$ 000)$ | NPV <br> $\left.(\$ 000)^{\prime}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 100 | 40 | 60 | 00 |  |  |  |
| C | 2 | 60 | 50 | 10 | $0 \% 24$ |  |  |  |
| B | 3 | 10 | $10 / 100=10 \%$ | 0 | $0^{\circ}$ |  |  |  |
| D | 4 | Total NPV earned |  |  |  |  |  | 44 |

Question 7: A company is considering the following 5 investment projects and has $\$ 100,000$ available for investment. All this projects are indivisiole. Determine the optimal project selection.

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ |
| :--- | :---: | :---: |
| A | 40 | 20 |
| B | $\mathbf{1 0 4}$ | 35 |
| C | $\mathbf{5 0}$ | $\mathbf{2 4}$ |
| D | $\mathbf{6 0}$ | $\mathbf{1 8}$ |

Sol:

| Projects | Initial Invesinht <br> $(\$ 000$ | NPV <br> $(\$ 000)$ | Profitability <br> Index (PI) | Ranks |
| :--- | :---: | :---: | :---: | :---: |
| A | 10 | 20 | $20 / 40=0.5$ | 1 |
| B | 100 | 35 | $35 / 100=0.35$ | 3 |
| C | 50 | 24 | $24 / 50=0.48$ | 2 |
| D | 60 | 18 | $18 / 60=0.3$ | 4 |

Capital Rationing:

| Projects | Ranks <br> A | Available Fund <br> $(\$ 000)$ | Fund Invested <br> $(\$ 000)$ | Fund Remaining <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 1 | 100 | 40 | 60 | 20 |  |
| C | 2 | 60 | 50 | 10 | 24 |  |
| B | 3 | 10 |  | 10 |  |  |
| D | 4 |  |  |  |  |  |

Question 8: A company is considering the following 5 investment projects and has $\$ 100,000$ available for investment. All this projects are divisible and project $A$ and $C$ are mutually exclusive. Determine the optimal project selection.

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ |
| :--- | :---: | :---: |
| A | $\mathbf{4 0}$ | $\mathbf{2 0}$ |
| B | $\mathbf{1 0 0}$ | $\mathbf{3 5}$ |
| C | $\mathbf{5 0}$ | $\mathbf{2 4}$ |
| D | $\mathbf{6 0}$ | $\mathbf{1 8}$ |

## Sol:

| Projects | Initial Investment <br> $(\$ 000)$ | NPV <br> $(\$ 000)$ | Profitability <br> Index (PI) | Ranks |
| :--- | :---: | :---: | :---: | :---: |
| A | 40 | 20 | $20 / 40=0.5$ | 1 |
| B | 100 | 35 | $35 / 100=0.35$ | 3 |
| C | 50 | 24 | $24 / 50=0.48$ | 2 |
| D | 60 | 18 | $18 / 60=0.3$ | 4 |

Capital Rationing:

| Project Mix | Fund Invested (\$000) | $\mathrm{NPV}(\$ 000)$ |
| :---: | :---: | :---: |
| A+C | Not Possible (mutually exclusive projects) |  |
| A+B | Not possible (require $\$ 140,000$ ) |  |
| A+D | 100 | $20+78=38$ |
| A | 40 | - 20 |
| B | 100 | 35 |
| B+C | Not possible (require \$150,000) 0 |  |
| B+D | Not possible (require $\$ 160,000$ ) |  |
| C | 50 | 24 |
| C+D | Not possible (require \$118,000) |  |
| D | 60 | 18 |

From the project mix we can see that if we move with Yanks and eliminate impossible options ( mutually exclusive projects or limited fund cases ave will get the best mix of projects that will provide maximum return. Here, project A plus -will provide the maximum NPV of $\$ 38,000$.

## Financial Management <br> Equity Valuation Problems and Solution

Question 1: A company promises to pay a fixed dividend of $\mathbf{\$ 1 0}$ per preferred share. If your required return from investment is $15 \%$ then how much should you offer to purchase this share?
Sol: Here, $\mathrm{D}=\$ 10$, required return, $\mathrm{r}=15 \%=0.15$, Share Price, $\mathrm{P}=$ ?

$$
P_{0}=\frac{D}{r}=\frac{\$ 10}{0.15}=\$ 66.67
$$

Question 2: ABC Textiles re-invests $50 \%$ of its earning on growth projects that yield an ROE of $\mathbf{2 0 \%}$. If company's year-end projected earnings per share (EPS) is $\$ 10$ and your required return from investment is $15 \%$ then how much should you offer per share to purchase ABC Textiles' share? How much would you offer if ABC act like a casi cow? Calculate Net Present Value of Growth Opportunity (NPVGO). Explain why ABC should act like a cash cow when its growth projects generate returns below the kequired return from investment.

Sol: Here, reinvestment or retention rate, $\mathrm{b}=50 \%=0.50, \mathrm{ROE}=20 \%=20, \mathrm{EPS}, \mathrm{E} 1=\$ 10$, required return, $\mathrm{r}=15 \%=0.15$

## Reinvestment Case:

Dividend payout ratio $=(100-50) \%=50 \%$,
$D_{1}=E_{1} x$ dividend payout ratio $=\$ 10 \times 50 \%=\$ 5$
Growth rate, $\mathrm{g}=\mathrm{ROE} \times \mathrm{b}=0.20 \times 0.50=0.10=10 \%$
Share price @ reinvestment case, $P_{0}=\frac{D_{1}}{r-g}=\frac{\$ 5}{0.150 .10}=\$ 100$

## Cash-cow Case:

Dividend payout ratio $=100 \%$, reinvestmentrate $=0 \%$, growth rate, $\mathrm{g}=0$
$D_{1}=E_{1} x$ dividend payout ratio $=\$ 10 \times 1000^{\circ}=\$ 10$
Share price @ cash-cow case, $P_{0}=\frac{\$ 10}{0.15}=\$ 66.67$
Net Present Value of Growth Qpportunity (NPVGO):
NPVGO = Share price @ remvestment case-Share price @ cash-cow case = \$100-\$66.67=\$33.33

## If project return is below required return:

ABC Textiles should act like a cash cow when its growth projects generate negative NPV i.e. project return is bolow the required return. A company should not accept any such projects that fail to achieve roquired return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company sould better act like a cash cow and extend all of its earnings as dividends.

Question 3: Beximco Pharmaceuticals follows a dividend payout policy of $40 \%$ on its êachings. The company has a historical record of achieving an average ROE of $\mathbf{1 5 \%}$. How much should you offer per share if its projected year-end dividend is 200 taka per share and your required return from investment is $\mathbf{1 2 \%}$. Calculate Net Present Value of Growth Opportunity (NPVGO).
Find NPVGO if required return increases to $16 \%$.
Sol: Reinvestment Case:
Here, dividend payout ratio $(\mathrm{DPR})=40 \%=0.40$
Reinvestment or retention rate, $\mathrm{b}=(100-40) \%=60 \%=0.60$
$\mathrm{ROE}=15 \%=0.15, \mathrm{D}_{1}=200 \mathrm{tk}=\mathrm{E} 1 \times \mathrm{DPR}=\mathrm{E} 1 \times 0.40$, i.e. $\mathrm{E} 1=200 / 0.40=500 \mathrm{tk}$
required return, $\mathrm{r}=12 \%=0.12$
Growth rate, $\mathrm{g}=\mathrm{ROE} \times \mathrm{b}=0.15 \times 0.60=0.09=9 \%$

Share price @ reinvestment case, $P_{0}=\frac{D_{1}}{r-g}=\frac{200}{0.12-0.09}=6,666.67 \mathrm{tk}$

## Cash-cow Case:

Dividend payout ratio $=100 \%$, reinvestment rate $=0 \%$, growth rate, $\mathrm{g}=0$
$D_{1}=E_{1}=500 t k$
Share price @ cash-cow case, $P_{0}=\frac{D_{1}}{r-g}=\frac{500}{0.12}=4,166.67 \mathrm{tk}$
Net Present Value of Growth Opportunity (NPVGO):
NPVGO = Share price @ reinvestmentcase - Share price @ cash-cow case = 6, 666.67-4, 166.67
$=2,500 \mathrm{tk}$

## if required return increases to $16 \%$ :

Share price@ reinvestment case, $P_{0}=\frac{D_{1}}{r-g}=\frac{200}{0.16-0.09}=2,857.14 \mathrm{tk}$
Share price @ cash-cow case, $P_{0}=\frac{D_{1}}{r-g}=\frac{500}{0.16}=3,125 \mathrm{tk}$
NPVGO = Share price @ reinvestmentcase-Share price @ cash-cow case = 2, 857.14-3,125
$=-267.86 t k$
Beximco Pharmaceuticals should act like a cash cow whenf growth projects generate negative NPV i.e. project return (15\%) is below the requiredretyrn (16\%). A company should not accept any such projects that fail to achieve required revurn because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better like a cash cow and extend all of its earnings as dividends.

Question 4: Olympic Industries follows a thidend payout policy of $\mathbf{2 5 \%}$ on its earnings. The company has a historical record of achieving an average ROE of $20 \%$. How much should you offer per share if its projected year-end dividend is 100 taka per share and your required return from investment is $\mathbf{2 2 \%}$. Calculate Net Present Value of Growth Opportunity (NPVGO).

## Sol: Reinvestment Case:

Here, dividend payout rate (DPR) $=25 \%=0.25$
Reinvestment or retentien rate, $\mathrm{b}=(100-25) \%=75 \%=0.75$
$\mathrm{ROE}=20 \%=0.2 Q \mathrm{D}_{1}=100 \mathrm{tk}=\mathrm{E} 1 \times \mathrm{DPR}=\mathrm{E} 1 \times 0.25$, i.e. $\mathrm{E} 1=100 / 0.25=400 \mathrm{tk}$
required return, $022 \%=0.22$
Growth rate $-\mathrm{ROE} \times \mathrm{b}=0.20 \times 0.75=0.15=15 \%$
Sharepro@ reinvestment case, $P_{0}=\frac{D_{1}}{r-g}=\frac{100}{0.22-0.15}=1,428.57 \mathrm{tk}$
Cash-cow Case:
Dividend payout ratio $=100 \%$, reinvestment rate $=0 \%$, growth rate, $\mathrm{g}=0$
$D_{1}=E_{1}=400 t \mathrm{k}$
Share price @ cash-cow case, $P_{0}=\frac{D_{1}}{r-g}=\frac{400}{0.22}=1,818.18 \mathrm{tk}$
Net Present Value of Growth Opportunity (NPVGO):
NPVGO = Share price @ reinvestment case - Share price @ cash-cow case =1,428.57tk-1,818.18tk
$=-389.61 \mathrm{tk}$
Olympic Industries should act like a cash cow when its growth projects generate negative NPV i.e. project return $(20 \%)$ is below the required return ( $22 \%$ ). A company should not accept any
such projects that fail to achieve required return because accepting this type of projects will put a downward pressure on the share price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends.

Question 5: XYZ Pharmaceuticals follows a dividend payout policy of $\mathbf{3 0 \%}$ on its earnings. The company has a historical record of achieving an average ROE of $10 \%$. Explain why XYZ can be a potential target for hostile takeover by other companies if investors' required return from investment is above $10 \%$.

Sol: XYZ Pharmaceuticals should act like a cash cow when required return is above project return ( $10 \%$ ). A company should not accept any such projects that fail to achieve required return because accepting this type of projects will put a downward pressure on the shafe price thereby reduce the value of the firm in the market. In such case the company should better act like a cash cow and extend all of its earnings as dividends.
For example, if XYZ' projected year-end EPS, E1 $=1000 \mathrm{Tk}$. then in:

## Reinvestment Case:

Here, dividend payout ratio $(\mathrm{DPR})=30 \%=0.30$
Reinvestment or retention rate, $\mathrm{b}=(100-30) \%=70 \%=0.70$
$\mathrm{D}_{1}=\mathrm{E} 1 \times \mathrm{DPR}=1000 \times 0.30=300 \mathrm{tk}$
$\mathrm{ROE}=10 \%=0.10$, required return, $\mathrm{r}=11 \%=0.11$
Growth rate, $\mathrm{g}=\mathrm{ROE} \times \mathrm{b}=0.10 \times 0.70=0.07=7 \%$
Share price @ reinvestment case, $P_{0}=\frac{D_{1}}{r-g}=\frac{300}{0.11-00^{7}}$

## Cash-cow Case:

Dividend payout ratio $=100 \%$, reinvestment rater $\%$, growth rate, $\mathrm{g}=0$
Shareprice @cash-cow case, $P_{0}=\frac{D_{1}=E_{1}}{r-g}=\frac{1 \sqrt{200}}{0.11}=9,090.9 \mathrm{tk}$
Net Present Value of Growth Opportusidy (NPVGO):
NPVGO $=$ Share price @ reinvestmet case - Share price @ cash-cow case $=7,500 t k-9,090.9 t k$ $=-1,590.9 \mathrm{tk}$

XYZ Pharmaceuticals woulabe subject to takeover, because another firm could buy $51 \%$ or more share of XYZ for tharket price of 7,500 taka per share and increase the value of the firm ( $9,090.9$ taka per share) by changing its investment policy (pay out all earnings as dividends) and make gain of $1,590.9$ taka per share.

Question 6: NTwe assume a stock has an expected dividend payout of $50 \%$, a required rate of rettin of $12 \%$, and an expected growth rate for dividends of $9 \%$, then find the P/E rationd the value of the stock per share for a current EPS of \$2.


## Financial Management Bond Valuation Problems and Solution

Question 1: How much should you pay today to purchase a $10 \%$ semi-annual coupon bond of 3 years maturity where the bond comes with face value of $\$ \mathbf{1 , 0 0 0}$ and yield to maturity $\mathbf{1 2 \%}$ ?
Sol: Here, Coupon payment, $\mathrm{C}=10 / 2 \%=5 \%$ of $\$ 1,000=\$ 50, \mathrm{r}=12 / 2 \%=6 \%=0.06$

| Time | Cash <br> Flow | Present Value |
| :--- | :--- | :--- |
| $\mathbf{1}$ | 50 | Present value, $P_{t}=F(1+r)^{-1}$ <br> $50(1+0.06)^{-1}=\$ 47.17$ |
| $\mathbf{2}$ | 50 | $50(1+0.06)^{-2}=\$ 44.5$ |
| $\mathbf{3}$ | 50 | $50(1+0.06)^{-3}=\$ 41.98$ |
| $\mathbf{4}$ | 50 | $50(1+0.06)^{-4}=\$ 39.6$ |
| $\mathbf{5}$ | 50 | $50(1+0.06)^{-5}=\$ 37.36$ |
| $\mathbf{6}$ | 1050 | $1050(1+0.06)^{-6}=\$ 740.2$ <br> Price, $\Sigma=950.82$ |

Question 2: Ford issued a 30 years maturity bond 28 years before where it promises to make $12 \%$ semi-annual coupon payment and a paymentof $\$ 1,000$ face value at maturity. How much should you pay today to purchase this dond if yield to maturity for 2 years Treasury note is $15 \%$ ?
Sol: Here, maturity left $=2$ years, coupon payment $\mathrm{C}=12 / 2 \%=6 \%$ of $\$ 1,000=\$ 60, \mathrm{r}=15 / 2$ $\%=7.5 \%=0.075$

| Time | Cash <br> Flow | PresentYalue <br> $\mathbf{1}$ <br> $\mathbf{2}$ |
| :--- | :--- | :--- |
| $\mathbf{3}$ | 60 | 60 |

Question 3: Son purchased a 50 years maturity bond with 2 years remaining at a price of \$867 wherethe bond promises to make $10 \%$ semi-annual coupon payment and a single paymentoof $\$ 1,000$ at maturity. Find the rate of yield to maturity (YTM) Sam will receive frombis investment in bond.

## Soll

This is a discount bond i.e. the bond is selling at lower price than its face value
A bond is sold at discount when coupon rate is lower than YTM. As the bond is
selling at discount, YTM must be higher than coupon rate of $10 \%$.
Here, maturity left $=2$ years, bond price $=\$ 867$, coupon payment, $\mathrm{C}=10 / 2 \%=5 \%$ of $\$ 1,000$ $=\$ 50$, YTM, $\mathrm{r}=$ ?
Make A Guess: $\$ 867$ investment is providing a total cash flow of ( $4 \times \$ 50+\$ 1000$ ) $=\$ 1200$
Interest earned $=\$ 1200-\$ 867=\$ 333$ in 2 years.
If we consider simple interest, $\$ 867 \times \mathrm{rx} 2=\$ 333$, then,

$$
\mathrm{r}=\frac{\$ 333}{\$ 867 \times 2}=0.192=19.2 \%
$$

| Time | Cash Flow | Net Present Value@20\% $r=20 / 2=10 \%=0.1$ | Net Present Value@17\% $r=17 / 2=8.5 \%=0.085$ |
| :---: | :---: | :---: | :---: |
| 0 | (867) | (867) | (867) |
| 1 | 50 | $\begin{gathered} \text { Present value, } P_{t}=F(1+r)^{-t} \\ 50(1+0.1)^{-1}=\$ 45.45 \end{gathered}$ | $\begin{aligned} \text { Present value, } P_{t} & =F(1+r)^{-t} \\ 50(1+0.085)^{-1} & =\$ 46.08 \end{aligned}$ |
| 2 | 50 | $50(1+0.1)^{-2}=\$ 41.32$ | $50(1+0.085)^{-2}=\$ 42.47$ |
| 3 | 50 | $50(1+0.1)^{-3}=\$ 37.56$ | $50(1+0.085)^{-3}=\$ 39$ |
| 4 | 1050 | $\begin{gathered} 1050(1+0.1)^{-4}=\$ 717.16 \\ \text { NPV, } \Sigma=841.5-\$ 867=-\$ 25.49 \end{gathered}$ | $\begin{gathered} 1050(1+0.085)^{-4}=\$(57.65 \\ \text { NPV, } \Sigma=\$ 885.35 \bigcirc 867=\$ 18.35 \end{gathered}$ |

$Y T M \quad(\mathrm{r})=L+\left[\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)\right]=0.17+\frac{\$ 18.35}{\$ 18.35-(-\$ 25.49)} x(0.20-0.97)$
$=0.1825=18.25 \%$
Question 4: Mr. Karim paid $\$ 750$ to purchase a 30 years matufity bond with 2.5 years remaining, where the bond promises to make $8 \%$ semi-annual coupon payment and a single payment of $\$ 1,000$ at maturity. Find the rate of yied to maturity (YTM) Mr. Karim will receive from his investment in bond.
Sol: Here, bond price $=\$ 750$, maturity left $=2.5$ years coupon payment, $\mathrm{C}=8 / 2 \%=4 \%$ of $\$ 1,000=\$ 40$, YTM, $\mathrm{r}=$ ?
Make A Guess: $\$ 750$ investment is providing a totala Interest earned $=\$ 1200-\$ 750=\$ 450$ in 2.5 years.
If we consider simple interest, $\$ 750 \mathrm{xrx} 2.5 \$ 450$, then,
$\mathrm{r}=\frac{\$ 450}{\$ 750 \times 2.5}=0.24=24 \%$

| Time | Cash <br> Flow | Net Present Value@24\% $r=24 / 2=12 \%=0.12$ | $\begin{gathered} \text { Net Present Value @20\% } \\ \mathbf{r}=\mathbf{2 0} / \mathbf{2}=\mathbf{1 0 \%}=\mathbf{0 . 1 0} \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| 0 | (750) | (750) | (750) |
| 1 | 40 | Present value, $P_{t}=F(1+r)^{-t}$ $40(1+0.12)^{-1}=\$ 35.71$ | $\begin{aligned} \text { Present value, } P_{t} & =F(1+r)^{-} \\ 40(1+0.10)^{-1} & =\$ 36.36 \end{aligned}$ |
| 2 | 40 | $40(1+0.12)^{-2}=\$ 31.88$ | $40(1+0.10)^{-2}=\$ 33.05$ |
| 3 | 0 | $40(1+0.12)^{-3}=\$ 28.47$ | $40(1+0.10)^{-3}=\$ 30.05$ |
| 4 |  | $40(1+0.12)^{-4}=\$ 25.42$ | $40(1+0.10)^{-4}=\$ 27.32$ |
| $\frac{5}{4}$ | 1040 | $\begin{gathered} 1040(1+0.12)^{-5}=\$ 590.12 \\ \text { NPV, } \Sigma \$ 711.61-\$ 750=-\$ 38.38 \end{gathered}$ | $\begin{gathered} 1040(1+0.10)^{-5}=\$ 645.75 \\ \text { NPV, } \Sigma=\$ 772.55-\$ 750=\$ 22.55 \end{gathered}$ |

$$
Y T M \quad(\mathrm{r})=L+\left[\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)\right]=0.20+\frac{\$ 22.55}{\$ 22.55-(-\$ 38.38)} x(0.24-0.20)
$$

$=0.2148=21.48 \%$
Question 5: Mr. Imdad is considering a $12 \%$ semi-annual coupon bond with 3 years maturity where face value is $\$ 5,000$ and yield to maturity, $y=8 \%$. How much Mr. Imdad should pay to purchase the bond? Find effective maturity of the bond. If the bond`s yield to maturity increases by 0.6 percentage points, calculate the percentage change in the bond price and therefore, the new price.
Sol: Here, coupon payment, $C=12 / 2 \%=6 \%$ of $\$ 5,000=\$ 300$, maturity $=3$ years,
$\mathrm{r}=8 / 2 \%=4 \%=0.04$

| Time (t) | Cash <br> Flow | Present Value | Weight <br> ( $\mathrm{W}_{\mathrm{t}}$ ) | Time $x$ Weight (t) $\times\left(W_{t}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 300 | $\begin{array}{r} \text { Present value, } P_{t}=F(1+r) \\ 300(1+0.04)^{-1}=\$ 288.46 \end{array}$ | $\frac{\$ 288.46}{5524.21}=0.052$ | 0.052 |
| 2 | 300 | $300(1+0.04)^{-2}=\$ 277.36$ | $\frac{\$ 277.36}{5524.21}=0.05$ | 0.1 |
| 3 | 300 | $300(1+0.04)^{-3}=\$ 266.7$ | $\frac{\$ 266.7}{5524.21}=0.048$ | 0.144 |
| 4 | 300 | $300(1+0.04)^{-4}=\$ 256.44$ | $\frac{\$ 256.44}{5524.21}=0.046$ | $0.185 \quad \rho^{\gamma}$ |
| 5 | 300 | $300(1+0.04)^{-5}=\$ 246.57$ | $\frac{\$ 246.57}{5524.21}=0.044$ | $0.2233^{\circ}$ |
| 6 | 5300 | $\begin{gathered} 5300(1+0.04)^{-6}=\$ 4188.66 \\ \text { Price, } \Sigma=5524.21 \\ \hline \end{gathered}$ | $\frac{\$ 4188.66}{5524.21}=0.758$ | $\begin{aligned} & \text { Duration, } \Sigma=5.255 \end{aligned}$ |

Here, percentage change in yield to maturity, $\Delta \mathrm{Y}=0.6 / 2 \%=0.3 \%$

$$
\begin{aligned}
& \text { Here modified duration, } D^{*}=D /(1+y)=5.255 / 1.04=502(\text { semi-annual }) \\
& \text { i.e. } \frac{\Delta P}{P}=-D^{*} \mathrm{x} \Delta y=-5.052 x 0.3 \%=-1.5156 \% \\
& \text { Bond price will decrease by } 1.51 \% \\
& \frac{\Delta P}{P}=-1.5156 \%=-0.015156 \\
& \text { i.e. } \Delta P=P(-0.015156)=\$ 5524.21(-15156)=-\$ 83.72 \\
& \text { Bond price will decrease by } \$ 83.72 \\
& \text { New price wil be }=\$ 5524.2 \text { \$83.72 }=\$ 5440.49
\end{aligned}
$$

Question 6: Mr. Azad is workig in Lanka Bangla Finance Company in Bangladesh. Lanka Bangla invests a signifieant amount of their capital in Treasury Securities issued by Bangladesh Bank. Bangladesh Bank is going to increase their lending rate from 5\% to $\mathbf{5 . 5 \%}$. Find out the impate of this change in interest rate on Lanka Bangla' Treasury security portfolio of $100,000,000$ if the semi-annual duration of the portfolio is 2.67.
Sol: Here, current due of portfolio or investment $=100,000,000$ taka, semi-annual duration, $\mathrm{D}=2.67$, initial yidd to maturity, $\mathrm{Y}=5 / 2 \%=2.5 \%=0.025$
Percentage chané in interest rate, $\Delta \mathrm{Y}=0.5 / 2 \%=0.25 \%$
Here modifred duration,$D^{*}=D /(1+y)=2.67 / 1.025=2.6048(\operatorname{sem} i-a n n u a l)$
i.e. $\frac{\Delta P}{\text { P }}$. $D^{*} \Delta y=-2.6048 \times 0.25 \%=-0.6512 \%$
PQtfolio value will decrease by $0.6512 \%$
$\frac{\Delta P}{P}=-0.6512 \%=-0.006512$
i.e. $\Delta P=P(-0.006512)=100,000,000(-0.006512)=-651,200$

Portfolio value will decrease by 651,200 taka
Question 7: Mr. Saikat is planning to invest in a 7\% semi-annual coupon bond with 2.5 years maturity where face value is $\mathbf{\$ 1 0 , 0 0 0}$ and yield to maturity, $\mathrm{y}=\mathbf{1 0 \%}$. How much Mr. Saikat should pay to purchase the bond? Find effective maturity of the bond. If the bond's yield to maturity decreases by 2.2 percentage points, calculate change in the bond price and therefore, the new price.

Sol: Here, coupon payment, $\mathrm{C}=7 / 2 \%=3.5 \%$ of $\$ 10,000=\$ 350$, maturity $=2.5$ years, $r=10 / 2 \%=5 \%=0.05$

| Time <br> $(\mathbf{t})$ | Cash <br> Flow | Present Value | Weight <br> $\left(\mathbf{W}_{\mathbf{t}}\right.$ | Time x Weight <br> $(\mathbf{t}) \mathbf{x}\left(\mathbf{W}_{\mathbf{t}}\right)$ |
| :--- | :--- | :--- | :--- | :---: |
| $\mathbf{1}$ | 350 | Present value, $P_{t}=F(1+r)^{-1}$ <br> $300(1+0.04)^{-1}=\$ 288.46$ | $\$ 288.46$ <br> 5524.21$=0.052$ | 0.052 |
| $\mathbf{2}$ | 350 | $300(1+0.04)^{-2}=\$ 277.36$ | $\frac{\$ 277.36}{5524.21}=0.05$ | 0.1 |
| $\mathbf{3}$ | 350 | $300(1+0.04)^{-3}=\$ 266.7$ | $\frac{\$ 266.7}{5524.21}=0.048$ | 0.144 |
| $\mathbf{4}$ | 350 | $300(1+0.04)^{-4}=\$ 256.44$ | $\frac{\$ 256.44}{5524.21}=0.046$ | $0.1850^{\circ}$ |
| $\mathbf{5}$ | 10,350 | $5300(1+0.04)^{-6}=\$ 4188.66$ <br> Price, $\Sigma=5524.21$ | $\frac{\$ 4188.66}{5524.21}=0.758$ | Dugation, $\Sigma=5.255$ |

Here, percentage change in yield to maturity, $\Delta \mathrm{Y}=0.6 / 2 \%=0.3 \%$
Here modified duration, $D^{*}=D /(1+y)=5.255 / 1.04=5.052$ (semianual)
i.e. $\frac{\Delta P}{P}=-D^{*} \mathrm{x} \Delta y=-5.052 x 0.3 \%=-1.5156 \%$

Bond price will decrease by $1.51 \%$
$\frac{\Delta P}{P}=-1.5156 \%=-0.015156$
i.e. $\Delta P=P(-0.015156)=\$ 5524.21(-0.015156) \$ 83.72$

Bond price will decrease by $\$ 83.72$
New price wil be $=\$ 5524.21-\$ 83.72 \$ 440.49$

## Financial Management

## Risk \& Return Problems and Solution

Question 1: Suppose you have invested $\mathbf{\$ 3 0 , 0 0 0}$ in the following four stocks:

| Security | Amount Invested (\$) | Beta |
| :---: | :---: | :---: |
| Stock A | $\mathbf{5 , 0 0 0}$ | $\mathbf{0 . 7 5}$ |
| Stock B | $\mathbf{1 0 , 0 0 0}$ | $\mathbf{1 . 1 0}$ |
| Stock C | $\mathbf{8 , 0 0 0}$ | $\mathbf{1 . 3 6}$ |
| Stock D | $\mathbf{7 , 0 0 0}$ | $\mathbf{1 . 8 8}$ |

The risk-free rate is $4 \%$ and the expected return on the market portfolio is $15 \%$. Based on the CAPM, what is the expected return on the above portfolio?
Sol:
Based on CAPM: $E\left(r_{p}\right)=r_{f}+\beta_{p}\left[E\left(r_{m}\right)-r_{f}\right]=0.04+\beta_{p}[0.15-0.04]$
we dont know the value of portfolio beta $\beta_{p}$; we need to find it using avalable information
$\beta_{p}=w_{A} \beta_{A}+w_{B} \beta_{B}+w_{C} \beta_{C}+w_{C} \beta_{C}=\frac{5,000}{30,000} x 0.75+\frac{10,000}{30,000} \times 1.10+\frac{8,080}{80,000} \times 1.36+\frac{7,000}{30,000} \times 1.88$
$=0.125+0.367+0.3627+0.4387=1.293$
Now,$E\left(r_{p}\right)=r_{f}+\beta_{p}\left[E\left(r_{m}\right)-r_{f}\right]=0.04+1.293[0.15-0.04\}=0.18223=18.223 \%$

Question 2: You have been provided the following data the securities of three firms and the market:

| Security | $\bar{R}$ | $\rho_{i, m}$ | $\beta_{i}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Stock A | $\mathbf{0 . 1 3}$ | 0,12 | (i) | $\mathbf{0 . 9 0}$ |
| Stock B | $\mathbf{0 . 1 6}$ | (ii) | $\mathbf{0 . 4}$ | $\mathbf{1 . 1 0}$ |
| Stock C | $\mathbf{0 . 2 5}$ | $\mathbf{0}^{\boldsymbol{n}}$ | $\mathbf{0 . 2 4}$ | $\mathbf{0 . 7 5}$ |
| Market | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 1}$ | (iv) | (iii) |
| Risk-free Asset $\left(r_{f}\right)$ | $\mathbf{0 . 0 5}$ | (vi) | (vii) | (viii) |

Assume the CAPM holds trae.
a) Fill in the missing yalues in the table.
b) Provide an eyadyation of the investment performance of the three firms
c) What is youfinvestment recommendation? Why?

Sol:
(i) Correlation ${ }^{\circ}$ efficient between stock A and market, $\rho_{A, m}=$ ?

$0.9=\frac{\rho_{\mathrm{A}, m} x 0.12}{0.1}$
or,$\rho_{\mathrm{A}, m} \times 0.12=0.9 \times 0.1$
$o r, \rho_{\mathrm{A}, m}=\frac{0.9 \times 0.1}{0.12}=0.75$
(ii) Standard deviation of stock $B, \sigma_{B}=$ ?

$$
\begin{aligned}
& \beta_{B}=\frac{\rho_{B, m} x \sigma_{B}}{\sigma_{m}} \\
& \text { or }, \sigma_{B}=\frac{\beta_{B} x \sigma_{m}}{\rho_{B, m}}=\frac{1.1 \times 0.1}{0.4}=0.275=27.5 \%
\end{aligned}
$$

(iii) $\beta_{c}=\frac{\rho_{C, m} x \sigma_{C}}{\sigma_{m}}=\frac{0.75 \times 0.24}{0.1}=1.8$
(iv) Correlation coefficient between market and market is 1
(v) Market Beta is 1
(vi) Risk-free asset is considered to have no risk; hence standard deviation $\sigma=0$
(vii) Correlation coefficient between risk free asset and market is zero because riso free asset have no risk associated with it and it is independent of market movement.
(viii) Risk free asset has no market risk or beta.
$b$ and $C$ :

| Stock | Expected Return | Required Return (CAPM) | Recommendation |  |
| :---: | :---: | :---: | :---: | :---: |
| A | $13 \%$ | $0.05+0.9(0.15-0.05)=14 \%$ | Overpriced | Sell |
| B | $16 \%$ | $0.05+1.1(0.15-0.05)=16 \%$ |  | Buy |
| B Correctly Priced | or <br> sell |  |  |  |
| C | $25 \%$ | $0.05+1.8(0.15-0.05)=23 \%$ | Underpriced | Buy |

## Question 3:

a) What is the expected return and variance on an equally weighted portfolio of the following three stocks?

| State of Economy | Probability of State of Economy | Expected Retrun |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Stock A | Stock B | Stock C |
| Boom |  | 0.07 | 0.15 | 0.33 |
| Bust | $\stackrel{5}{5}$ | 0.13 | 0.03 | -0.06 |

Sol:

| State of Economy | Prob. | Expected Return |  |  | Portfolio Return (r) | $\left(\mathrm{r}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Stack A | Stock B | Stock C |  |  |
| Boom | 0.8 | 0.07 | 0.15 | 0.33 | $(0.07+0.15+0.33) / 3=0.1834$ | 0.0336 |
| Bust | 0.2 | 0.13 | 0.03 | -0.06 | 0.03334 | 0.001112 |

Expected portolio return, $\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=0.8 \times 0.1834+0.2 \times 0.03334=0.153388=15.3 \%$
Variance, $\rho_{p}^{\prime}=0.8 \times 0.0336+0.2 \times 0.001112-(0.153388)^{2}=0.003574$
Standare (Deviation, $\sigma_{p}=0.05978=5.978 \%$
b)What is the expected return and variance of a portfolio invested $20 \%$ each in $A$ and $B$, and the remaining in C ?

| State of <br> Economy | Prob. | Expected Return |  | Portfolio Return (r) | $\left(\mathbf{r}^{2}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Boom |  | $\mathbf{A}$ | $\mathbf{0 . 0 7}$ |  |  | $\mathbf{0 . 3 3}$ |
| $\mathbf{0 . 2 x} \mathbf{0 . 0 7}+\mathbf{0 . 2 x} \mathbf{0 . 1 5 + 0 . 6 x . 3 3}=\mathbf{0 . 2 4 2}$ | $\mathbf{0 . 0 5 8 5 6 4}$ |  |  |  |  |
| Bust | $\mathbf{0 . 2}$ | $\mathbf{0 . 1 3}$ | $\mathbf{0 . 0 3}$ | $-\mathbf{0 . 0 6}$ | $\mathbf{- 0 . 0 0 4}$ | $\mathbf{0 . 0 0 0 0 1 6}$ |

Expected portfolio return, $\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=0.8 \times 0.242+0.2 \times(-0.004)=0.1928=19.28 \%$
Variance, $\sigma_{p}^{2}=0.8 \times 0.058564+0.2 \times 0.000016-(0.1928)^{2}=0.00968$
Standard Deviation, $\sigma_{p}=0.0984=9.84 \%$

Question 4: Your portfolio is invested $\mathbf{3 0 \%}$ each in stock A and C and the remaining in $B$ ? What is the expected return, variance and standard deviation of the portfolio?

| State of Economy | Prob. | Expected Return |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |
| Boom | 0.20 | 0.30 | 0.45 | 0.33 |
| Good | 0.35 | 0.12 | 0.10 | 0.15 |
| Poor |  | 0.01 | -0.15 | -0.05 |
| Bust | 0.15 | -0.06 | -0.30 | -0.0 |

Sol:

| State of Economy | Prob. | Expected Return |  |  | Portfolio Return (r) | $\partial^{\left(r^{2}\right)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |  |  |
| Boom | 0.20 | 0.30 | 0.45 | 0.33 | $0.3 \times 0.3+0.4 \times 0.45+0.3 \times .33=0.369$ | 0.136161 |
| Good | 0.35 | 0.12 | 0.10 | 0.15 | 0.121 | 0.014641 |
| Poor | 0.30 | 0.01 | -0.15 | -0.05 | -0.072 | 0.005184 |
| Bust | 0.15 | -0.06 | -0.30 | -0.09 | -0.165 ${ }^{\circ}$ | 0.027225 |

Exp. Port. return, $\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=0.2 \times 0.369+0.35 \times 0.121+0.30 \times(-0.072)+0 .(0) \times(-0.165)=0.0698$ =6.98\%
Variance, $\sigma_{p}^{2}=0.2 \times 0.136161+0.35 \times 0.014641+0.30 \times 0.00584+0.15 \times 0.027225-$ $(0.0698)^{2}=0.0331$
Standard Deviation, $\sigma_{p}=0.182=18.2 \%$
Question 5: You own a stock portfolio invested $25 \%$ in stoek $Q, 20 \%$ in stock $R, 15 \%$ in
stock S, and the remaining in stock T? The betas for these four stocks are $0.75,1.90,1.38$ and 1.16 respectively. What is the portfolio beta?
Sol:

$$
\text { Portfolio beta } \beta_{p}=w_{Q} \beta_{Q}+w_{R} \beta_{R}+w_{S} \beta_{S}+w_{T} \beta_{T}-25 x 0.75+0.2 \times 1.9+0.15 \times 1.38+0.4 \times 1.16=1.2385
$$

Question 6: You own a portfolio equalky jnvested in a risk-free asset and two stocks. If one of the stocks has a beta of 1.85 and the total portfolio is equally as risky as the market, what must the beta be forthe other stock in your portfolio?
Sol:


Beta of theother stock is 1.15
Question 7: A stock has a beta of $\mathbf{1 . 2 5}$, the expected return on the market is $\mathbf{1 2 \%}$ and the risfree rate is $5 \%$. According to CAPM, what must the expected return on this stock bef
Sol: Based on CAPM : $E(r)=r_{f}+\beta_{i}\left[E\left(r_{m}\right)-r_{f}\right]=0.05+1.25[0.12-0.05]=0.1375=13.75 \%$
Question 8: A stock has an expected return of $\mathbf{1 4 . 2 \%}$, the risk free rate is $\mathbf{4 \%}$ and the market risk premium is $\mathbf{7 \%}$. According to CAPM, what must the beta of this stock be?
Sol: Based on CAPM : $E(r)=r_{f}+\beta_{i}\left[E\left(r_{m}\right)-r_{f}\right]=0.04+\beta x 0.07$

```
or, \(0.142=0.04+\beta x 0.07\)
or, \(0.142-0.04=\beta \times 0.07\)
```

or,$\beta=\frac{0.102}{0.07}=1.457$

Question 9: A stock has an expected return of $10.5 \%$, its beta is 0.73 , and the risk free rate is $5.5 \%$. According to CAPM, what must the expected return on market be?
Sol: B ased on CAPM : $E(r)=r_{f}+\beta_{i}\left[E\left(r_{m}\right)-r_{f}\right]=0.055+0.73\left[E\left(r_{m}\right)-0.055\right]$
or, $0.105=0.055+0.73\left[E\left(r_{m}\right)-0.055\right]$
or,$E\left(r_{m}\right)-0.055=\frac{0.105-0.055}{0.73}$
or, $E\left(r_{m}\right)=\frac{0.105-0.055}{0.73}+0.055=0.12349=12.349 \%$
Question 10: A stock has an expected return of $16.2 \%$, a beta of 1.75 , and the expected return on the market is $\mathbf{1 1 \%}$. According to CAPM, what must the risk free rate be?
Sol: Based on CAPM : $E(r)=r_{f}+\beta_{i}\left[E\left(r_{m}\right)-r_{f}\right]=r_{f}+1.75\left[0.11-r_{f}\right]=r_{f}+0.1925-1.75$ or, $0.162=0.1925-0.75 r_{f}$
or, $r_{f}=\frac{0.1925-0.162}{0.75}=0.0407=4.07 \%$
Question 11: A stock has a beta of 0.92 , and an expected return of $10.30 \%$. A risk free asset currently earns $5 \%$.
a) Based on CAPM, what is the expected return on a portfolfothat is equally invested in the two assets?
Sol: Based on CAPM : $E(r)=r_{f}+\beta_{i}\left[E\left(r_{m}\right)-r_{f}\right]=0.05+0,92\left[E\left(r_{m}\right)-0.05\right]$
or, $0.103=0.05+0.92\left[E\left(r_{m}\right)-0.05\right]$
or, $E\left(r_{m}\right)-0.05=\frac{0.103-0.05}{0.92}$
or,$E\left(r_{m}\right)=\frac{0.103-0.05}{0.92}+0.05=0.10$
Portfolio beta $\beta_{p}=1=w_{1} \beta_{1}+w_{2} \beta_{2}=\frac{1}{2} \times 0+\frac{1}{2} \times 0.92=0.46$
B ased on CAPM: $E\left(r_{p}\right)=r_{f}^{+} \beta_{p}\left[E\left(r_{m}\right)-r_{f}\right]=0.05+0.46[0.1076-0.05]=0.0765=7.65 \%$

## If it is not based on- A'PM:

Expected Portfoligeturn, $\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=0.5 \times 0.103+0.5 \times 0.05=0.0765=7.65 \%$
b) If a portfoliosof the two assets has a beta of 0.50 , what are the portfolio weights?

B ased op APM : $E\left(r_{p}\right)=r_{f}+\beta_{p}\left[E\left(r_{m}\right)-r_{f}\right]=0.05+0.5[0.1076-0.05]=0.0788=7.88 \%$

$$
0.0788=w \times 0.05+(1-w) \times 0.103
$$

$$
0.0788=0.05 w+0.103-0.103 w
$$

Or, $0.053 w=0.0242$

$$
o r, w=\frac{0.0242}{0.053}=0.4566=45.66 \%
$$

Investment in risk free asset is $45.66 \%$ and the remaining $54.34 \%$ is in the stock.
c) If a portfolio of the two assets has an expected return of $\mathbf{9 \%}$, what is its beta?

$$
\begin{aligned}
& \text { B ased on CAPM: } 0.09=r_{f}+\beta_{p}\left[E\left(r_{m}\right)-r_{f}\right]=0.05+\beta_{p}[0.1076-0.05] \\
& \text { or, } \beta_{p}=\frac{0.09-0.05}{0.1076-0.05}=0.6945
\end{aligned}
$$

d) If a portfolio of the two assets has a beta of 1.84 , what are the portfolio weights? How do you interpret the weights for the two assets in this case? Explain.

Portfolio beta $\beta_{p}=1.84=w_{1} \beta_{1}+w_{2} \beta_{2}=w x 0+(1-w) x 0.92$
or, $1-w=\frac{1.84}{0.92}=2$
or, $w=1-\frac{1.84}{0.92}=-1$
Investment in the stock is $200 \%$ of capital, which is made possible by borrowing $100 \%$ of capital at risk free rate of $5 \%$. Suppose, if an investor has $\$ 10,000$ to invest in these two assets then he will invest $\$ 20,000$ in stock by borrowing $\$ 10,000$ at risk free rate of $5 \%$.
Question 12: Stock Y has a beta of 1.35 and an expected return of $14 \%$. Stock $Z$ has a beta of 0.85 and an expected return of $11.5 \%$. The market risk premium is $6.8 \%$.
a) If the risk free rate is $5.5 \%$, are these stocks correctly priced?

Sol: For the two stocks to be correctly priced their reward to risk ratio shouldde same.
Rew ard-risk ratio of stock $Y=\frac{0.14-0.055}{1.35}=0.063$
Rew ard-risk ratio of stock $Z=\frac{0.115-0.055}{0.85}=0.0705$
Rew ard-risk ratio of stock $Y \neq R e w a r d-r i s k$ ratio of stock $Z$
Stock Y and Zare not correctly priced.
b) What would the risk free rate have to be for the tho stocks to be correctly priced?

Rew ard-risk ratio of stock $Y=R e w$ ard-risk ratio gfistock $Z$
or,$\frac{0.14-r_{f}}{1.35}=\frac{0.115-r_{f}}{0.85}$
or, $0.85 \times 0.14-0.85 r_{f}=1.35 \times 0.115-$
or, $1.35 r_{f}-0.85 r_{f}=1.35 \times 0.115-0.85 \times 0.14$
or, $0.5 r_{f}=0.03625$
or, $r_{f}=0.0725=7.25 \%$

## Question 13:

a) If your portfolioisinvested $40 \%$ each in $A$ and $B$ and the remaining in $C$, what is


| State of <br> Economy | Prob. | Expected Return |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |
| Boom | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 2 0}$ | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 6 0}$ |
| Normal | $\mathbf{0 . 4 0}$ | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 1 2}$ | $\mathbf{0 . 0 5}$ |
| Bust |  | $\mathbf{0 . 0 1}$ | $\mathbf{- 0 . 2 5}$ | $\mathbf{- 0 . 5 0}$ |

Sol

| State of <br> Economy | Prob. | Expected Return |  | Portfolio Return (r) | $\left(\mathbf{r}^{2}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{B}$ | $\mathbf{C}$ |  |  |  |
| Boom | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 2 0}$ | $\mathbf{0 . 3 5}$ | $\mathbf{0 . 6 0}$ | $\mathbf{0 . 4 x} \mathbf{0 . 2 + 0 . 4 x} \mathbf{0 . 3 5 + 0 . 2 x 0 . 6}=\mathbf{0 . 3 4}$ | $\mathbf{0 . 1 1 5 6}$ |
| Normal | $\mathbf{0 . 4 0}$ | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 1 2}$ | $\mathbf{0 . 0 5}$ | $\mathbf{0 . 1 1 8}$ | $\mathbf{0 . 0 1 3 9 2 4}$ |
| Bust | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 0 1}$ | $\mathbf{- 0 . 2 5}$ | $\mathbf{- 0 . 5 0}$ | $\mathbf{- 0 . 1 9 6}$ | $\mathbf{0 . 0 3 8 4 1 6}$ |

Exp. Port. return, $\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=0.35 \times 0.34+0.4 \times 0.118+0.25 \times(-0.196)=0.1172=11.72 \%$
Variance, $\sigma_{p}^{2}=0.35 \times 0.1156+0.4 \times 0.013924+0.25 \times 0.038416-(0.1172)^{2}=0.04189$
Standard Deviation, $\sigma_{p}=0.20469=20.47 \%$
b) If the expected T-bill rate is $\mathbf{3 . 8 0 \%}$, what is the expected risk premium on the portfolio?
Expected risk premium $=11.72 \%-3.80 \%=7.92 \%$
c) If the expected inflation rate is $\mathbf{3 . 5 0 \%}$, what are the approximate and exact expected real returns on the portfolio? What are the approximate and exact expected real risk premiums on the portfolio?
Approximate expected real return $=11.72 \%-3.50 \%=8.22 \%$
Exact expected real return (Fisher Equation):
$1+0.1172=(1+0.0350)(1+r)$
or, $1+r=\frac{1.1172}{1.0350}$
$o r, r=\frac{1.1172}{1.0350}-1=0.07942=7.942 \%$
Approximate expected real risk premium $=7.92 \%-3.50 \%=4.42 \%$
Exact expected real risk premium $=\frac{1.0792}{1.0350}-1=0.0427=4.27 \%$
Question 14: You want to create a portfolio equally as risky as the market, and you have $\$ 1,000,000$ to invest. Given this information, fill in the rest of the following dable:

| Asset | Investment (\$) | Beta |
| :--- | :---: | :---: |
| Stock A | $\mathbf{1 8 0 , 0 0 0}$ | $\mathbf{0 . 7 5}$ |
| Stock B | 290,000 | 1.36 |
| Stock C |  | R45 |
| Risk Free Asset |  |  |

Sol:

$$
\begin{aligned}
& w_{A}=\frac{180}{1000}=0.18 ; \quad w_{B}=\frac{290}{1000}=0.29 ; \\
& \text { Portfolio beta } \beta_{p}=1=w_{A} \beta_{A}+w_{B} \beta_{B}+w_{C} \beta_{C} \\
& \text { or, } 1-0.18 x 0.75-0.29 x 1.3=1.45 x \\
& \text { or, } x=\frac{0.488}{1.45} \\
& \text { Investment in Asset } \mathrm{C}=\frac{0.488}{145} \times 1,000,000=\$ 336,551.72 \\
& \text { Investment in Risk-fyeed }
\end{aligned}
$$

Question 15: You have $\mathbf{\$ 1 0 0 , 0 0 0}$ to invest in a portfolio containing stock $X$, stock $Y$, and a risk-free asset. deu must invest all of your money. Your goal is to create a portfolio that has an expected return of $10.7 \%$ and that has only $\mathbf{8 0 \%}$ of the risk of the overall market. If $X$ has $\mathbf{a f}$ expected return of $17.2 \%$ and a beta of 1.8 , $Y$ has an expected return of $8.75 \%$ and a beta of 0.5 , and the risk free rate is $7 \%$, how much money will you invest in stock?

```
w
Portfolio beta }\mp@subsup{\beta}{p}{}=1\times80%=0.8=1.8X+0.5Y+(1-X - Y)0
i.e.1.8 X + 0.5Y = 0.8
```

$\qquad$

```
                                    Eq(1)
Portfolio return, E (r r})=0.107=0.172X+0.0875Y+(1-X - Y)0.07
or,0.107 = 0.172X + 0.0875Y+0.07-0.07X - 0.07Y
or,0.037 = 0.102X + 0.0175Y
or,0.102X+0.0175Y=0.037
\[
\begin{aligned}
& \mathrm{Eq}(1) \times 0.0175: 0.0315 \mathrm{X}+0.00875 Y=0.014 \\
& -\mathrm{Eq}(2) \times 0.5: \quad 0.051 \mathrm{X}+0.00875 Y=0.0185
\end{aligned}
\]
\[
\begin{aligned}
& 0.0315 \mathrm{X}-0.051 \mathrm{X}=0.014-0.0185 \\
& \text { or, }-0.0195 X=-0.0045 \\
& \text { or, } X=\frac{-0.0045}{-0.0195}=\frac{45}{195} \\
& \text { Investment in } X=\frac{45}{195} \times \$ 100,000=\$ 23,076.92
\end{aligned}
\]

Question 16: You have invested \(40 \%\) of your capital in stock \(A\) and the remaifing in stock B. Historical return over last 5 years shows the following pattern:
\begin{tabular}{|c|c|c|}
\hline Year & \begin{tabular}{c} 
Stock A \\
\((\mathbf{x})\)
\end{tabular} & \begin{tabular}{c} 
Stock B \\
\((\mathbf{y})\)
\end{tabular} \\
\hline 2014 & \(\mathbf{2 5 \%}\) & \(\mathbf{1 5 \%}\) \\
\hline 2013 & \(\mathbf{3 2 \%}\) & \(\mathbf{1 0 \%}\) \\
\hline \(\mathbf{2 0 1 2}\) & \(\mathbf{1 5 \%}\) & \(\mathbf{3 0 \%}\) \\
\hline 2011 & \(\mathbf{2 0 \%}\) & \(\mathbf{- 5 \%}\) \\
\hline
\end{tabular}

Find the expected portfolio return and variance.
\[
\begin{aligned}
& \sum x^{2}=0.2274 ; \quad \sum y^{2}=0.125 ; \quad \sum x=0.92 ; \quad \sum y=8 ; / \sum x y=0.1045 \\
& \bar{x}=0.23 ; \quad \bar{y}=0.125 ; \quad \sigma_{x}=0.06284 ; \quad \sigma_{y}=0.1255
\end{aligned}
\]
\[
\text { Portfolio return, } \mathrm{E}\left(\mathrm{r}_{p}\right)=0.4 \times 0.23+0.6 \times 0.125=0.167=16.7 \%
\]


Sol:
\[
\begin{aligned}
& =\frac{0.1045-\frac{0.92 x 0.5}{}}{\sqrt{\left(0.2274-\frac{0.92^{2}}{4}\right)}=-0.3341} \\
& \text { Portfolio std. } \left.25-\frac{0.5^{2}}{4}\right) \\
& =\sqrt{0.4^{2} \times 0.06284^{2}+0.6^{2} \times 0.125^{2}+2 \times 0.4 \times 0.6 \times 0.06284 \times 0.125 \times(-0.3341)} \\
& =0.8 \times 069=7.069 \%
\end{aligned}
\]

Question 17: Security \(F\) has an expected return of \(\mathbf{1 0 \%}\) and a standard deviation of \(\mathbf{2 6 \%}\) peryear. Security \(G\) has an expected return of \(17 \%\) and a standard deviation of \(58 \%\) per year.
a) What is the expected return on a portfolio composed of \(\mathbf{3 0 \%}\) of security F and \(70 \%\) of security \(\mathbf{G}\) ?
b) If the correlation between the returns of security \(F\) and security \(G\) is 0.25 , what is the standard deviation of the portfolio described in part (a)?
Portfolio return, \(\mathrm{E}\left(\mathrm{r}_{p}\right)=0.3 \times 0.10+0.7 \times 0.17=0.149=14.9 \%\)
Portfolio std. dev., \(\sigma=\sqrt{0.3^{2} \times 0.26^{2}+0.7^{2} \times 0.58^{2}+2 \times 0.3 \times 0.7 \times 0.26 \times 0.58 \times 0.25}=0.4321=43.21 \%\)

Question 18: Suppose the expected returns of stocks \(A\) and \(B\) are \(13 \%\) and \(19 \%\) respectively whereas standard deviations are \(38 \%\) and \(62 \%\) respectively. Calculate expected return and standard deviation of a portfolio that is composed of \(45 \%\) of \(A\) and \(\mathbf{5 5 \%}\) of \(B\) when the correlation between the returns on \(A\) and \(B\) is 0.5 .
Sol:
Portfolio return, \(\mathrm{E}\left(\mathrm{r}_{p}\right)=0.45 \times 0.13+0.55 \times 0.19=0.163=16.3 \%\)
Portfolio std. dev., \(\sigma=\sqrt{0.45^{2} \times 0.38^{2}+0.55^{2} \times 0.62^{2}+2 \times 0.45 \times 0.55 \times 0.38 \times 0.62 \times 0.5}=0.4514=45.14 \%\)
Question 19: You have been provided the following data on the securities of three firms and the market:
\begin{tabular}{|c|c|c|c|c|}
\hline Security & \(\bar{R}\) & \(\sigma_{i}\) & \(\rho_{i, m}\) & \\
\hline Stock A & 0.10 & 0.27 & (i) & 0.85 \\
\hline Stock B & 0.14 & (ii) & 0.5 & 150 \\
\hline Stock C & 0.17 & 0.7 & 0.35 & (iii) \\
\hline Market & 0.12 & 0.2 & (iv) & (v) \\
\hline Risk-free Asset ( \(r_{f}\) ) & 0.05 & (vi) & (vii) & (viii) \\
\hline
\end{tabular}

\section*{Assume the CAPM holds true:}
a) Fill in the missing values in the table.
b) Are these stocks A, B and C correctly priced?
c) What is your investment recommendation? Why? Sol:
(i) Correlation coefficient between stock A and market, \(\rho_{A, m}=\) ?
\(\beta_{A}=\frac{\rho_{\mathrm{A}, m} x \sigma_{A}}{\sigma_{m}}\)
or, \(0.85=\frac{\rho_{\mathrm{A}, m} \times 0.27}{0.2}\)
\[
\text { or }, \rho_{\mathrm{A}, m}=\frac{0.85 \times 0.2}{0.27}=0.6296
\]
(ii) Standard deviation stock \(B, \sigma_{B}=\) ?
\(\beta_{B}=\frac{\rho_{B, m} x \sigma_{B}}{\sigma_{m}} \gamma\)

\(=\frac{\rho_{C, m} x \sigma_{c}}{\sigma_{m}}=\frac{0.35 \times 0.7}{0.2}=1.225\)
(iv) Correlation coefficient between market and market is 1
(v) Market Beta is 1
(vi) Risk-free asset is considered to have no risk; hence standard deviation \(\sigma=0\)
(vii) Correlation coefficient between risk free asset and market is zero because risk free asset have no risk associated with it and it is independent of market movement.
(viii) Risk free asset has no market risk or beta.
\begin{tabular}{|c|c|c|c|c|}
\hline Stock & Expected Return & \multicolumn{2}{|c|}{ Required Return (CAPM) } & \multicolumn{2}{c|}{ Recommendation } \\
\hline A & \(10 \%\) & \(0.05+0.85(0.12-0.05)=10.95 \%\) & Overpriced & Sell \\
\hline B & \(14 \%\) & \(0.05+1.5(0.12-0.05)=15.5 \%\) & Overpriced & Sell \\
\hline C & \(17 \%\) & \(0.05+1.225(0.12-0.05)=13.575 \%\) & Underpriced & Buy \\
\hline
\end{tabular}

Question 20: The market portfolio has an expected return of \(\mathbf{1 2 \%}\) and a standard deviation of \(\mathbf{1 9 \%}\). The risk free rate is \(5 \%\).
a) What is the expected return on a well diversified portfolio with a standard deviation of 7\%?
Sol:
For a well diversified portfolio,
Rew ard-risk ratio of market \(=\) Reward-risk ratio of portfolio
or, \(\frac{0.12-0.05}{0.19}=\frac{r-0.05}{0.07}\)
\(o r, r-0.05=\frac{0.07 \times 0.07}{0.19}\)
\(o r, r=\frac{0.07 \times 0.07}{0.19}+0.05=0.07579=7.579 \%\)
b) What is the standard deviation of a well diversified portfolio with an expected return of \(\mathbf{2 0 \%}\) ?
For a well diversified portfolio,
Reward-risk ratio of market \(=\) Reward-risk ratio of portfolio
\[
\begin{aligned}
& o r, \frac{0.12-0.05}{0.19}=\frac{0.2-0.05}{\sigma} \\
& o r, \sigma=\frac{0.2-0.05}{0.12-0.05} \times 0.19=0.40714=40.714 \%
\end{aligned}
\]

Question 21: A portfolio that combines the risk free asset and the market portfolio has an expected return of \(9 \%\) and a standard deviation of \(13 \%\). The risk free rate is \(5 \%\), and the expected return on the market portfolio is \(12 \%\). Assume the CAPM holds. What expected rate of return would a security earn if it had a 0.45 correlation with the market portfolio and a standard deviation of \(40 \%\) ?
Sol:
```

Portfolio return, E (r
or,0.09 = 0.05w+0.12-0.12
or,0.07w=0.03
or,w}=\frac{0.03}{0.07}=\frac{3}{7
Portfolio variace, \sigma P

```
or, \(0.0 \sim \sigma^{\circ}=\frac{16}{49} x \sigma_{m}^{2}\)
or,\(\sigma_{m}^{2}=\frac{0.0169 \times 49}{16}\)
or, \(\sigma_{m}=\sqrt{\frac{0.0169 \times 49}{16}}=0.2275=22.75 \%\)
Beta of a security, \(\beta_{i}=\frac{\rho_{i, m} x \sigma_{i}}{\sigma_{m}}=\frac{0.45 \times 0.4}{0.2275}=0.7912\)
Expected return of a security, \(\mathrm{E}(\mathrm{r})=r_{f}+\beta_{i}\left(r_{m}-r_{f}\right)=0.05+0.7912(0.12-0.05)=0.10538=10.538 \%\)

Question 22: Suppose the risk free rate is \(\mathbf{4 . 8 \%}\) and the market portfolio has an expected return of \(\mathbf{1 1 . 4 \%}\). The market portfolio has a variance of \(\mathbf{0 . 0 4 2 9}\). Portfolio Z has a correlation coefficient with the market of 0.39 and a variance of 0.1783 . According to the CAPM, what is the expected return on portfolio \(Z\) ?
Sol:
Beta of Portfolio \(Z, \quad \beta_{z}=\frac{\rho_{z, m} x \sigma_{z}}{\sigma_{m}}=\frac{0.39 x \sqrt{0.1783}}{\sqrt{0.0429}}=0.795\)
Expected return on Portfolio Z, \(\mathrm{E}\left(\mathrm{r}_{z}\right)=r_{f}+\beta_{z}\left(r_{m}-r_{f}\right)=0.048+0.795(0.114-0.048)=0.10047=10.047 \%\)
Question 23: Suppose market risk premium is \(\mathbf{7 . 5 \%}\) and the risk free rate is \(\mathbf{4 \%}\). Which of the following stock has the most systematic risk? Which one has the most unsystenatic risk? Which stock is riskier? Explain.
\begin{tabular}{|c|c|c|c|}
\hline State of Economy & Probability & Return from Stock A & Return from Stock B \\
\hline Recession & 0.15 & 0.09 & -0.30 \\
\hline Normal & \(\mathbf{0 . 5 5}\) & \(\mathbf{0 . 4 2}\) & 00.2 \\
\hline Good & \(\mathbf{0 . 3 0}\) & \(\mathbf{0 . 2 6}\) & 0.44 \\
\hline
\end{tabular}

\section*{Sol:}

Expected return from stock \(A, E\left(r_{A}\right)=0.15 \times 0.09+0.55 \times 0.42+0.30 \times 0.26=0.3225=32.25 \%\)
Standard deviation of stock \(A, \sigma_{A}=\sqrt{0.15 \times 0.09^{2}+0.55 x 0.42^{2}+0.3 \times 26^{2}-0.3225^{2}}=0.1204=12.04 \%\)
\(E\left(r_{A}\right)=0.3225=0.04+\beta_{A}(0.075)\)
or, \(\beta_{A}=\frac{0.3225-0.04}{0.075}=3.77\)

Expected return from stock \(B, E\left(r_{B}\right)=0.15 \times(-0.30)+0.55 \times 0.12+0.30 \times 0.44=0.153=15.3 \%\)
Standard deviation of stock \(B, \sigma_{B}=\sqrt{0.15 \times(0) 0)^{2}+0.55 \times 0.12^{2}+0.3 \times 0.44^{2}-0.153^{2}}=0.2368=23.68 \%\)
\(E\left(r_{B}\right)=0.153=0.04+\beta_{B}(0.075)\)
or, \(\beta_{B}=\frac{0.153-0.04}{0.075}=1.507\)
Standard deviation ( \(\sigma\) ) represen total risk (systematic + unsystematic risk) whereas beta ( \(\beta\) ) represents systematic risk.
Total risk of stock \(A\) is \(1 g\) whan stock \(B\), however systematic risk ( \(\beta\) ) is found higher for stock A compared to stock \(B\)
Thus, unsystematic risk stock A is lower than stock B, although stock A is more risky than stock B because unsystem atic risk can be diversified away

Question 24: Suppose you observe the following situation:
\begin{tabular}{|c|c|c|}
\hline Security & Beta & Expected Return \\
\hline A & \(\mathbf{1 . 4}\) & \(\mathbf{0 . 1 5 0}\) \\
\hline B & \(\mathbf{0 . 9}\) & \(\mathbf{0 . 1 1 5}\) \\
\hline
\end{tabular}

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk free rate?
Sol:
Correctly priced securities should have equal rew ard-risk ratio.
Rew ard-risk ratio of \(A=R e w a r d-r i s k\) ratio of \(B\)
or, \(\frac{0.15-r_{f}}{1.4}=\frac{0.115-r_{f}}{0.9}\)
or, \(0.9 \times 0.15-0.9 r_{f}=1.4 \times 0.115-1.4 r_{f}\)
or, \(0.5 r_{f}=0.161-0.135\)
or, \(r_{f}=\frac{0.026}{0.5}=0.052=5.2 \%\)
\(0.15=0.052+1.4\left(r_{m}-0.052\right)\)
or,\(r_{m}-0.052=\frac{0.15-0.052}{1.4}\)
or, \(r_{m}=\frac{0.15-0.052}{1.4}+0.052=0.122=12.2 \%\)
Expected return on the market \(=12.2 \%\)

\section*{Financial Management Cost of Capital Problems and Solutions}

Question 1: The Dybvig Corporation's common stock has a beta of 1.15. If the risk free rate is \(4.5 \%\) and the expected return on the market is \(11 \%\), what is Dybbig's cost of equity capital?
Cost of Equity \(=r_{f}+\beta\left(r_{m}-r_{f}\right)=0.045+1.15(0.11-0.045)=0.11975=11.975 \%\)
Question 2: The Devon Co. just issued a dividend of \$2.4 per share on its common stock. The company is expected to maintain a constant \(5.5 \%\) growth rate in its dividends indefinitely. If the stock sells for \(\mathbf{\$ 5 2}\) a share, what is the company's cost of equity?
\[
\text { Cost of Equity }=\frac{D_{1}}{p_{0}}+g=\frac{D_{0}(1+g)}{p_{0}}+g=\frac{\$ 2.4(1+0.055)}{\$ 52}+0.055=0.10369=
\]


Question 3: Stock in Country Road Industries has a beta of \(\mathbf{0 . 8 5}\). The market risk premium is \(8 \%\), and T-bills are currently yielding \(5 \%\). The companger most recent dividend was \(\$ 1.6\) per share, and dividends are expected to grow ata \(6 \%\) annual rate indefinitely. If the stock sells for \(\$ 37\) per share, what is your best estimate of the company's cost of equity?
Cost of Equity (CAPM \()=r_{f}+\beta\left(r_{m}-r_{f}\right)=0.05+0.85 x 0.080 .118=11.8 \%\)
Cost of Equity (D DM \()=\frac{D_{1}}{p_{0}}+g=\frac{D_{0}(1+g)}{p_{0}}+g=\frac{\$ 1 .(\nmid 0.06)}{\sigma^{2} \$ 37}+0.06=0.10583=10.583 \%\)
Cost of Equity \(=\frac{0.118+0.10583}{2}=0.11192 \sim 1.192 \%\)
Question 4: Advance Inc., is trying to determine its cost of debt. The firm has a debt issue outstanding with 12 years to maturia that is quoted at \(95 \%\) of face value. The issue makes semiannual payments and has a coupon rate of \(8 \%\) annually. What is Advance's pretax cost of debt? If the tax rate \(1 s 35 \%\), what is after tax cost of debt?
The pretax cost of debt is the TM of the company's bond
Bond price \(=95 \%\) of Face alue \(\$ 1,000=\$ 950\)
This is a discount bogi.e. the bond is selling at lower price than its face value.
A bond is sold drscount when coupon rate is lower than YTM. As the bond is
selling at discount, YTM must be higher than coupon rate of \(8 \%\).
NPV @ra\% \(=-\$ 950+\sum_{t=1}^{24} 40 x(1+0.045)^{-t}+1000 x(1+0.045)^{-24}=-\$ 22.47\)
NPV @ \(8.5 \%=-\$ 950+\sum_{t=1}^{24} 40 x(1+0.0425)^{-t}+1000 x(1+0.0425)^{-24}=\$ 12.84\)
\(Y T M=L+\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)=0.085+\frac{12.84}{12.84-(-22.47)} x(0.09-0.085)=0.0868=8.68 \%\)
Pretax cost of debt \(=8.68 \% ; \quad\) After tax cost of debt \(=0.0868(1-0.35)=0.05642=5.642 \%\)

Question 5: Shanken Corp. issued a 30 years, \(7 \%\) semiannual bond 7 years ago. The bond currently sells for 108 percent of its face value. The company's tax rate is \(\mathbf{3 5 \%}\).
a) What is the pretax cost of debt?

The pretax cost of debt is the YTM of the company's bond
Bond price \(=108 \%\) of Face Value \(\$ 1,000=\$ 1,080\)
This is a premium bond i.e. the bond is selling at higher price than its face value.
A bond is sold with premium when coupon rate is higher than YTM. As the bond is
selling at premium, Y TM must be lower than coupon rate of \(7 \%\).
\(\mathrm{NPV} @ 6.5 \%=-\$ 1,080+\sum_{t=1}^{46} 35 x(1+0.0325)^{-t}+1000 x(1+0.0325)^{-46}=-\$ 20.74\)
\(\mathrm{NPV} @ 6 \%=-\$ 1,080+\sum_{t=1}^{46} 35 x(1+0.03)^{-t}+1000 x(1+0.03)^{-46}=\$ 43.87\)
\(Y T M=L+\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)=0.06+\frac{\$ 43.87}{\$ 43.87-(-\$ 20.74)} x(0.085-0.06)=0.0634=6.34 \%\)
Pretax cost of debt \(=6.34 \%\);
b) What is the after tax cost of debt?

After tax cost of debt \(=0.0634(1-0.35)=0.0412=4.12\)
c) Which is more relevant, the pretax or the after tax cost of debt? Why?

After tax cost of debt is more relevant because that islhe actual cost to the company.
d) Suppose the book value of the debt isste is \(\mathbf{\$ 6 0}\) million. In addition, the company has a second debt issue on the market a zero coupon bond with seven years left to maturity; the book value of this issue is \(\$ 80\) million and the bonds sell for \(73 \%\) of par. What is the company's totall book value of debt? The total market value? What is your best estimate ofthe after tax cost of debt now?
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Pretax cost of debt $=4.6 \%$;

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Weighted Avg. Pretax Cost of Debt \(=\frac{\$ 64.8 \mathrm{million}}{\$ 123.2 \mathrm{million}} x 0.0634+\frac{\$ 58.4 \mathrm{million}}{\$ 123.2 \mathrm{million}} x 0.046=5.51 \%\)
After tax cost of debt of the company \(=0.0551(1-0.35)=0.0358=3.58 \%\)

Question 6: Fama's LIamas has a weighted average cost of capital of \(9.8 \%\). The company's cost of equity is \(15 \%\), and its cost of debt is \(7.5 \%\). The tax rate is \(\mathbf{3 5 \%}\). What is Fama's debt-equity ratio?
\(\mathrm{W} \mathrm{ACC}=0.098=w_{e} r_{e}+w_{d} r_{d}(1-T)=w_{e} x 0.15+\left(1-w_{e}\right) \times 0.075 x(1-0.35)\)
or, \(0.098=0.15 w_{e}+0.04875-0.04875 w_{e}\)
or, \(0.04925=0.10125 w_{\text {e }}\)
or,\(w_{e}=\frac{0.04925}{0.10125}=0.4864\)
i.e. \(w_{d}=1-0.4864=0.5136=51.36 \%\)

Debt-Equity ratio \(=\frac{0.5136}{0.4864}=1.055\)
Question 7: Filer manufacturing has 7.5 million shares of comnion stock outstanding. The current share price is \(\$ 49\), and the book value per share is \(\$ 4\). Filer Manufacturing also has two bond issues outstanding. The first bond issue hasaface value of \(\mathbf{\$ 6 0}\) million, pays \(\mathbf{7 \%}\) semi-annual coupon and sells for \(\mathbf{9 3 \%}\) of par. Thesecond issue has a face value of \(\mathbf{\$ 5 0}\) million, pays \(6.5 \%\) semiannual coupon, and sells for \(\mathbf{9 6 . 5 \%}\) of par. The first issue matures in 10 years, the second in 6 years.
a) What are Filer's capital structure weights on a book value basis?

Book value of common stock \(=7.5 \times\) million \(=\$ 30\) million
Book value of bond \(=\$ 60+\$ 50-\$ 110\) million
Proportion of debt in capiptructure, \(w_{d}=\frac{110}{110+30}=0.7857=78.57 \%\)
Proportion of equity capital structure, \(w_{e}=\frac{30}{110+30}=0.2142=21.42 \%\)
b) What are Fiter's capital structure weights on a market value basis?

Market valueg common stock \(=7.5 x \$ 49\) million \(=\$ 367.5\) million
Market varue of bond \(=\$ 60 \times 93 \%+\$ 50 \times 96.5 \%=\$ 104.05\) million
Pro Ytion of debt in capital structure, \(w_{d}=\frac{104.05}{104.05+367.5}=0.2206=22.06 \%\)
Proportion of equity in capital structure, \(w_{e}=\frac{367.5}{104.05+367.5}=0.7793=77.93 \%\)
c) Which are more relevant, the book value or market value weights? Why?

The market value weights are more relevant. The market value represents the true value of the firm and its capital structure.
d) Suppose the company's stock has a beta of 1.2 , the risk free rate is \(\mathbf{5 . 2 \%}\), and the market risk premium is \(\mathbf{7 \%}\). Assume that the overall cost of debt is the weighted average of the two outstanding debt issues. The tax rate is \(35 \%\). What is the company's WACC?
The pretax cost of a bond is the YTM of it
Y TM of Bond 1:
Bond price \(=93 \%\) of Face Value \(\$ 1,000=\$ 930\)
This is a discount bond i.e. the bond is selling at lower price than its face value.
A bond is sold at discount when coupon rate is lower than YTM. As the bond is selling at discount, Y TM must be higher than coupon rate of \(7 \%\).
\(\mathrm{NPV} @ 8 \%=-\$ 930+\sum_{t=1}^{20} 35 x(1+0.04)^{-t}+1000 x(1+0.04)^{-20}=\$ 2.04\)
\(\mathrm{NPV} @ 8.5 \%=-\$ 930+\sum_{t=1}^{20} 35 x(1+0.0425)^{-t}+1000 x(1+0.0425)^{-20}\)
\(Y T M=L+\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)=0.08+\frac{\$ 2.04}{\$ 2.04-(-\$ 29.7)}\)
Pretax cost of debt1 \(=8.03 \%\)
YTM of Bond 2:
Bond price \(=96.5 \%\) of Face Value \(\$ 1,000=\$ 965\)
This is a discount bond i.e. the bond is selling lower price than its face value.
A bond is sold at discount when coupon is lower than YTM. As the bond is selling at discount, YTM must be higheg than coupon rate of \(6.5 \%\).
\(\mathrm{NPV} @ 7 \%=-\$ 965+\sum_{t=1}^{12} 32.5 x\left(\mathrm{c} \mathrm{C}_{0.035)^{-t}+1000 x(1+0.035)^{-12}=\$ 10.84}\right.\)
\(\mathrm{NPV} @ 7.5 \%=-\$ 965+{ }^{12} 2.5 x(1+0.0375)^{-t}+1000 x(1+0.0375)^{-12}=-\$ 12.61\)
\(Y T M=L+\frac{N P V}{N P V-N P V_{H}} x(H-L)=0.07+\frac{\$ 10.84}{\$ 10.84-(-\$ 12.61)} x(0.075-0.07)=0.0723=7.23 \%\)
Pretax cost ofdebt2 \(=7.23 \%\);
Weightad verage Pretax Cost of Debt, \(r_{d}=\frac{55.8}{104.05} \times 0.0803+\frac{48.25}{104.05} \times 0.0723=0.0766=7.66 \%\)
of equity, \(r_{e}=0.052+1.2(0.07)=0.136=13.6 \%\)
Weighted Average Cost of Capital (WACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=0.7793 \times 0.136+0.2206 \times 0.0766(1-0.35)=11.696 \%\)

Question 8: Kose Inc., has a target debt equity ratio of \(\mathbf{0 . 6 5}\). Its WACC is \(\mathbf{1 1 . 2 \%}\), and the tax rate is \(\mathbf{3 5 \%}\).
a) If Kose's cost of equity is \(\mathbf{1 5 \%}\), what is its pretax cost of debt?
\(\frac{\text { Debt }}{\text { Equity }}=\frac{0.65}{1} ; \quad\) Proportion of debt,\(w_{d}=\frac{0.65}{1+0.65}=\frac{13}{33}\)
Proportion of equity, \(w_{e}=\frac{20}{33}\)
\(\mathrm{W} \mathrm{ACC}=0.112=w_{e} r_{e}+w_{d} r_{d}(1-T)=\frac{20}{33} x 0.15+\frac{13}{33} x r_{d} x(1-0.35)\)
or, \(0.112=\frac{3+8.45 r_{d}}{33}\)
or, \(3+8.45 r_{d}=3.696\)
or, \(r_{d}=0.0823=8.23 \%\)
Pretax cost of debt \(=8.23 \%\)
b) If instead you know that the after tax cost of debt is \(6.4 \%\), what is the cost of equity?
\(\frac{\text { Debt }}{\text { Equity }}=\frac{0.65}{1} ; \quad\) Proportion of debt, \(w_{d}=\frac{0.65}{1+0.65}=\)
Proportion of equity, \(w_{e}=\frac{20}{33}\)
\(\mathrm{W} \mathrm{ACC}=0.112=w_{e} r_{e}+w_{d} r_{d}(1-T)=\frac{2}{3-3} x r_{e}+\frac{13}{33} x 0.064\)
or, \(0.112=\frac{20 r_{e}+0.832}{33}\)
or, \(20 r_{e}+0.832=3.696\)
or, \(r_{e}=0.1432=14.3\)
Cost of equity \(=14.92 \%\)
Question 9: Gives the following information for Huntington Power Co. find the WACC.
Assume the company's tax rate is \(35 \%\), market risk premium \(7 \%\) and risk free rate \(6 \%\).
Debt
\(5,0008 \%\) semiannual coupon bonds of \(\$ 1,000\) par value, 20 years maturity remaining, selling for \(103 \%\) of par
Commonstock \(\mathbf{1 6 0 , 0 0 0}\) shares outstanding, selling for \(\$ 57\) per share; the beta is 1.10.
M Whet value of common stock \(=160,000 \times \$ 57=\$ 9,120,000\)
Market value of bond \(=5,000 \times \$ 1,000 \times 103 \%=\$ 5,150,000\)
Proportion of debt in capital structure, \(w_{d}=\frac{\$ 5,150,000}{\$ 5,150,000+\$ 9,120,000}=0.36089=36.089 \%\)
Proportion of equity in capital structure, \(w_{e}=\frac{\$ 9,120,000}{\$ 5,150,000+\$ 9,120,000}=0.6391=63.91 \%\)

The pretax cost of a bond is the YTM of it
Bond price \(=103 \%\) of Face V alue \(\$ 1,000=\$ 1,030\)
This is a premium bond i.e. the bond is selling at higher price than its face value.
A bond is sold at premium when coupon rate is higher than YTM. As the bond is
selling at premium, YTM must be lower than coupon rate of \(8 \%\).
\(\mathrm{NPV} @ 7.5 \%=-\$ 1,030+\sum_{t=1}^{40} 40 x(1+0.0375)^{-t}+1000 x(1+0.0375)^{-40}=\$ 21.37\)
\(\mathrm{NPV} @ 8 \%=-\$ 1,030+1000=-\$ 30\)
\(Y T M=L+\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)=0.075+\frac{\$ 21.37}{\$ 21.37-(-\$ 30)} x(0.08-0.075)=0.0708=7.708 \%\)
Cost of equity, \(r_{e}=0.06+1.1(0.07)=0.137=13.7 \%\)
Weighted Average Cost of Capital (W ACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=0.6391 \times 0.137+0.36089 \times 0.07708(1-0.35)=10.563 \%\)

Question 10: Titan Mining Corporation has 8.5 million shares of common stock outstanding and \(200,0007.5 \%\) semiannual bonds outstandiog with par value of \(\$ 1,000\) each. The common stock currently sells for \(\$ 34\) per shareand has a beta of 1.2 and the bonds have 15 years to maturity and sell for \(93 \%\) of par. The market risk premium is \(\mathbf{7 \%}\), T-bills are yielding \(5 \%\), and company's tax rateis \(35 \%\).
a) What is the firm's market value capital stgueture?

Market value of common stock \(=8.5\) million \(\$ 34=\$ 289\) million
Market value of bond \(=200,000 x \$ 1,0\) ga \(93 \%=\$ 186 \mathrm{million}\)
Proportion of debt in capital strucfine, \(w_{d}=\frac{\$ 186 \mathrm{million}}{\$ 289 \mathrm{~m} \text { illion }+\$ 186 \mathrm{~m} \text { illion }}=0.3915=39.15 \%\) Proportion of equity in capita structure, \(w_{e}=\frac{\$ 289 \text { million }}{\$ 289 \mathrm{million}+\$ 186 \mathrm{~m} \text { illion }}=0.6084=60.84 \%\)
b) If Titan Miningis evaluating a new investment project that has the same risk as the firm's typical project, what rate should the firm use to discount the project's cash flows

The preta cost of a bond is the YTM of it
B ondPrice \(=93 \%\) of Face Value \(\$ 1,000=\$ 930\)
This is a discount bond i.e. the bond is selling at lower price than its face value. A bond is sold at discount when coupon rate is lower than YTM. As the bond is selling at discount, YTM must be higher than coupon rate of \(7.5 \%\).
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$\mathrm{NPV} @ 7.5 \%=-\$ 930+\$ 1000=\$ 70$
$\mathrm{NPV} @ 8.5 \%=-\$ 930+\sum_{t=1}^{30} 37.5 x(1+0.0425)^{-t}+1000 x(1+0.0425)^{-30}=-\$ 13.89$
$Y T M=L+\frac{N P V_{L}}{N P V_{L}-N P V_{H}} x(H-L)=0.075+\frac{\$ 70}{\$ 70-(-\$ 13.89)} x(0.085-0.075)=0.08334=8.334 \%$
Cost of equity, $r_{e}=0.05+1.2(0.07)=0.134=13.4 \%$
Weighted Average Cost of Capital (W ACC) $=w_{e} r_{e}+w_{d} r_{d}(1-T)$
$=0.6084 \times 0.134+0.3915 \times 0.08334(1-0.35)=10.273 \%$

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Question 11: Suppose your company needs \(\mathbf{\$ 2 0}\) million to build a new assembly line. Nour target debt-equity ratio is 0.75 . The floatation cost for issuing new equity is \(8 \%\) drit the floatation cost for debt is only \(5 \%\). Your boss has decided to fund the project by borrowing money because the floatation costs are lower and the needed funds are relatively small.
a) What do you think about the rationale behind borrowing the entire amount?

If the firm finance the project using all debt, it will have to issue more equry in future to achieve the target capital structure and vice-versa.Therefore, it doesn't matter whether firm finance the project using all debt or all equity, we have to calculate weighted average floatation cog find out the amount to be raised to acquire the required amount for the project.
b) What is your company's weighted average floatation cost, assuming all equity is raised externally?
\(\frac{\text { Debt }}{\text { Equity }}=\frac{0.75}{1} ; \quad\) Proportion of debt,\(w_{d}=\frac{0}{0.75}=\frac{3}{7}\)
Proportion of equity, \(w_{e}=\frac{4}{7}\)
Weighted Average Floatation Cot \(=w_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=\frac{4}{7} x 0.08 x(1-0)+\frac{3}{7} x 0.05 \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
\(=0.06714=6.714 \%\)
c) What is the true cost of building the new assembly line after taking floatation costs into accoutht?
Amount to beriised \(=\frac{\$ 20 \text { million }}{1-0.06714}=\$ 21.44\) million
Question 12: Southern Alliance Company needs to raise \(\$ 45\) million to start a new project and \(\bar{\omega} \mathbf{H}\) raise the money by selling new bonds. The company will generate no internal equity for the foreseeable future. The company has a target capital structure of \(\mathbf{6 5 \%}\) common stock, \(5 \%\) preferred stock, and \(\mathbf{3 0 \%}\) debt. Floatation costs for issuing common stock are \(9 \%\), for new preferred stock \(6 \%\), and for new debt \(3 \%\). What is the true initial cost figure Southern should use when evaluating its project?

Proportion of debt, \(w_{d}=0.3\)
Proportion of common equity, \(w_{e}=0.65\)
Proportion of preferred equity, \(w_{p}=0.05\)
Weighted Average Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}+w_{p} f_{p}\)
\(=0.65 \times 0.09 \times(1-0)+0.3 \times 0.03+0.05 \times 0.06 ; \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
\(=0.0705=7.05 \%\)
Amount to be raised \(=\frac{\$ 45 \text { million }}{1-0.0705}=\$ 48.41 \mathrm{million}\)
Question 13: Och, Inc. is considering a project that will result in initial after tax cash savings of \(\$ 3.5\) million at the end of the first year, and these savings will grow at \(\sigma_{\text {rate }}\) of \(5 \%\) per year indefinitely. The firm has a target debt-equity ratio of 0.65 , a cos of equity of \(15 \%\), and an after tax cost of debt of \(5.5 \%\). The cost saving proposan is somewhat riskier than the usual projects the firm undertakes; management uses the subjective approach and applies an adjustment factor of \(+2 \%\) to the cost of eapital for such risky projects. Under what circumstances should Och take on the project
\(\frac{\text { Debt }}{\text { Equity }}=\frac{0.65}{1} ; \quad\) Proportion of debt,\(w_{d}=\frac{0.65}{1+0.65}=\frac{13}{33}\)
Proportion of equity, \(w_{e}=\frac{20}{33}\)
Weighted Average Cost of Capital (WACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=\frac{20}{33} x 0.15+\frac{13}{33} \times 0.055=0.11257=11.2\)
Net Present Value(NPV) of Projection
\(\mathrm{NPV}=-C+\frac{\$ 3.5 \text { million }}{0.11257+0.02-0.05} \geq 0\)
or,\(\frac{\$ 3.5 \text { million }}{0.11257+0.02-0.0}\)
or, \$42.385million
Och should ondake on the project if it's initial investment is \(\$ 42.385 \mathrm{~m}\) illion or less.
Question 14; Coodbye Inc. recently issued new securities to finance a new TV show. The project cost \(\$ 15\) million, and the company paid \(\$ 850,000\) in floatation costs. In addition, the equi \(\hat{\boldsymbol{y}}\) issued had a floatation cost of \(\mathbf{7 \%}\) of the amount raised, whereas the debt issuedhad a floatation cost of \(3 \%\) of the amount raised. If Goodbye issued new securities inthe same proportion as its target capital structure, what is the company's target debtequity ratio?

Floatation cost \(=\frac{\$ 850,000}{\$ 15 \text { million }+\$ 850,000}=0.0536=5.36 \%\)
Weighted Average Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}=0.0536\)
or, \(w x 0.07 x(1-0)+(1-w) x 0.03=0.0536 ; \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
or, \(0.04 w=0.0536-0.03\)
or, \(w_{e}=0.59069=59.069 \%\)
Debt-Equity ratio \(=\frac{1-0.59069}{0.59069}=0.6929\)
Question 15: Photochronograph Corporation (PC), manufactures time series photographic equipment. It is currently at its target debt-equity ratio of 0.7. It's considering building a new \(\$ 45\) million manufacturing facility. This new planfis expected to generate after tax cash flows of \(\mathbf{\$ 6 . 2}\) million a year in perpetuity. Therompany raises all equity from outside financing. There are three financing options:
1) A new issue of common stock: The floatation costs of the new common stock would be \(8 \%\) of the amount raised. The required return on the company's new equity is \(14 \%\).
2) A new issue of 20 year bonds: The floatation costs of the new bonds would be \(4 \%\) of the proceeds. If the company issues these new bohds at an annual coupon rate of \(8 \%\), they will sell at par.
3) Increased use of accounts payable financing. Because this financing is part of the company's ongoing daily business, it hag no floatation costs, and the company assigns it a cost that is the same as the overall firm WACC. Management has a target ratio of accounts payable todong term debt of 0.20 (assume there is no difference between the pretax and (after tax accounts payable cost.)
What is the NPV of the new plant? Assume that PC has a 35\% tax rate.



Proportion accounts payable in capital structure, \(w_{a p}=\frac{0.2}{1.2} \times \frac{7}{17}=\frac{7}{102}\)
Prgertion of bond (long term debt) in capital structure, \(w_{d}=\frac{1}{1.2} \times \frac{7}{17}=\frac{35}{102}\)
Cost of Equity, \(r_{e}=14 \%=0.14\)
Cost of bond, \(r_{d}=\) YTM of the bond \(=8 \%\)
Cost of accounts payable, \(r_{a p}=\) W ACC

Weighted Average Cost of Capital(WACC), \(r_{W A C C}=w_{e} r_{e}+w_{d} r_{d}(1-T)+w_{a p} r_{a p}\)
\(=\frac{10}{17} \times 0.14+\frac{35}{102} \times 0.08 \times(1-0.35)+\frac{7}{102} \times r_{\text {WACC }}\)
\(o r, r_{W A C C}\left(1-\frac{7}{102}\right)=\frac{10}{17} \times 0.14+\frac{35}{102} \times 0.08 \times(1-0.35)\)
or, \(r_{W A C C}=0.10758=10.758 \%\)
Weighted Average Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}+w_{a p} f_{a p}\)
\(=\frac{10}{17} \times 0.08+\frac{35}{102} \times 0.04+\frac{7}{102} \times 0=0.06078=6.078 \%\);
Net Present V alue(NPV) of Project:
\(\mathrm{NPV}=-\frac{\$ 45 \text { million }}{(1-0.06078)}+\frac{\$ 6.2 \text { million }}{0.10758}\)
\(=-\$ 47.912 \mathrm{million}+\$ 57.631 \mathrm{million}=\$ 9.719 \mathrm{million}\)
Question 16: Trower Corp. has a debt equity ratio of 1.2. The@ompany is considering a new plant that will cost \(\$ 145\) million to build. When the company issues new equity, it incurs a floatation cost of \(\mathbf{8 \%}\). The floatation cost on new debt is \(\mathbf{3 . 5 \%}\).
a) What is the initial cost of the plant if the compano raises all equity externally?
\(\frac{\text { Debt }}{\text { Equity }}=\frac{1.2}{1} ; \quad\) Proportion of debt,\(w_{d}=\frac{1.2}{1.2}\)
Proportion of equity, \(w_{e}=\frac{5}{11}\)
Weighted Average Floatation Cost \(\boldsymbol{w}_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=\frac{5}{11} \times 0.08(1-0)+\frac{6}{11} x 0.035 ;\) [k=\% of finance from internal source \(]\)
\(=0.05545=5.545 \%\)
Amount to be raised \(\frac{\$ 145 \text { million }}{1-0.05545}=\$ 153.513\) million
b) What if it pically uses \(\mathbf{6 0 \%}\) retained earnings?

Weighted A verage Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=\frac{5}{11} \times 0.08(1-0.6)+\frac{6}{11} x 0.035 ; \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
\(=0.3363=3.363 \%\)
Amount to be raised \(=\frac{\$ 145 \text { million }}{1-0.03363}=\$ 150.047 \mathrm{~m}\) illion
c) What if all equity investments are financed through retained earnings?

Weighted Average Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=\frac{5}{11} \times 0.08(1-1)+\frac{6}{11} \times 0.035 ; \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
\(=0.01909=1.909 \%\)
Amount to be raised \(=\frac{\$ 145 \text { million }}{1-0.01909}=\$ 147.822\) million

Question 17: Suppose a firm has both a current and a target debt-equity ratio of \(\mathbf{0 . 6}\), a cost of debt of \(5.15 \%\), and a cost of equity of \(10 \%\). The corporate tax rate is \(34 \%\). The firm is considering taking on a warehouse renovation costing \(\$ \mathbf{6 0}\) million that is expected to yield cost savings of \(\mathbf{\$ 1 2}\) million a year for six years. Should the firm take on the warehouse renovation?
Sol:
\(\frac{\text { Debt }}{\text { Equity }}=\frac{0.6}{1} ; \quad\) Proportion of debt, \(w_{d}=\frac{0.6}{1+0.6}=0.375\)
Proportion of equity, \(w_{e}=\frac{1}{1+0.6}=0.625\)
Weighted Average Cost of Capital (W ACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=0.625 \times 0.1+0.375 \times 0.0515 x(1-0.34)=0.075246=7.5246 \%\)
Net Present Value(NPV) of W arehouse Renovation Project:
\(\mathrm{NPV}=-60+\frac{12}{0.075246}-\frac{12}{0.075246}(1+0.075246)^{-6}=-\$ 3.716 \mathrm{~m} 0 \mathrm{~m}\)
NPV is negative which indicates that renovation project fails
generate enough cash to fulfill required cost of capital. Heace, reject the project.
Question 18: Consider a firm whose debt has a market \&alue of \(\$ 40\) million and whose stock has a market value of \(\$ 60\) million (3million outstarding shares of stock, each selling for \(\$ 20\) per share). The firm pays a \(5 \%\) rate of interest on its new debt and has a beta of 1.41. The corporate tax rate is \(34 \%\). The risk prenium on the market is \(\mathbf{9 . 5 \%}\) and that the current Treasury bill rate is \(1 \%\). What cost of capital the firm should use to discount its future cash flows?
Proportion of debt, \(w_{d}=40 \%=0.4\); Proportion of equity, \(w_{e}=60 \%\)
B ased on CAPM, Cost of equity, \({ }^{\circ} r_{f}+\beta\left(r_{m}-r_{f}\right)=0.01+1.41 x 0.095=0.14395=14.395 \%\)
W eighted Average Cost of Cpital (WACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=0.6 \times 0.14395+0.4 \times 0.0510 .34)=0.09957=10 \%\)
The firm should use cos capital of \(10 \%\) to discount its future cash flows.
Question 19: The Werinstein Corporation has a target capital structure of \(\mathbf{8 0 \%}\) equity and \(\mathbf{2 0 \%}\) debt. The f10atation costs for equity issues are \(20 \%\) of the amount raised; the floatation costs \(\%\) debt issues are \(6 \%\). If Weinstein needs \(\$ 65\) million for a new manufacturingfacility, how much it will have to raise?
Weighted - verage Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=0.80 .2 x(1-0)+0.2 \times 0.06 ; \quad[\mathrm{k}=\%\) of finance from internal source \(]\)
- \(\uparrow\) - \(72=17.2 \%\)

Amount to be raised \(=\frac{\$ 65 m}{1-0.172}=\$ 78.502 \mathrm{million}\)
Question 20: Tripleday Printing Company is currently at its target debt-equity ratio of \(\mathbf{1 0 0 \%}\). It is considering building a new \(\mathbf{\$ 5 0 0 , 0 0 0}\) printing plant in Kansas. This new plant is expected to generate after tax cash flows of \(\$ 73,150\) per year forever. The tax rate is \(34 \%\). To finance the project, the firm is considering all debt or all equity option. The issuance costs of the new common stock and new debt would be about \(\mathbf{1 0 \%}\) and \(\mathbf{2 \%}\) respectively. The required return on the company's new equity is \(20 \%\) and cost of new debt is \(10 \%\). Should the firm accept the project?
\(\frac{\text { Debt }}{\text { Equity }}=\frac{1}{1} ; \quad\) Proportion of debt,\(w_{d}=\frac{1}{1+1}=0.5\)
Proportion of equity, \(w_{e}=0.5\)
Weighted Average Cost of Capital (W ACC) \(=w_{e} r_{e}+w_{d} r_{d}(1-T)\)
\(=0.5 \times 0.2+0.5 \times 0.1 \times(1-0.34)=0.133=13.3 \%\)
If the firm finance the project using all debt, it will have to issue more equity in future to achieve the target capital structure and vice-versa.

Therefore, it doesn't matter whether the firm finance the project using all debt or all equity, we have to calculate weighted average floatation cost to find out the amount to be raised to acquire the required amount for the project.

Weighted Average Floatation Cost \(=w_{e} f_{e}(1-k)+w_{d} f_{d}\)
\(=0.5 \times 0.1 \times(1-0)+0.5 \times 0.02 ; \quad[\mathrm{k}=\%\) of finance from internal sourcle \(]\)
\(=0.06=6 \%\)
Amount to be raised \(=\frac{\$ 500,000}{1-0.06}=\$ 531,914.89\)
Net Present Value(NPV) of Printing Plant Project:
\(\mathrm{NPV}=-\$ 531,914.89+\frac{\$ 73,150}{0.133}=\$ 18,085.11\)
NPV is positive which indicates that printion plaject will be able to generate enough cash to fulfill required cost of capital. Hence, accept the project.

\section*{Financial Management}

\section*{Additional Problems}

\section*{Risk and Return}
1. Determine geometric return from the following numbers:
\begin{tabular}{ll} 
Year & return \\
2004 & \(16.4 \%\) \\
2005 & 27.45 \\
2006 & 33.22 \\
2007 & -2.07 \\
2008 & 21.22 \\
2009 & 42.09 \\
2010 & 55.34 \\
2011 & -7.77 \\
2012 & -10.25 \\
2013 & 11.88
\end{tabular}
2. Using the returns in preceding problem, determine the averae return and standard deviation of returns.
3. Using the following information about possible retasis on an investment next year, determine the expected return and standard devaition of returns:
\begin{tabular}{llll} 
Scenario & Return & & Probabidity \\
Highly optimistic & \(55 \%\) & & NO \\
Optimistic & 30 & & 00 \\
Average & 22 & .35 \\
Below average & 15 & & .20 \\
Pessimistic & -8 & & .15
\end{tabular}
4. Identify which project is preferia, given the following information about the four pairs of projects, \(A\) and \(B, C\) and \(B, E\) and \(F\), and \(G\) and \(H\).

5. In addition to the values provided in the previous problem, the following information are provided:

Correlation between returns of A and \(\mathrm{B}=+.5\)
Correlation between returns of C and \(\mathrm{D}=+.1\)
Correlation between returns of E and \(\mathrm{F}=0\)
Correlation between returns of G and \(\mathrm{H}=-.5\)
(a) If you form a portfolio of two assets as paired above with \(60 \%\) of assets invested in the first asset, what are the standard deviations?
(b) Between the pair AB and GH , which pair provides a better combination?
6. Explain what is systematic risk and what is diversifiable risk.
7. Explain how bond prices react to a change in interest rate.
8. Explain how bond prices change with time everything else remaining the same.
9. The following information is given for stock M :
\begin{tabular}{lrc} 
Scenario & Return & Probability \\
Recession & \(-6 \%\) & .25 \\
Normal & \(28 \%\) & .50 \\
Expansion & \(54 \%\) & .25
\end{tabular}

M has a beta of .85 . Market return is 26 percent and risk free rate is 11 percent. (a) What is the expected return of M ? (b) What is the required return of M ? (c) M overpriced or underpriced?
10. What is the yield to maturity for a bond that has 5 years le to maturity, has 12.5 percent coupon and a face value of Taka 1,000 , and narket price is Taka 933.74 ?
11. What is the expected capital gains if the investor deordes to hold the bond for another year? (assume everything else remains unchanged
12. Hawthorn Corporation is growing rapidly and so far, has not paid any dividend. It will pay its first dividend next year and the expected dividend is Taka 16 per share.
Dividend will grow at \(35 \%, 30 \%, 18 \%\) in the subsequent years, and then is expected to settle at a growth rate of 12 percent.
a. If Hawthorn shareholders refare a return of 25 percent, what would be equilibrium price of Hawthorn?
b. If a shareholder holds his shares for one more year, what will be expected price of the share?
c. What is the cument yield on the shares now?

\section*{13. Indicate true or false}
a) Everything else remaining the same, a discount bond should increase in value as maturity approaches.
b) Everything else remaining the same, a higher growth rate will result in a higher value stock.
e. A zero coupon bond will never sell at a premium.
14. Which of the following statements is correct?
a. Slope of the CML is beta.
b. In a fully diversified portfolio, systematic risk is completely diversified away.
c. In forming a two asset portfolio, negative correlation between the two returns is less desirable than no correlation between the two returns.
d. A steep characteristic line is indicative of high systematic risk (high beta) asset.
15. You are going to make your first stock investment and you are considering the information provided for stock X and stock Y .

Stock X Stock Y
\begin{tabular}{lcc} 
Expected Return & \(22.5 \%\) & \(22.5 \%\) \\
Standard deviation of returns & \(38 \%\) & \(44 \%\) \\
Beta & 1.8 & 1.25
\end{tabular}
(a) As your first investment, which stock should you choose and why?
(b) If a well diversified investor wishes to add one of these two stocks to his portfolio which one he should choose and why?
16. Determine the fair value of a stock when the following information is known. Expected dividend: Cash 13.50. The required return is 26 percent and grow irate in dividend is 18 percent.
17. Use the following information to answer the following 3 questions
a) A bond is selling at Taka 980. It pays a coupon of 13 pereaty and has a face value of Taka 1000. It will mature in 8 years. What is the curent yield on the bond?
b) If the market yield declines to 10 percent, everythimgelse remaining the same, at what price should the bond sell?
c) What is the percent change in the bond's pree?
18. If the risk-free rate is 9 percent and marketwemium is 12 percent, what is the required return on a stock that has a beta 1.25 ?
19. The following is the characteristic lineof Stock Max.
\(\mathrm{K}_{\mathrm{xi}}=1.3+1.2 \mathrm{k}_{\mathrm{mi}}\)
Market return is 28 percent. Risk free rate is 11 percent. What is the required return of Max?
20. Previous problem coontinued. What is the required return on a stock that has a beta of 1 ?
21. Crossings Cofporation issued 10 year bonds 2 years ago. The bond carries an annual coupon of © percent paid annually, has a face value of Taka 1,000, and is currently sellingtor Taka 932.
(a), hat is the yield to maturity for the bond?

What is the expected price of the bond a year later assuming everything else remains the same?
(c) What is the expected capital gains and current yield for an investor who owns the bond?
22. Sovarn Corporation is growing rapidly and, so far, has not paid any dividend. It will not pay any dividend next year but is expected to pay a dividend at the end of \(2^{\text {nd }}\) year of an amount of Taka 10.00 . Dividend will grow at \(25 \%, 20 \%, 17 \%\) in the subsequent years, and then is expected to settle at a growth rate of 12 percent.
a. If Sovarn's shareholders require a return of 24 percent, what would be equilibrium price of Sovarn?
b. If a shareholder holds his share for one more year, what will be expected price of the share one year later?
c. What is the current dividend yield on the shares now?
23. The expected return on stock X is 28 percent and expected return on stock Y is 32 percent. The standard deviation of X is 36 percent and standard deviation of Y is 40 percent. The correlation between the returns of X and Y is +.60 . You invested a total of Taka \(1,00,000,40\) percent of the money in stock X.
(a) What is the standard deviation of your portfolio?
(b) What is the expected return on the portfolio?
24. Indicate true or false
a) A change in interest rate should have a greater affect on the value of two year compared with a value of a 20 year bond.
b) A required condition for the Gordon Growth Model is that the discountgate, \(k\), must be equal to or greater than the growth rate, \(g\).
25. Which of the following statements is correct?
a) Slope of the characteristic line of stock \(X\) is the beta of stoc
b) In a fully diversified portfolio, unsystematic risk is complefely diversified away.
c) In forming a two asset portfolio, negative correlation between the two returns is less desirable than no correlation between the two returne
d) A steep slope of the CML (compared with a relatrvely flat) is indicative of market demanding higher compensation for higher sodematic risk.
26. Primavara Corporation is expected to annetrice a dividend of Taka 35 per share reflecting a new growth rate of \(15 \%\), 6 nvestors require a return of 32 percent, what is the fair value of the stock?
27. If the risk-free rate is 11 percerind market risk premium is 15 percent, what is the required return on a stock thathas a beta of 1.20 ?
28. The following is the chardeteristic line of Stock Max. \(\mathrm{K}_{\mathrm{xi}}=1.3+1.85 \mathrm{~K}_{\mathrm{mi}}\)
Market return is 26 percent. Risk free rate is 11 percent.
(a) What is the required return of Max?
(b) Whats the required return on a stock that has a beta of 1 ?
29. Theatest dividend paid by Somtax Corporation was Taka 18. The Return on Equity (ROE) is typically 26 percent. Somtax has a history of paying out 60 percent of its profits. Beta of Somtax is 1.25 . Current Risk free rate is 10 percent. Market Risk premium is 16 percent.
(a) What is the fair price of Somtax?
(b) What is likely to happen if the market price of the stock increases to Taka 125?
(c) Ignore your answer to (b) above. What is likely to happen if inflation expectation increases by 2 percent points? What will be the new price?
30. Hechinger Corporation has a target capital structure of 40 percent debt and 60 percent equity. The firm pays out 40 percent of its earnings in dividends. The current EPS of the firm is Taka 80 per share. Current share price in the market is Taka 200 and the
growth rate is 14 percent. The YTM on its bonds is 16 percent and its marginal tax rate is 40 percent. If new equity is issued, flotation cost will be 15 percent.
a) Determine the cost of equity, \(\mathrm{k}_{\mathrm{e}}\), assuming no new shares are issued.
b) Determine the cost of equity, \(\mathrm{k}_{\mathrm{s}}\), assuming new shares are issued.
c) What is the WACC/MCC assuming new shares are issued.
31. Deaucent Corporation has a target capital structure of 35 percent debt and 65 percent equity. It is expected to pay a dividend of Taka 30 per share. Current share price in the market is Taka 225 and the growth rate is 15 percent. The YTM on its bonds is 16 percent and its marginal tax rate is 37 percent. If new equity is issued, flotation cost will be 20 percent.
a) Determine the cost of equity, \(\mathrm{k}_{\mathrm{e}}\), assuming no new shares are issued.
b) Determine the cost of equity, \(\mathrm{k}_{\mathrm{s}}\), assuming new shares are issued.
c) What is the WACC/MCC assuming new shares are issued.
32. Terra X Corporation has a target capital structure of 30 debt and 70equity. Retained earnings for the year is not enough to finance the new projects and the company is planning to issue new shares to raise new equity capital. The 8 ompany is expected to grow at 10 percent per year. It's last dividend was Taka 18 per share. Currently shares are selling at Taka 112 per share. If new shares are issued, the shares will net Taka 102 per share.
(a) What is the cost of retained earnings?
(b) What is the cost of new equity?
(c) The yield to maturity of the company bonds is \(15 \%\). Tax rate is \(40 \%\). Assuming new shares are issued, what is the marginal cost of capital of the company?
33. Trifura Corporation has a targetcapital structure of 30 debt and 70 equity. The beta of the company is 1.4. Current esk-free rate is 10 percent. Average return on market index is 28 percent.
(a) What is the cost ofretained earnings?
(b) The yield to maturity of the company's bonds is \(15 \%\). Tax rate is \(40 \%\). Determine the weighted average cost of capital of the company.
34. The follorwing are the cash flows for two mutually exclusive projects under considQation. The projects are considered average risk projects. The company's WACC is 16 percent.
\begin{tabular}{llllll} 
& Year 0 & Year 1 & Year 2 & Year 3 & Year 4 \\
Cash flows of A & \((12,000)\) & 6,000 & 5,000 & 4,000 & 3,000 \\
Cash flows of B & \((12,000)\) & 3,000 & 6,000 & 6,000 & 4,000
\end{tabular}
a. Determine the IRR values for the two projects.
b. Determine the cross-over point.
c. Draw a figure of NPV profiles to prove that project B creates more value. (Figures don't have to be exact, use discount rates of \(10 \%, 15 \%, 20 \%\), and \(25 \%\) )
35. Project \(X\) and \(Y\) are mutually exclusive. Project \(X\) has an IRR of 32.5 percent and Project \(Y\) has an IRR of 27.6 percent. Both projects happen to have identical NPV at 17.4 percent. If the firms WACC is 20 percent, and both projects belong in the average risk class, which project should be selected? Draw a figure to support your answer.
36. What are the reasons for computing a modified IRR?
37. (a) Using a reinvestment rate of 18 percent and a cost of capital of \(15 \%\), determine the modified IRR with the following 8 year cash flow stream.
\begin{tabular}{lllcc} 
Year 0 & Year 1 & Year 2 & Year 3 \\
\(-100,000\) & 30,000 & \(-30,000\) & 40,000 & Year 4 through 8
\end{tabular} \(0^{0}\)
(b) Should you accept or reject the project?
38. Indicate True or False
a) Floatation cost increases the marginal cost of capital.
b) Modified IRR is based on the assumption that project flows reinvested at the IRR rate.
c) Smaller projects have a tendency of producing higher CR values.
d) The weighted average cost of capital (WACC) is enerally based on the assumption of an optimal capital structure and that every Takaraised as long term capital is done maintaining the long term optimal capital mincture mix.
39. Project \(X\) and \(Y\) are mutually exclusi eadnd have equal risk. Project \(X\) has an IRR of 24.5 percent and Project \(Y\) has an IRK of 28.6 percent. Both projects happen to have a net present value of Taka 3, 20 at 16.4 percent. If the firms WACC is 20 percent, and both projects belong in same risk class, which project should be selected? Your answer must be based on fight reason. Draw a figure to show your justification.
40. Determine the IRR QT project that has an initial outlay of Taka 40,00,000, generates no cash flow for first 5 years, and then generates a cash flow of Taka 80,00,000 at the end of the 6 year and Taka \(75,00,000\) at the end of \(7^{\text {th }}\) year.
41. (a) Using reinvestment rate of 20 percent, determine the modified IRR with the followng cash flow stream.
\begin{tabular}{cccccc} 
Year 0 & Year 1 & Year 2 & Year 3 & Year 4 & Year 5 \\
\(-40,000\) & 30,000 & \(-15,000\) & 40,000 & 25,000 & 25,000
\end{tabular}
(b) State the decision rule using a discount rate of 20 percent.
(c) State the decision.
42. The following are the cash flows for two mutually exclusive projects under consideration. The projects are considered average risk projects. The company's WACC is 14 percent.
\begin{tabular}{llllll} 
& Year 0 & Year 1 & Year 2 & Year 3 & Year 4 \\
Cash flows of A & \((12,000)\) & 6,000 & 5,000 & 4,000 & 3,000 \\
Cash flows of B & \((12,000)\) & 3,000 & 6,000 & 6,000 & 4,000
\end{tabular}
(a) Determine the IRR values for the two projects.
(b) Determine the cross-over point.

Draw a figure of NPV profiles to prove that project B creates more value. (Figures doe have to be exact)

Table A-1 Future Value Interest Factors for One Dollar Compounded at \(\boldsymbol{k}\) Percent for \(\boldsymbol{n}\) Periods: FVIF \(_{k, n}=(1+k)^{\boldsymbol{n}}\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Period & 1\% & 2\% & 3\% & 4\% & 5\% & 6\% & 7\% & 8\% & 9\% & 10\% & 11\% & 12\% & 13\% & 14\% & 15\% & 16\% & 20\% & 24\% & 25\% & 30\% \\
\hline 1 & 1.0100 & 1.0200 & 1.0300 & 1.0400 & 1.0500 & 1.0600 & 1.0700 & 1.0800 & 1.0900 & 1.1000 & 1.1100 & 1.1200 & 1.1300 & 1.1400 & 1.1500 & 1.1600 & 1.2000 & 1.2400 & 1.2500 & 1.3000 \\
\hline 2 & 1.0201 & 1.0404 & 1.0609 & 1.0816 & 1.1025 & 1.1236 & 1.1449 & 1.1664 & 1.1881 & 1.2100 & 1.2321 & 1.2544 & 1.2769 & 1.2996 & 1.3225 & 1.3456 & 1.4400 & 1.5376 & 1.5625 & 1.6900 \\
\hline 3 & 1.0303 & 1.0612 & 1.0927 & 1.1249 & 1.1576 & 1.1910 & 1.2250 & 1.2597 & 1.2950 & 1.3310 & 1.3676 & 1.4049 & 1.4429 & 1.4815 & 1.5209 & 1.5609 & 1.7280 & 1.9066 & 1.9531 & 2.1970 \\
\hline 4 & 1.0406 & 1.0824 & 1.1255 & 1.1699 & 1.2155 & 1.2625 & 1.3108 & 1.3605 & 1.4116 & 1.4641 & 1.5181 & 1.5735 & 1.6305 & 1.6890 & 1.7490 & 1.8106 & 2.0736 & 2.3642 & 2.4414 & 2.8561 \\
\hline 5 & 1.0510 & 1.1041 & 1.1593 & 1.2167 & 1.2763 & 1.3382 & 1.4026 & 1.4693 & 1.5386 & 1.6105 & 1.6851 & 1.7623 & 1.8424 & 1.9254 & 2.0114 & 2.1003 & 2.4883 & 2.9316 & 3.0518 & 3.7129 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 6 & 1.0615 & 1.1262 & 1.1941 & 1.2653 & 1.3401 & 1.4185 & 1.5007 & 1.5869 & 1.6771 & 1.7716 & 1.8704 & 1.9738 & 2.0820 & 2.1950 & 2.3131 & 2.4364 & 2.9860 & 3.6352 & 3.8147 & 4.8268 \\
\hline 7 & 1.0721 & 1.1487 & 1.2299 & 1.3159 & 1.4071 & 1.5036 & 1.6058 & 1.7138 & 1.8280 & 1.9487 & 2.0762 & 2.2107 & 2.3526 & 2.5023 & 2.6600 & 2.8262 & 3.5832 & 4.5077 & 4.7684 & 6.2749 \\
\hline 8 & 1.0829 & 1.1717 & 1.2668 & 1.3686 & 1.4775 & 1.5938 & 1.7182 & 1.8509 & 1.9926 & 2.1436 & 2.3045 & 2.4760 & 2.6584 & 2.8526 & 3.0590 & 3.2784 & 4.2998 & 5.5895 & 5.9605 & 8.1573 \\
\hline 9 & 1.0937 & 1.1951 & 1.3048 & 1.4233 & 1.5513 & 1.6895 & 1.8385 & 1.9990 & 2.1719 & 2.3579 & 2.5580 & 2.7731 & 3.0040 & 3.2519 & 3.5179 & 3.8030 & 5.1598 & 6.9310 & 7.4506 & 10.604 \\
\hline 10 & 1.1046 & 1.2190 & 1.3439 & 1.4802 & 1.6289 & 1.7908 & 1.9672 & 2.1589 & 2.3674 & 2.5937 & 2.8394 & 3.1058 & 3.3946 & 3.7072 & 4.0456 & 4.4114 & 6.1917 & 8.5944 & 9.3132 & 13.786 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 11 & 1.1157 & 1.2434 & 1.3842 & 1.5395 & 1.7103 & 1.8983 & 2.1049 & 2.3316 & 2.5804 & 2.8531 & 3.1518 & 3.4785 & 3.8359 & 4.2262 & 4.6524 & 5.1173 & 7.4301 & 10.657 & 11.642 & 17.922 \\
\hline 12 & 1.1268 & 1.2682 & 1.4258 & 1.6010 & 1.7959 & 2.0122 & 2.2522 & 2.5182 & 2.8127 & 3.1384 & 3.4985 & 3.8960 & 4.3345 & 4.8179 & 5.3503 & 5.9360 & 8.9161 & 13.215 & 14.552 & 23.298 \\
\hline 13 & 1.1381 & 1.2936 & 1.4685 & 1.6651 & 1.8856 & 2.1329 & 2.4098 & 2.7196 & 3.0658 & 3.4523 & 3.8833 & 4.3635 & 4.8980 & 5.4924 & 6.1528 & 6.8858 & 10.699 & 16.386 & 18.190 & 30.288 \\
\hline 14 & 1.1495 & 1.3195 & 1.5126 & 1.7317 & 1.9799 & 2.2609 & 2.5785 & 2.9372 & 3.3417 & 3.7975 & 4.3104 & 4.8871 & 5.5348 & 6.2613 & 7.0757 & 7.9875 & 12.839 & 20.319 & 22.737 & 39.374 \\
\hline 15 & 1.1610 & 1.3459 & 1.5580 & 1.8009 & 2.0789 & 2.3966 & 2.7590 & 3.1722 & 3.6425 & 4.1772 & 4.7846 & 5.4736 & 6.2543 & 7.1379 & 8.1371 & 9.2655 & 15.407 & 25.196 & 28.422 & 51.186 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 16 & 1.1726 & 1.3728 & 1.6047 & 1.8730 & 2.1829 & 2.5404 & 2.9522 & 3.4259 & 3.9703 & 4.5950 & 5.3109 & 6.1304 & 7.0673 & 8.1372 & 9.3576 & 10.748 & 18.488 & 31.243 & 35.527 & 66.542 \\
\hline 17 & 1.1843 & 1.4002 & 1.6528 & 1.9479 & 2.2920 & 2.6928 & 3.1588 & 3.7000 & 4.3276 & 5.0545 & 5.8951 & 6.8660 & 7.9861 & 9.2765 & 10.761 & 12.468 & 22.186 & 38.741 & 44.409 & 86.504 \\
\hline 18 & 1.1961 & 1.4282 & 1.7024 & 2.0258 & 2.4066 & 2.8543 & 3.3799 & 3.9960 & 4.7171 & 5.5599 & 6.5436 & 7.6900 & 9.0243 & 10.575 & 12.375 & 14.463 & 26.623 & 48.039 & 55.511 & 112.455 \\
\hline 19 & 1.2081 & 1.4568 & 1.7535 & 2.1068 & 2.5270 & 3.0256 & 3.6165 & 4.3157 & 5.1417 & 6.1159 & 7.2633 & 8.6128 & 10.197 & 12.056 & 14.232 & 16.777 & 31.948 & 59.568 & 69.389 & 146.192 \\
\hline 20 & 1.2202 & 1.4859 & 1.8061 & 2.1911 & 2.6533 & 3.2071 & 3.8697 & 4.6610 & 5.6044 & 6.7275 & 8.0623 & 9.6463 & 11.523 & 13.743 & 16.367 & 19.461 & 38.338 & 73.864 & 86.736 & 190.050 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 21 & 1.2324 & 1.5157 & 1.8603 & 2.2788 & 2.7860 & 3.3996 & 4.1406 & 5.0338 & 6.1088 & 7.4002 & 8.9492 & 10.804 & 13.021 & 15.668 & 18.822 & 22.574 & 46.005 & 91.592 & 108.420 & 247.065 \\
\hline 22 & 1.2447 & 1.5460 & 1.9161 & 2.3699 & 2.9253 & 3.6035 & 4.4304 & 5.4365 & 6.6586 & 8.1403 & 9.9336 & 12.100 & 14.714 & 17.861 & 21.645 & 26.186 & 55.206 & 113.574 & 135.525 & 321.184 \\
\hline 23 & 1.2572 & 1.5769 & 1.9736 & 2.4647 & 3.0715 & 3.8197 & 4.7405 & 5.8715 & 7.2579 & 8.9543 & 11.026 & 13.552 & 16.627 & 20.362 & 24.891 & 30.376 & 66.247 & 140.831 & 169.407 & 417.539 \\
\hline 24 & 1.2697 & 1.6084 & 2.0328 & 2.5633 & 3.2251 & 4.0489 & 5.0724 & 6.3412 & 7.9111 & 9.8497 & 12.239 & 15.179 & 18.788 & 23.212 & 28.625 & 35.236 & 79.497 & 174.631 & 211.758 & 542.801 \\
\hline 25 & 1.2824 & 1.6406 & 2.0938 & 2.6658 & 3.3864 & 4.2919 & 5.4274 & 6.8485 & 8.6231 & 10.835 & 13.585 & 17.000 & 21.231 & 26.462 & 32.919 & 40.874 & 95.396 & 216.542 & 264.698 & 705.641 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 30 & 1.3478 & 1.8114 & 2.4273 & 3.2434 & 4.3219 & 5.7435 & 7.6123 & 10.063 & 13.268 & 17.449 & 22.892 & 29.960 & 39.116 & 50.950 & 66.212 & 85.850 & 237.376 & 634.820 & 807.794 & * \\
\hline 35 & 1.4166 & 1.9999 & 2.8139 & 3.9461 & 5.5160 & 7.6861 & 10.677 & 14.785 & 20.414 & 28.102 & 38.575 & 52.800 & 72.069 & 98.100 & 133.176 & 180.314 & 590.668 & * & * & * \\
\hline 36 & 1.4308 & 2.0399 & 2.8983 & 4.1039 & 5.7918 & 8.1473 & 11.424 & 15.968 & 22.251 & 30.913 & 42.818 & 59.136 & 81.437 & 111.834 & 153.152 & 209.164 & 708.802 & * & * & * \\
\hline 40 & 1.4889 & 2.2080 & 3.2620 & 4.8010 & 7.0400 & 10.286 & 14.974 & 21.725 & 31.409 & 45.259 & 65.001 & 93.051 & 132.782 & 188.884 & 267.864 & 378.721 & * & * & * & * \\
\hline 50 & 1.6446 & 2.6916 & 4.3839 & 7.1067 & 11.467 & 18.420 & 29.457 & 46.902 & 74.358 & 117.391 & 184.565 & 289.002 & 450.736 & 700.233 & * & * & * & * & * & * \\
\hline
\end{tabular}

Table A-2 Future Value Interest Factors for a One-Dollar Annuity Compouned at \(k\) Percent for \(n\) Periods: FVIFA \(_{k, n}=\left[(1+k)^{n}-1\right] / k\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Period & 1\% & 2\% & 3\% & 4\% & 5\% & 6\% & 7\% & 8\% & 9\% & 10\% & 11\% & 12\% & 13\% & 14\% & 15\% & 16\% & 20\% & 24\% & 25\% & 30\% \\
\hline 1 & 1.0000 & 1.0200 & 1.0300 & 1.0400 & 1.0500 & 1.0600 & 1.0700 & 1.0800 & 1.0900 & 1.1000 & 1.1100 & 1.1200 & 1.1300 & 1.1400 & 1.1500 & 1.1600 & 1.2000 & 1.2400 & 1.2500 & 1.3000 \\
\hline 2 & 2.0100 & 2.0200 & 2.0300 & 2.0400 & 2.0500 & 2.0600 & 2.0700 & 2.0800 & 2.0900 & 2.1000 & 2.1100 & 2.1200 & 2.1300 & 2.1400 & 2.1500 & 2.1600 & 2.2000 & 2.2400 & 2.2500 & 2.3000 \\
\hline 3 & 3.0301 & 3.0604 & 3.0909 & 3.1216 & 3.1525 & 3.1836 & 3.2149 & 3.2464 & 3.2781 & 3.3100 & 3.3421 & 3.3744 & 3.4069 & 3.4396 & 3.4725 & 3.5056 & 3.6400 & 3.7776 & 3.8125 & 3.9900 \\
\hline 4 & 4.0604 & 4.1216 & 4.1836 & 4.2465 & 4.3101 & 4.3746 & 4.4399 & 4.5061 & 4.5731 & 4.6410 & 4.7097 & 4.7793 & 4.8498 & 4.9211 & 4.9934 & 5.0665 & 5.3680 & 5.6842 & 5.7656 & 6.1870 \\
\hline 5 & 5.1010 & 5.2040 & 5.3091 & 5.4163 & 5.5256 & 5.6371 & 5.7507 & 5.8666 & 5.9847 & 6.1051 & 6.2278 & 6.3528 & 6.4803 & 6.6101 & 6.7424 & 6.8771 & 7.4416 & 8.0484 & 8.2070 & 9.0431 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 6 & 6.1520 & 6.3081 & 6.4684 & 6.6330 & 6.8019 & 6.9753 & 7.1533 & 7.3359 & 7.5233 & 7.7156 & 7.9129 & 8.1152 & 8.3227 & 8.5355 & 8.7537 & 8.9775 & 9.9299 & 10.980 & 11.259 & 12.756 \\
\hline 7 & 7.2135 & 7.4343 & 7.6625 & 7.8983 & 8.1420 & 8.3938 & 8.6540 & 8.9228 & 9.2004 & 9.4872 & 9.7833 & 10.089 & 10.405 & 10.730 & 11.067 & 11.414 & 12.916 & 14.615 & 15.073 & 17.583 \\
\hline 8 & 8.2857 & 8.5830 & 8.8923 & 9.2142 & 9.5491 & 9.8975 & 10.260 & 10.637 & 11.028 & 11.436 & 11.859 & 12.300 & 12.757 & 13.233 & 13.727 & 14.240 & 16.499 & 19.123 & 19.842 & 23.858 \\
\hline 9 & 9.3685 & 9.7546 & 10.159 & 10.583 & 11.027 & 11.491 & 11.978 & 12.488 & 13.021 & 13.579 & 14.164 & 14.776 & 15.416 & 16.085 & 16.786 & 17.519 & 20.799 & 24.712 & 25.802 & 32.015 \\
\hline 10 & 10.462 & 10.950 & 11.464 & 12.006 & 12.578 & 13.181 & 13.816 & 14.487 & 15.193 & 15.937 & 16.722 & 17.549 & 18.420 & 19.337 & 20.304 & 21.321 & 25.959 & 31.643 & 33.253 & 42.619 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 11 & 11.567 & 12.169 & 12.808 & 13.486 & 14.207 & 14.972 & 15.784 & 16.645 & 17.560 & 18.531 & 19.561 & 20.655 & 21.814 & 23.045 & 24.349 & 25.733 & 32.150 & 40.238 & 42.566 & 56.405 \\
\hline 12 & 12.683 & 13.412 & 14.192 & 15.026 & 15.917 & 16.870 & 17.888 & 18.977 & 20.141 & 21.384 & 22.713 & 24.133 & 25.650 & 27.271 & 29.002 & 30.850 & 39.581 & 50.895 & 54.208 & 74.327 \\
\hline 13 & 13.809 & 14.680 & 15.618 & 16.627 & 17.713 & 18.882 & 20.141 & 21.495 & 22.953 & 24.523 & 26.212 & 28.029 & 29.985 & 32.089 & 34.352 & 36.786 & 48.497 & 64.110 & 68.760 & 97.625 \\
\hline 14 & 14.947 & 15.974 & 17.086 & 18.292 & 19.599 & 21.015 & 22.550 & 24.215 & 26.019 & 27.975 & 30.095 & 32.393 & 34.883 & 37.581 & 40.505 & 43.672 & 59.196 & 80.496 & 86.949 & 127.913 \\
\hline 15 & 16.097 & 17.293 & 18.599 & 20.024 & 21.579 & 23.276 & 25.129 & 27.152 & 29.361 & 31.772 & 34.405 & 37.280 & 40.417 & 43.842 & 47.580 & 51.660 & 72.035 & 100.815 & 109.687 & 167.286 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 16 & 17.258 & 18.639 & 20.157 & 21.825 & 23.657 & 25.673 & 27.888 & 30.324 & 33.003 & 35.950 & 39.190 & 42.753 & 46.672 & 50.980 & 55.717 & 60.925 & 87.442 & 126.011 & 138.109 & 218.472 \\
\hline 17 & 18.430 & 20.012 & 21.762 & 23.698 & 25.840 & 28.213 & 30.840 & 33.750 & 36.974 & 40.545 & 44.501 & 48.884 & 53.739 & 59.118 & 65.075 & 71.673 & 105.931 & 157.253 & 173.636 & 285.014 \\
\hline 18 & 19.615 & 21.412 & 23.414 & 25.645 & 28.132 & 30.906 & 33.999 & 37.450 & 41.301 & 45.599 & 50.396 & 55.750 & 61.725 & 68.394 & 75.836 & 84.141 & 128.117 & 195.994 & 218.045 & 371.518 \\
\hline 19 & 20.811 & 22.841 & 25.117 & 27.671 & 30.539 & 33.760 & 37.379 & 41.446 & 46.018 & 51.159 & 56.939 & 63.440 & 70.749 & 78.969 & 88.212 & 98.603 & 154.740 & 244.033 & 273.556 & 483.973 \\
\hline 20 & 22.019 & 24.297 & 26.870 & 29.778 & 33.066 & 36.786 & 40.995 & 45.762 & 51.160 & 57.275 & 64.203 & 72.052 & 80.947 & 91.025 & 102.444 & 115.380 & 186.688 & 303.601 & 342.945 & 630.165 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 21 & 23.239 & 25.783 & 28.676 & 31.969 & 35.719 & 39.993 & 44.865 & 50.423 & 56.765 & 64.002 & 72.265 & 81.699 & 92.470 & 104.768 & 118.810 & 134.841 & 225.026 & 377.465 & 429.681 & 820.215 \\
\hline 22 & 24.472 & 27.299 & 30.537 & 34.248 & 38.505 & 43.392 & 49.006 & 55.457 & 62.873 & 71.403 & 81.214 & 92.503 & 105.491 & 120.436 & 137.632 & 157.415 & 271.031 & 469.056 & 538.101 & * \\
\hline 23 & 25.716 & 28.845 & 32.453 & 36.618 & 41.430 & 46.996 & 53.436 & 60.893 & 69.532 & 79.543 & 91.148 & 104.603 & 120.205 & 138.297 & 159.276 & 183.601 & 326.237 & 582.630 & 673.626 & * \\
\hline 24 & 26.973 & 30.422 & 34.426 & 39.083 & 44.502 & 50.816 & 58.177 & 66.765 & 76.790 & 88.497 & 102.174 & 118.155 & 136.831 & 158.659 & 184.168 & 213.978 & 392.484 & 723.461 & 843.033 & * \\
\hline 25 & 28.243 & 32.030 & 36.459 & 41.646 & 47.727 & 54.865 & 63.249 & 73.106 & 84.701 & 98.347 & 114.413 & 133.334 & 155.620 & 181.871 & 212.793 & 249.214 & 471.981 & 898.092 & * & * \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 30 & 34.785 & 40.568 & 47.575 & 56.085 & 66.439 & 79.058 & 94.461 & 113.283 & 136.308 & 164.494 & 199.021 & 241.333 & 293.199 & 356.787 & 434.745 & 530.312 & * & * & * & * \\
\hline 35 & 41.660 & 49.994 & 60.462 & 73.652 & 90.320 & 111.435 & 138.237 & 172.317 & 215.711 & 271.024 & 341.590 & 431.663 & 546.681 & 693.573 & 881.170 & * & * & * & * & * \\
\hline 36 & 43.077 & 51.994 & 63.276 & 77.598 & 95.836 & 119.121 & 148.913 & 187.102 & 236.125 & 299.127 & 380.164 & 484.463 & 618.749 & 791.673 & * & * & * & * & * & * \\
\hline 40 & 48.886 & 60.402 & 75.401 & 95.026 & 120.800 & 154.762 & 199.635 & 259.057 & 337.882 & 442.593 & 581.826 & 767.091 & * & * & * & * & * & * & * & * \\
\hline 50 & 64.463 & 84.579 & 112.797 & 152.667 & 209.348 & 290.336 & 406.529 & 573.770 & 815.084 & * & * & * & * & * & * & * & * & * & * & * \\
\hline
\end{tabular}

Table A-3 Present Value Interest Factors for One Dollar Discounted at \(k\) Percent for \(n\) Periods: \(P V I F_{k, n}=1 /(1+k)^{n}\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Period & 1\% & 2\% & 3\% & 4\% & 5\% & 6\% & 7\% & 8\% & 9\% & 10\% & 11\% & 12\% & 13\% & 14\% & 15\% & 16\% & 20\% & 24\% & 25\% & 30\% \\
\hline 1 & 0.9901 & 0.9804 & 0.9709 & 0.9615 & 0.9524 & 0.9434 & 0.9346 & 0.9259 & 0.9174 & 0.9091 & 0.9009 & 0.8929 & 0.8850 & 0.8772 & 0.8696 & 0.8621 & 0.8333 & 0.8065 & 0.8000 & 0.7692 \\
\hline 2 & 0.9803 & 0.9612 & 0.9426 & 0.9246 & 0.9070 & 0.8900 & 0.8734 & 0.8573 & 0.8417 & 0.8264 & 0.8116 & 0.7972 & 0.7831 & 0.7695 & 0.7561 & 0.7432 & 0.6944 & 0.6504 & 0.6400 & 0.5917 \\
\hline 3 & 0.9706 & 0.9423 & 0.9151 & 0.8890 & 0.8638 & 0.8396 & 0.8163 & 0.7938 & 0.7722 & 0.7513 & 0.7312 & 0.7118 & 0.6931 & 0.6750 & 0.6575 & 0.6407 & 0.5787 & 0.5245 & 0.5120 & 0.4552 \\
\hline 4 & 0.9610 & 0.9238 & 0.8885 & 0.8548 & 0.8227 & 0.7921 & 0.7629 & 0.7350 & 0.7084 & 0.6830 & 0.6587 & 0.6355 & 0.6133 & 0.5921 & 0.5718 & 0.5523 & 0.4823 & 0.4230 & 0.4096 & 0.3501 \\
\hline 5 & 0.9515 & 0.9057 & 0.8626 & 0.8219 & 0.7835 & 0.7473 & 0.7130 & 0.6806 & 0.6499 & 0.6209 & 0.5935 & 0.5674 & 0.5428 & 0.5194 & 0.4972 & 0.4761 & 0.4019 & 0.3411 & 0.3277 & 0.2693 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 6 & 0.9420 & 0.8880 & 0.8375 & 0.7903 & 0.7462 & 0.7050 & 0.6663 & 0.6302 & 0.5963 & 0.5645 & 0.5346 & 0.5066 & 0.4803 & 0.4556 & 0.4323 & 0.4104 & 0.3349 & 0.2751 & 0.2621 & 0.2072 \\
\hline 7 & 0.9327 & 0.8706 & 0.8131 & 0.7599 & 0.7107 & 0.6651 & 0.6227 & 0.5835 & 0.5470 & 0.5132 & 0.4817 & 0.4523 & 0.4251 & 0.3996 & 0.3759 & 0.3538 & 0.2791 & 0.2218 & 0.2097 & 0.1594 \\
\hline 8 & 0.9235 & 0.8535 & 0.7894 & 0.7307 & 0.6768 & 0.6274 & 0.5820 & 0.5403 & 0.5019 & 0.4665 & 0.4339 & 0.4039 & 0.3762 & 0.3506 & 0.3269 & 0.3050 & 0.2326 & 0.1789 & 0.1678 & 0.1226 \\
\hline 9 & 0.9143 & 0.8368 & 0.7664 & 0.7026 & 0.6446 & 0.5919 & 0.5439 & 0.5002 & 0.4604 & 0.4241 & 0.3909 & 0.3606 & 0.3329 & 0.3075 & 0.2843 & 0.2630 & 0.1938 & 0.1443 & 0.1342 & 0.0943 \\
\hline 10 & 0.9053 & 0.8203 & 0.7441 & 0.6756 & 0.6139 & 0.5584 & 0.5083 & 0.4632 & 0.4224 & 0.3855 & 0.3522 & 0.3220 & 0.2946 & 0.2697 & 0.2472 & 0.2267 & 0.1615 & 0.1164 & 0.1074 & 0.0725 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 11 & 0.8963 & 0.8043 & 0.7224 & 0.6496 & 0.5847 & 0.5268 & 0.4751 & 0.4289 & 0.3875 & 0.3505 & 0.3173 & 0.2875 & 0.2607 & 0.2366 & 0.2149 & 0.1954 & 0.1346 & 0.0938 & 0.0859 & 0.0558 \\
\hline 12 & 0.8874 & 0.7885 & 0.7014 & 0.6246 & 0.5568 & 0.4970 & 0.4440 & 0.3971 & 0.3555 & 0.3186 & 0.2858 & 0.2567 & 0.2307 & 0.2076 & 0.1869 & 0.1685 & 0.1122 & 0.0757 & 0.0687 & 0.0429 \\
\hline 13 & 0.8787 & 0.7730 & 0.6810 & 0.6006 & 0.5303 & 0.4688 & 0.4150 & 0.3677 & 0.3262 & 0.2897 & 0.2575 & 0.2292 & 0.2042 & 0.1821 & 0.1625 & 0.1452 & 0.0935 & 0.0610 & 0.0550 & 0.0330 \\
\hline 14 & 0.8700 & 0.7579 & 0.6611 & 0.5775 & 0.5051 & 0.4423 & 0.3878 & 0.3405 & 0.2992 & 0.2633 & 0.2320 & 0.2046 & 0.1807 & 0.1597 & 0.1413 & 0.1252 & 0.0779 & 0.0492 & 0.0440 & 0.0254 \\
\hline 15 & 0.8613 & 0.7430 & 0.6419 & 0.5553 & 0.4810 & 0.4173 & 0.3624 & 0.3152 & 0.2745 & 0.2394 & 0.2090 & 0.1827 & 0.1599 & 0.1401 & 0.1229 & 0.1079 & 0.0649 & 0.0397 & 0.0352 & 0.0195 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 16 & 0.8528 & 0.7284 & 0.6232 & 0.5339 & 0.4581 & 0.3936 & 0.3387 & 0.2919 & 0.2519 & 0.2176 & 0.1883 & 0.1631 & 0.1415 & 0.1229 & 0.1069 & 0.0930 & 0.0541 & 0.0320 & 0.0281 & 0.0150 \\
\hline 17 & 0.8444 & 0.7142 & 0.6050 & 0.5134 & 0.4363 & 0.3714 & 0.3166 & 0.2703 & 0.2311 & 0.1978 & 0.1696 & 0.1456 & 0.1252 & 0.1078 & 0.0929 & 0.0802 & 0.0451 & 0.0258 & 0.0225 & 0.0116 \\
\hline 18 & 0.8360 & 0.7002 & 0.5874 & 0.4936 & 0.4155 & 0.3503 & 0.2959 & 0.2502 & 0.2120 & 0.1799 & 0.1528 & 0.1300 & 0.1108 & 0.0946 & 0.0808 & 0.0691 & 0.0376 & 0.0208 & 0.0180 & 0.0089 \\
\hline 19 & 0.8277 & 0.6864 & 0.5703 & 0.4746 & 0.3957 & 0.3305 & 0.2765 & 0.2317 & 0.1945 & 0.1635 & 0.1377 & 0.1161 & 0.0981 & 0.0829 & 0.0703 & 0.0596 & 0.0313 & 0.0168 & 0.0144 & 0.0068 \\
\hline 20 & 0.8195 & 0.6730 & 0.5537 & 0.4564 & 0.3769 & 0.3118 & 0.2584 & 0.2145 & 0.1784 & 0.1486 & 0.1240 & 0.1037 & 0.0868 & 0.0728 & 0.0611 & 0.0514 & 0.0261 & 0.0135 & 0.0115 & 0.0053 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 21 & 0.8114 & 0.6598 & 0.5375 & 0.4388 & 0.3589 & 0.2942 & 0.2415 & 0.1987 & 0.1637 & 0.1351 & 0.1117 & 0.0926 & 0.0768 & 0.0638 & 0.0531 & 0.0443 & 0.0217 & 0.0109 & 0.0092 & 0.0040 \\
\hline 22 & 0.8034 & 0.6468 & 0.5219 & 0.4220 & 0.3418 & 0.2775 & 0.2257 & 0.1839 & 0.1502 & 0.1228 & 0.1007 & 0.0826 & 0.0680 & 0.0560 & 0.0462 & 0.0382 & 0.0181 & 0.0088 & 0.0074 & 0.0031 \\
\hline 23 & 0.7954 & 0.6342 & 0.5067 & 0.4057 & 0.3256 & 0.2618 & 0.2109 & 0.1703 & 0.1378 & 0.1117 & 0.0907 & 0.0738 & 0.0601 & 0.0491 & 0.0402 & 0.0329 & 0.0151 & 0.0071 & 0.0059 & 0.0024 \\
\hline 24 & 0.7876 & 0.6217 & 0.4919 & 0.3901 & 0.3101 & 0.2470 & 0.1971 & 0.1577 & 0.1264 & 0.1015 & 0.0817 & 0.0659 & 0.0532 & 0.0431 & 0.0349 & 0.0284 & 0.0126 & 0.0057 & 0.0047 & 0.0018 \\
\hline 25 & 0.7798 & 0.6095 & 0.4776 & 0.3751 & 0.2953 & 0.2330 & 0.1842 & 0.1460 & 0.1160 & 0.0923 & 0.0736 & 0.0588 & 0.0471 & 0.0378 & 0.0304 & 0.0245 & 0.0105 & 0.0046 & 0.0038 & 0.0014 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 30 & 0.7419 & 0.5521 & 0.4120 & 0.3083 & 0.2314 & 0.1741 & 0.1314 & 0.0994 & 0.0754 & 0.0573 & 0.0437 & 0.0334 & 0.0256 & 0.0196 & 0.0151 & 0.0116 & 0.0042 & 0.0016 & 0.0012 & * \\
\hline 35 & 0.7059 & 0.5000 & 0.3554 & 0.2534 & 0.1813 & 0.1301 & 0.0937 & 0.0676 & 0.0490 & 0.0356 & 0.0259 & 0.0189 & 0.0139 & 0.0102 & 0.0075 & 0.0055 & 0.0017 & 0.0005 & * & * \\
\hline 36 & 0.6989 & 0.4902 & 0.3450 & 0.2437 & 0.1727 & 0.1227 & 0.0875 & 0.0626 & 0.0449 & 0.0323 & 0.0234 & 0.0169 & 0.0123 & 0.0089 & 0.0065 & 0.0048 & 0.0014 & * & * & * \\
\hline 40 & 0.6717 & 0.4529 & 0.3066 & 0.2083 & 0.1420 & 0.0972 & 0.0668 & 0.0460 & 0.0318 & 0.0221 & 0.0154 & 0.0107 & 0.0075 & 0.0053 & 0.0037 & 0.0026 & 0.0007 & * & * & * \\
\hline 50 & 0.6080 & 0.3715 & 0.2281 & 0.1407 & 0.0872 & 0.0543 & 0.0339 & 0.0213 & 0.0134 & 0.0085 & 0.0054 & 0.0035 & 0.0022 & 0.0014 & 0.0009 & 0.0006 & * & * & * & * \\
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Table A-4 Present Value Interest Factors for a One-Dollar Annuity Discounted at \(k\) Percent for \(n\) Periods: PVIFA \(=\left[1-1 /(1+k)^{n}\right] / k\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Period & 1\% & 2\% & 3\% & 4\% & 5\% & 6\% & 7\% & 8\% & 9\% & 10\% & 11\% & 12\% & 13\% & 14\% & 15\% & 16\% & 20\% & 24\% & 25\% & 30\% \\
\hline 1 & 0.9901 & 0.9804 & 0.9709 & 0.9615 & 0.9524 & 0.9434 & 0.9346 & 0.9259 & 0.9174 & 0.9091 & 0.9009 & 0.8929 & 0.8850 & 0.8772 & 0.8696 & 0.8621 & 0.8333 & 0.8065 & 0.8000 & 0.7692 \\
\hline 2 & 1.9704 & 1.9416 & 1.9135 & 1.8861 & 1.8594 & 1.8334 & 1.8080 & 1.7833 & 1.7591 & 1.7355 & 1.7125 & 1.6901 & 1.6681 & 1.6467 & 1.6257 & 1.6052 & 1.5278 & 1.4568 & 1.4400 & 1.3609 \\
\hline 3 & 2.9410 & 2.8839 & 2.8286 & 2.7751 & 2.7232 & 2.6730 & 2.6243 & 2.5771 & 2.5313 & 2.4869 & 2.4437 & 2.4018 & 2.3612 & 2.3216 & 2.2832 & 2.2459 & 2.1065 & 1.9813 & 1.9520 & 1.8161 \\
\hline 4 & 3.9020 & 3.8077 & 3.7171 & 3.6299 & 3.5460 & 3.4651 & 3.3872 & 3.3121 & 3.2397 & 3.1699 & 3.1024 & 3.0373 & 2.9745 & 2.9137 & 2.8550 & 2.7982 & 2.5887 & 2.4043 & 2.3616 & 2.1662 \\
\hline 5 & 4.8534 & 4.7135 & 4.5797 & 4.4518 & 4.3295 & 4.2124 & 4.1002 & 3.9927 & 3.8897 & 3.7908 & 3.6959 & 3.6048 & 3.5172 & 3.4331 & 3.3522 & 3.2743 & 2.9906 & 2.7454 & 2.6893 & 2.4356 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 6 & 5.7955 & 5.6014 & 5.4172 & 5.2421 & 5.0757 & 4.9173 & 4.7665 & 4.6229 & 4.4859 & 4.3553 & 4.2305 & 4.1114 & 3.9975 & 3.8887 & 3.7845 & 3.6847 & 3.3255 & 3.0205 & 2.9514 & 2.6427 \\
\hline 7 & 6.7282 & 6.4720 & 6.2303 & 6.0021 & 5.7864 & 5.5824 & 5.3893 & 5.2064 & 5.0330 & 4.8684 & 4.7122 & 4.5638 & 4.4226 & 4.2883 & 4.1604 & 4.0386 & 3.6046 & 3.2423 & 3.1611 & 2.8021 \\
\hline 8 & 7.6517 & 7.3255 & 7.0197 & 6.7327 & 6.4632 & 6.2098 & 5.9713 & 5.7466 & 5.5348 & 5.3349 & 5.1461 & 4.9676 & 4.7988 & 4.6389 & 4.4873 & 4.3436 & 3.8372 & 3.4212 & 3.3289 & 2.9247 \\
\hline 9 & 8.5660 & 8.1622 & 7.7861 & 7.4353 & 7.1078 & 6.8017 & 6.5152 & 6.2469 & 5.9952 & 5.7590 & 5.5370 & 5.3282 & 5.1317 & 4.9464 & 4.7716 & 4.6065 & 4.0310 & 3.5655 & 3.4631 & 3.0190 \\
\hline 10 & 9.4713 & 8.9826 & 8.5302 & 8.1109 & 7.7217 & 7.3601 & 7.0236 & 6.7101 & 6.4177 & 6.1446 & 5.8892 & 5.6502 & 5.4262 & 5.2161 & 5.0188 & 4.8332 & 4.1925 & 3.6819 & 3.5705 & 3.0915 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 11 & 10.368 & 9.7868 & 9.2526 & 8.7605 & 8.3064 & 7.8869 & 7.4987 & 7.1390 & 6.8052 & 6.4951 & 6.2065 & 5.9377 & 5.6869 & 5.4527 & 5.2337 & 5.0286 & 4.3271 & 3.7757 & 3.6564 & 3.1473 \\
\hline 12 & 11.255 & 10.575 & 9.9540 & 9.3851 & 8.8633 & 8.3838 & 7.9427 & 7.5361 & 7.1607 & 6.8137 & 6.4924 & 6.1944 & 5.9176 & 5.6603 & 5.4206 & 5.1971 & 4.4392 & 3.8514 & 3.7251 & 3.1903 \\
\hline 13 & 12.134 & 11.348 & 10.635 & 9.9856 & 9.3936 & 8.8527 & 8.3577 & 7.9038 & 7.4869 & 7.1034 & 6.7499 & 6.4235 & 6.1218 & 5.8424 & 5.5831 & 5.3423 & 4.5327 & 3.9124 & 3.7801 & 3.2233 \\
\hline 14 & 13.004 & 12.106 & 11.296 & 10.563 & 9.8986 & 9.2950 & 8.7455 & 8.2442 & 7.7862 & 7.3667 & 6.9819 & 6.6282 & 6.3025 & 6.0021 & 5.7245 & 5.4675 & 4.6106 & 3.9616 & 3.8241 & 3.2487 \\
\hline 15 & 13.865 & 12.849 & 11.938 & 11.118 & 10.380 & 9.7122 & 9.1079 & 8.5595 & 8.0607 & 7.6061 & 7.1909 & 6.8109 & 6.4624 & 6.1422 & 5.8474 & 5.5755 & 4.6755 & 4.0013 & 3.8593 & 3.2682 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 16 & 14.718 & 13.578 & 12.561 & 11.652 & 10.838 & 10.106 & 9.4466 & 8.8514 & 8.3126 & 7.8237 & 7.3792 & 6.9740 & 6.6039 & 6.2651 & 5.9542 & 5.6685 & 4.7296 & 4.0333 & 3.8874 & 3.2832 \\
\hline 17 & 15.562 & 14.292 & 13.166 & 12.166 & 11.274 & 10.477 & 9.7632 & 9.1216 & 8.5436 & 8.0216 & 7.5488 & 7.1196 & 6.7291 & 6.3729 & 6.0472 & 5.7487 & 4.7746 & 4.0591 & 3.9099 & 3.2948 \\
\hline 18 & 16.398 & 14.992 & 13.754 & 12.659 & 11.690 & 10.828 & 10.059 & 9.3719 & 8.7556 & 8.2014 & 7.7016 & 7.2497 & 6.8399 & 6.4674 & 6.1280 & 5.8178 & 4.8122 & 4.0799 & 3.9279 & 3.3037 \\
\hline 19 & 17.226 & 15.678 & 14.324 & 13.134 & 12.085 & 11.158 & 10.336 & 9.6036 & 8.9501 & 8.3649 & 7.8393 & 7.3658 & 6.9380 & 6.5504 & 6.1982 & 5.8775 & 4.8435 & 4.0967 & 3.9424 & 3.3105 \\
\hline 20 & 18.046 & 16.351 & 14.877 & 13.590 & 12.462 & 11.470 & 10.594 & 9.8181 & 9.1285 & 8.5136 & 7.9633 & 7.4694 & 7.0248 & 6.6231 & 6.2593 & 5.9288 & 4.8696 & 4.1103 & 3.9539 & 3.3158 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 21 & 18.857 & 17.011 & 15.415 & 14.029 & 12.821 & 11.764 & 10.836 & 10.017 & 9.2922 & 8.6487 & 8.0751 & 7.5620 & 7.1016 & 6.6870 & 6.3125 & 5.9731 & 4.8913 & 4.1212 & 3.9631 & 3.3198 \\
\hline 22 & 19.660 & 17.658 & 15.937 & 14.451 & 13.163 & 12.042 & 11.061 & 10.201 & 9.4424 & 8.7715 & 8.1757 & 7.6446 & 7.1695 & 6.7429 & 6.3587 & 6.0113 & 4.9094 & 4.1300 & 3.9705 & 3.3230 \\
\hline 23 & 20.456 & 18.292 & 16.444 & 14.857 & 13.489 & 12.303 & 11.272 & 10.371 & 9.5802 & 8.8832 & 8.2664 & 7.7184 & 7.2297 & 6.7921 & 6.3988 & 6.0442 & 4.9245 & 4.1371 & 3.9764 & 3.3254 \\
\hline 24 & 21.243 & 18.914 & 16.936 & 15.247 & 13.799 & 12.550 & 11.469 & 10.529 & 9.7066 & 8.9847 & 8.3481 & 7.7843 & 7.2829 & 6.8351 & 6.4338 & 6.0726 & 4.9371 & 4.1428 & 3.9811 & 3.3272 \\
\hline 25 & 22.023 & 19.523 & 17.413 & 15.622 & 14.094 & 12.783 & 11.654 & 10.675 & 9.8226 & 9.0770 & 8.4217 & 7.8431 & 7.3300 & 6.8729 & 6.4641 & 6.0971 & 4.9476 & 4.1474 & 3.9849 & 3.3286 \\
\hline & & & & & & & & & & & & & & & & & & & & \\
\hline 30 & 25.808 & 22.396 & 19.600 & 17.292 & 15.372 & 13.765 & 12.409 & 11.258 & 10.274 & 9.4269 & 8.6938 & 8.0552 & 7.4957 & 7.0027 & 6.5660 & 6.1772 & 4.9789 & 4.1601 & 3.9950 & 3.3321 \\
\hline 35 & 29.409 & 24.999 & 21.487 & 18.665 & 16.374 & 14.498 & 12.948 & 11.655 & 10.567 & 9.6442 & 8.8552 & 8.1755 & 7.5856 & 7.0700 & 6.6166 & 6.2153 & 4.9915 & 4.1644 & 3.9984 & 3.3330 \\
\hline 36 & 30.108 & 25.489 & 21.832 & 18.908 & 16.547 & 14.621 & 13.035 & 11.717 & 10.612 & 9.6765 & 8.8786 & 8.1924 & 7.5979 & 7.0790 & 6.6231 & 6.2201 & 4.9929 & 4.1649 & 3.9987 & 3.3331 \\
\hline 40 & 32.835 & 27.355 & 23.115 & 19.793 & 17.159 & 15.046 & 13.332 & 11.925 & 10.757 & 9.7791 & 8.9511 & 8.2438 & 7.6344 & 7.1050 & 6.6418 & 6.2335 & 4.9966 & 4.1659 & 3.9995 & 3.3332 \\
\hline 50 & 39.196 & 31.424 & 25.730 & 21.482 & 18.256 & 15.762 & 13.801 & 12.233 & 10.962 & 9.9148 & 9.0417 & 8.3045 & 7.6752 & 7.1327 & 6.6605 & 6.2463 & 4.9995 & 4.1666 & 3.9999 & 3.3333 \\
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[^0]:    16. Kaisar has a piece of land and he planted mahogany tree saplings at a cost of Taka $1,00,000$. He expects to sell the trees for Taka $2,00,000$ at the end of seven years. What rate of return will he get? (Answer without using calculator or any tables)
    17. Kashem approached Hashem for a loan of Taka $12,00,000$. Kashem told Hashem that he will pay him double the amount in five years. What rate of return is Kashem promising? (Exact Rate)
    18. Kashfia had borrowed Taka 65,000 from Habib four years ago. She paid him Taka $1,00,000$ today as a full pay-off. What interest rate did Kashfia pay?
    19. Keka wants to accumulate Taka $12,00,000$ by depositing Taka $1,00,000$ in an account every year at the end of the year, the first deposit to be made at the end of this year. The
