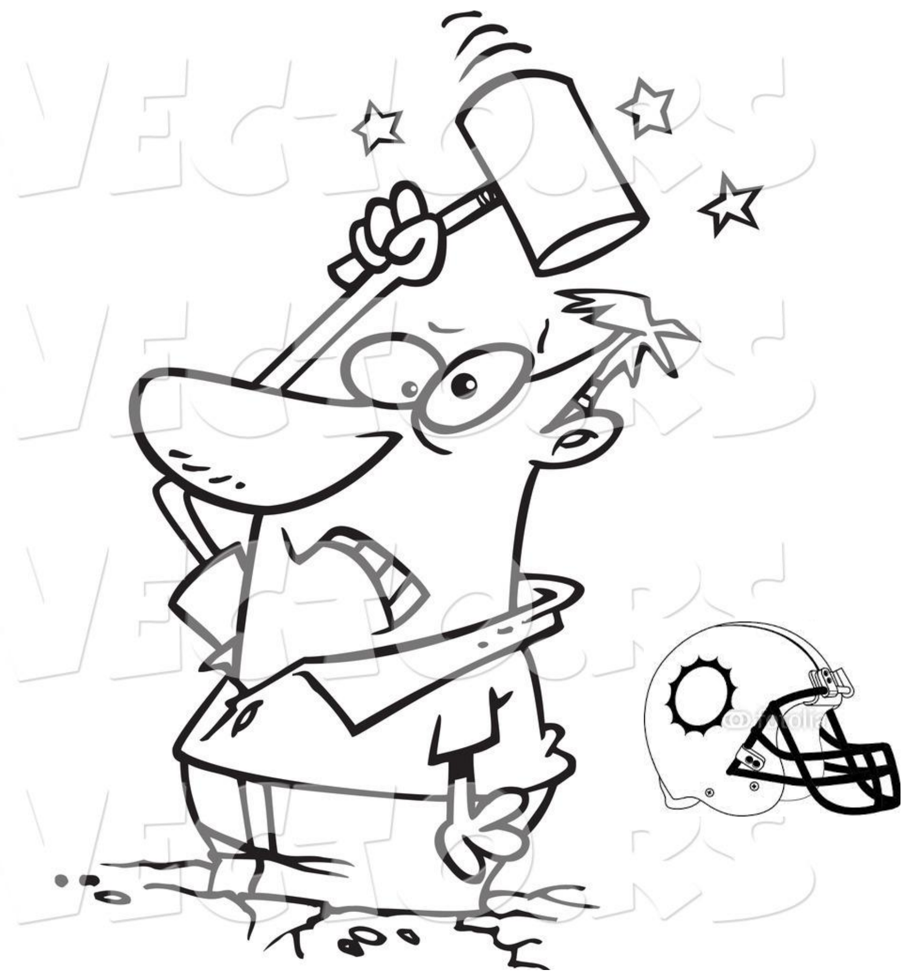
# Applied Mathematics

## for Business & Economics Students



Notes Prepared By:

Mohammad Kamrul Arefin

# Notes on Business Mathematics II

# for BMT201 Group 1 & 2 Students

## Table of Contents

**Applied Mathematics**

**Practice Questions and Answers # 4**

**Problem Set 7-5:**

Find $f'(x)$. Do not leave negative exponents in answers.

**Question No. 1**:

$f(x) = 2 + k$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[2+k]$

$= \frac{d}{dx}(2) + \frac{d}{dx}(k) = 0$

**Question No. 2**:

$f(x) = x$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x] = 1$

**Question No. 3**:

$f(x) = x/2$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x/2]$

$= \frac{d}{dx}[\frac{1}{2}.x] = \frac{1}{2}\frac{d}{dx}[x] = \frac{1}{2}.1 = \frac{1}{2}$

**Question No. 4**:

$f(x) = 2x + 3$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[2x+3]$

$= \frac{d}{dx}(2x) + \frac{d}{dx}(3) = 2\frac{d}{dx}(x) + 0 = 2$

**Question No. 5**:

$$f(x) = x/3 + 4$$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x/3 + 4]$

$$= \frac{d}{dx}(\frac{x}{3}) + \frac{d}{dx}(4) = \frac{1}{3}\frac{d}{dx}(x) + 0 = \frac{1}{3}.1 = \frac{1}{3}$$

**Question No. 6**:

$$f(x) = 2/3 - 3(x/2) = 2/3 - 3x/2$$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[2/3 - 3x/2]$

$$= \frac{d}{dx}(2/3) - \frac{d}{dx}(3x/2) = 0 - \frac{d}{dx}(\frac{3}{2}x) = -\frac{3}{2}\frac{d}{dx}(x) = -\frac{3}{2}$$

**Question No. 7**:

$$f(x) = 3x^2 + 2x - 5$$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^2 + 2x - 5]$

$$= \frac{d}{dx}(3x^2) + \frac{d}{dx}(2x) - \frac{d}{dx}(5) = 3\frac{d}{dx}(x^2) + 2\frac{d}{dx}(x) - 0$$

$$= 3(2x^{2-1}) + 2(1) = 6x + 2$$

**Question No. 8**:

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + 12$$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[\frac{x^3}{3} - \frac{x^2}{2} + x + 12]$

$$= \frac{d}{dx}(\frac{x^3}{3}) - \frac{d}{dx}(\frac{x^2}{2}) + \frac{d}{dx}(x) + \frac{d}{dx}(12)$$

$$= \frac{1}{3}\frac{d}{dx}(x^3) - \frac{1}{2}\frac{d}{dx}(x^2) + 1 + 0$$

$$= \frac{1}{3}(3.x^{3-1}) - \frac{1}{2}(2.x^{2-1}) + 1 = x^2 - x + 1$$

**Question No. 12**:

$$f(x) = ax^2 + bx + c$$

Derivative of f(x) = $f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[ax^2 + bx + c]$

$$= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)$$

$$= a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + 0$$

$$= a(2.x^{2-1}) + b = 2ax + b$$

**Question No. 20**:

$$\frac{d}{dy}(10y^2 - 4x + 7)$$

$$= \frac{d}{dy}(10y^2) - \frac{d}{dy}(4x) + \frac{d}{dy}(7)$$

$$= 10\frac{d}{dy}(y^2) - 4\frac{d}{dy}(x) + 0 = 10(2y) - 0 + 0 = 20y$$

**Question No. 22**:

$$\frac{d}{dx}(3pw^2 - 2p^3)$$

$$= \frac{d}{dx}(3pw^2) - \frac{d}{dx}(2p^3)$$

$$= 0 + 0 = 0$$

**Question No. 24**:

$$\frac{d}{dm}(\frac{a}{m} - 3m^2 + 5a^3)$$

$$= \frac{d}{dm}(\frac{a}{m}) - \frac{d}{dm}(3m^2) + \frac{d}{dm}(5a^3) = a\frac{d}{dm}(\frac{1}{m}) - 3\frac{d}{dm}(m^2) + 0$$

$$= a\frac{d}{dm}(m^{-1}) - 3(2m) = a(-1m^{-1-1}) - 6m = -am^{-2} - 6m = -\frac{a}{m^2} - 6m$$

**Find the slope of the tangent to each of the following curves at the indicated value of x:**

**Question No. 25**:

$f(x) = 3$

$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3] = 0$

$Slope$ at x=1; $f'(1) = 0$

**Question No. 29**:

$f(x) = 3x^2 - 2x + 5$

$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^2 - 2x + 5]$

$= \frac{d}{dx}(3x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(5) = 3(2x) - 2(1) + 0 = 6x - 2$

$Slope$ at x=0.5; $f'(0.5) = 6(0.5) - 2 = 1$

**Question No. 30**:

$f(x) = 3x^2 - 2x + 5$

$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^2 - 2x + 5]$

$= \frac{d}{dx}(3x^2) - \frac{d}{dx}(2x) + \frac{d}{dx}(5) = 3(2x) - 2(1) + 0 = 6x - 2$

$Slope$ at x=0.5; $f'(0.5) = 6(0.5) - 2 = 1$

**Question No. 33**:

$$f(x) = 8x^{1/3} + x$$

$$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[8x^{1/3} + x]$$

$$= \frac{d}{dx}(8x^{1/3}) + \frac{d}{dx}(x) = 8(\frac{1}{3}x^{\frac{1}{3}-1}) + 1 = \frac{8}{3}x^{-\frac{2}{3}} + 1 = \frac{8}{3}\frac{1}{x^{\frac{2}{3}}} + 1 = \frac{8}{3}\frac{1}{\sqrt[3]{x^2}} + 1$$

$$Slope \text{ at x=8; } f'(8) = \frac{8}{3}\frac{1}{(8)^{\frac{2}{3}}} + 1 = \frac{8}{3}\frac{1}{(2^3)^{\frac{2}{3}}} + 1 = \frac{8}{3}\frac{1}{2^{3.\frac{2}{3}}} + 1$$

$$= \frac{8}{3}\frac{1}{2^2} + 1 = \frac{8}{3}\frac{1}{4} + 1 = \frac{2}{3} + 1 = \frac{5}{3}$$

**Find the value of x for which the slope is 0**

**Question No. 35**:

$$f(x) = 10 - 3x^2 + 3x$$

$$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[10 - 3x^2 + 3x]$$

$$= \frac{d}{dx}(10) - \frac{d}{dx}(3x^2) + \frac{d}{dx}(3x) = 0 - 3(2x) + 3(1) = -6x + 3$$

$$Slope \ = 0$$

$$or, -6x + 3 = 0$$

$$or, -6x = -3$$

$$or, x = 1/2$$

Slope will be 0 when x=1/2

**Question No. 37**:

$$f(x) = 3x^{1/3} - 4x;\ \text{x>0}$$

$$Slope = f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[3x^{1/3} - 4x]$$

$$= \frac{d}{dx}(3x^{1/3}) - \frac{d}{dx}(4x) = 3(\frac{1}{3}x^{\frac{1}{3}-1}) - 4(1) = x^{-\frac{2}{3}} - 4 = \frac{1}{x^{\frac{2}{3}}} - 4$$

$$Slope \ = 0$$

$$or, \frac{1}{x^{\frac{2}{3}}} - 4 = 0$$

$$or, \frac{1}{x^{\frac{2}{3}}} = 4$$

$$or, x^{\frac{2}{3}} = \frac{1}{4}$$

$$or, x^{\frac{2}{3}.3} = (\frac{1}{4})^3$$

$$or, x^2 = \frac{1}{64}$$

$$or, x = \pm\frac{1}{8}$$

as x>0, hence slope is 0 when x=1/8

**Question No. 41**: **If the total cost of producing y yards of Yardall is,**

**C(y) = 0.001y$^{2}$+2y+500, Find the marginal cost at outputs of a) 1,000 yards b)2,000 yards**

$$\text{Marginal Cost} = C'(y) = \frac{d}{dy}[C(y)] = \frac{d}{dy}[0.001y^2 + 2y + 500]$$

$$= \frac{d}{dy}(0.001y^2) + \frac{d}{dy}(2y) + \frac{d}{dy}(500) = 0.001(2y) + 2(1) + 0 = 0.002y + 2$$

$a$) Marginal Cost at outputs=1000 yards

$C'(1000) = 0.002(1000) + 2 = \$4$ per yard

$b$) Marginal Cost at outputs=2000 yards

$C'(2000) = 0.002(2000) + 2 = \$6$ per yard

**Question No. 41**: **If the total cost of producing t tons of Tonal is,**

**C(t) = 0.0005t$^{3}$-0.3t$^{2}$+100t+30,000,**

**Find the marginal cost at outputs of a) 100 tons b)200 tons**

$$\text{Marginal Cost} = C'(t) = \frac{d}{dt}[C(t)] = \frac{d}{dt}[0.0005t^3 - 0.3t^2 + 100t + 30000]$$

$$= \frac{d}{dt}(0.0005t^3) - \frac{d}{dt}(0.3t^2) + \frac{d}{dt}(100t) + \frac{d}{dt}(30000)$$

$$= 0.0005(3t^2) - 0.3(2t) + 100(1) + 0 = 0.0015t^2 - 0.6t + 100$$

$a$) Marginal Cost at outputs=100 tons

$C'(100) = 0.0015(100)^2 - 0.6(100) + 100 = \$55$ per ton

$b$) Marginal Cost at outputs=200 tons

$C'(200) = 0.0015(200)^2 - 0.6(200) + 100 = \$40$ per ton

**Problem Set 7-6:**

Find $f'(x)$. Do not leave negative exponents in answers.

**Question No. 1**:

$$f(x) = (6x-5)^5$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(6x-5)^5] = 5(6x-5)^{5-1}\frac{d}{dx}(6x-5)$$

$$= 5(6x-5)^4[\frac{d}{dx}(6x) - \frac{d}{dx}(5)]$$

$$= 5(6x-5)^4[6\frac{d}{dx}(x) - 0] = 5(6x-5)^4[6] = 30(6x-5)^4$$

**Question No. 2**:

$$f(x) = (2x+6)^5$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(2x+6)^5] = 5(2x+6)^{5-1}\frac{d}{dx}(2x+6)$$

$$= 5(2x+6)^4[\frac{d}{dx}(2x) + \frac{d}{dx}(6)]$$

$$= 5(2x+6)^4[2] = 10(2x+6)^4$$

**Question No. 3**:

$$f(x) = (2x)^3$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(2x)^3] = 3(2x)^{3-1}\frac{d}{dx}(2x)$$

$$= 3(2x)^2(2) = 24x^2$$

**Question No. 4**:

$$f(x)=(6x)^{\frac{1}{3}}$$

$$f'(x)=\frac{d}{dx}[f(x)]=\frac{d}{dx}[(6x)^{\frac{1}{3}}]=\frac{1}{3}(6x)^{\frac{1}{3}-1}\frac{d}{dx}(6x)$$

$$=\frac{1}{3}(6x)^{-\frac{2}{3}}(6)=2\frac{1}{(6x)^{\frac{2}{3}}}=\frac{2}{\sqrt[3]{(6x)^2}}$$

**Question No. 5**:

$$f(x)=(4x)^{\frac{1}{2}}$$

$$f'(x)=\frac{d}{dx}[f(x)]=\frac{d}{dx}[(4x)^{\frac{1}{2}}]=\frac{1}{2}(4x)^{\frac{1}{2}-1}\frac{d}{dx}(4x)$$

$$=\frac{1}{2}(4x)^{-\frac{1}{2}}(4)=2\frac{1}{(4x)^{\frac{1}{2}}}=\frac{2}{\sqrt{(4x)}}=\frac{2}{2\sqrt{x}}=\frac{1}{\sqrt{x}}$$

**Question No. 6**:

$$f(x)=(9x)^{\frac{4}{3}}$$

$$f'(x)=\frac{d}{dx}[f(x)]=\frac{d}{dx}[(9x)^{\frac{4}{3}}]=\frac{4}{3}(9x)^{\frac{4}{3}-1}\frac{d}{dx}(9x)$$

$$=\frac{4}{3}(9x)^{\frac{4}{3}-1}(9)=12(9x)^{\frac{1}{3}}$$

**Question No. 7**:

$$f(x)=(8x-3)^{\frac{3}{2}}$$

$$f'(x)=\frac{d}{dx}[f(x)]=\frac{d}{dx}[(8x-3)^{\frac{3}{2}}]=\frac{3}{2}(8x-3)^{\frac{3}{2}-1}\frac{d}{dx}(8x-3)$$

$$=\frac{3}{2}(8x-3)^{\frac{3}{2}-1}\{\frac{d}{dx}(8x)-\frac{d}{dx}(3)\}=\frac{3}{2}(8x-3)^{\frac{1}{2}}(8)=12(8x-3)^{\frac{1}{2}}=12\sqrt{8x-3}$$

**Question No. 8**:

$$f(x) = (12x - 9)^{\frac{5}{3}}$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(12x-9)^{\frac{5}{3}}] = \frac{5}{3}(12x-9)^{\frac{5}{3}-1}\frac{d}{dx}(12x-9)$$

$$= \frac{5}{3}(12x-9)^{\frac{2}{3}}(12) = 20(12x-9)^{\frac{2}{3}} = 20\sqrt[3]{(12x-9)^2}$$

**Question No. 9**:

$$f(x) = (3x^2 - 6x + 2)^{\frac{5}{2}}$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(3x^2-6x+2)^{\frac{5}{2}}]$$

$$= \frac{5}{2}(3x^2-6x+2)^{\frac{5}{2}-1}\frac{d}{dx}(3x^2-6x+2)$$

$$= \frac{5}{2}(3x^2-6x+2)^{\frac{3}{2}}(6x-6) = \frac{5}{2}(3x^2-6x+2)^{\frac{3}{2}}6(x-1)$$

$$= 15(x-1)(3x^2-6x+2)^{\frac{3}{2}} = 15(x-1)\sqrt{(3x^2-6x+2)^3}$$

**Question No. 10**:

$$f(x) = (x^3 - 3x^2 + 6x)^{\frac{4}{3}}$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(x^3-3x^2+6x)^{\frac{4}{3}}]$$

$$= \frac{4}{3}(x^3-3x^2+6x)^{\frac{4}{3}-1}\frac{d}{dx}(x^3-3x^2+6x)$$

$$= \frac{4}{3}(x^3-3x^2+6x)^{\frac{1}{3}}(3x^2-6x+6)$$

$$= \frac{4}{3}(x^3-3x^2+6x)^{\frac{1}{3}}3(x^2-2x+2) = 4(x^2-2x+2)(x^3-3x^2+6x)^{\frac{1}{3}}$$

**Question No. 11**:

$$f(x) = (2x-3)^{\frac{1}{2}}$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(2x-3)^{\frac{1}{2}}]$$

$$= \frac{1}{2}(2x-3)^{\frac{1}{2}-1}\frac{d}{dx}(2x-3)$$

$$= \frac{1}{2}(2x-3)^{-\frac{1}{2}}(2) = (2x-3)^{-\frac{1}{2}} = \frac{1}{(2x-3)^{\frac{1}{2}}} = \frac{1}{\sqrt{(2x-3)}}$$

**Question No. 12**:

$$f(x) = (3x^2+5)^{\frac{2}{3}}$$

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[(3x^2+5)^{\frac{2}{3}}]$$

$$= \frac{2}{3}(3x^2+5)^{\frac{2}{3}-1}\frac{d}{dx}(3x^2+5)$$

$$= \frac{2}{3}(3x^2+5)^{-\frac{1}{3}}(6x) = 4x(3x^2+5)^{-\frac{1}{3}} = \frac{4x}{(3x^2+5)^{\frac{1}{3}}}$$

**Question No. 13**:

$$f(x)=\frac{4}{2x-3}=4\frac{1}{2x-3}=4(2x-3)^{-1}$$

$$f'(x)=\frac{d}{dx}[f(x)]=\frac{d}{dx}[4(2x-3)^{-1}]=4\frac{d}{dx}[(2x-3)^{-1}]$$

$$=4(-1)(2x-3)^{-1-1}\frac{d}{dx}(2x-3)=-4(2x-3)^{-2}(2)$$

$$=-8(2x-3)^{-2}=-\frac{8}{(2x-3)^2}$$

Another Method

$$f'(x)=\frac{d}{dx}(\frac{4}{2x-3})=\frac{(2x-3)\frac{d}{dx}(4)-4\frac{d}{dx}(2x-3)}{(2x-3)^2}$$

$$=\frac{(2x-3)(0)-4(2)}{(2x-3)^2}=-\frac{8}{(2x-3)^2}$$

**Do the following at your home:**

**Question No. 14**: $f(x)=\frac{6}{3x-5}$ **Question No. 17**: $f(x)=\frac{9}{(3x-5)^2}$

**Question No. 18**: $f(x)=\frac{12}{(2x+10)^3}$

**Question No. 15**:

$$f(x)=(\frac{1}{x}-2)^2$$

$$f'(x)==\frac{d}{dx}[(\frac{1}{x}-2)^2]=2(\frac{1}{x}-2)^{2-1}\frac{d}{dx}(\frac{1}{x}-2)$$

$$=2(\frac{1}{x}-2)\frac{d}{dx}(x^{-1}-2)=2(\frac{1}{x}-2)(-x^{-1-1}-0)=2(\frac{1}{x}-2)(-x^{-2})$$

$$=2(\frac{1}{x}-2)(-x^{-2})=-2\frac{1}{x^2}(\frac{1}{x}-2)$$

**Question No. 16**:

$$f(x) = (5 - \frac{1}{x^2})^3$$

$$f'(x) == \frac{d}{dx}[(5 - \frac{1}{x^2})^3] = 3(5 - \frac{1}{x^2})^{3-1}\frac{d}{dx}(5 - \frac{1}{x^2})$$

$$= 3(5 - \frac{1}{x^2})^2\frac{d}{dx}(5 - x^{-2}) = 3(5 - \frac{1}{x^2})^2\{0 - (-2)x^{-2-1})$$

$$= 3(5 - \frac{1}{x^2})^2(2x^{-3}) = 6\frac{1}{x^3}(5 - \frac{1}{x^2})^2$$

**Question No. 19**:

$$f(x) = 5x + \frac{10}{(3x + 2)}$$

$$f'(x) = \frac{d}{dx}[5x + \frac{10}{(3x + 2)}] = \frac{d}{dx}(5x) + \frac{d}{dx}\{\frac{10}{(3x + 2)}\}$$

$$= 5 + \frac{(3x + 2)\frac{d}{dx}(10) - 10\frac{d}{dx}(3x + 2)}{(3x + 2)^2} = 5 + \frac{0 - 10(3)}{(3x + 2)^2}$$

$$= 5 - \frac{30}{(3x + 2)^2}$$

**Do the following at your home:**

**Question No. 20**: $f(x) = 0.1x + \frac{5}{5 - 0.2x}$

**Question No. 21**: $f(x) = 3x + \frac{1}{(5 + 2x)^{\frac{1}{2}}}$

**Question No. 22**: $f(x) = \frac{1}{(3x - 7)^{\frac{1}{2}}} - 2x$

**Question No. 23**:

$$\text{Find } g'(2) \text{ if } g(x) = 10x + \frac{18}{(5+2x)^{1/2}}$$

$$g'(x) = \frac{d}{dx}[10x + \frac{18}{(5+2x)^{1/2}}] = \frac{d}{dx}(10x) + \frac{d}{dx}\{\frac{18}{(5+2x)^{1/2}}\}$$

$$= 10 + \frac{(5+2x)^{\frac{1}{2}} \frac{d}{dx}(18) - 18\frac{d}{dx}\{(5+2x)^{\frac{1}{2}}\}}{\{(5+2x)^{\frac{1}{2}}\}^2}$$

$$= 10 + \frac{0 - 18\frac{1}{2}(5+2x)^{\frac{1}{2}-1} \frac{d}{dx}(5+2x)}{(5+2x)}$$

$$= 10 - \frac{18.\frac{1}{2}(5+2x)^{-\frac{1}{2}}(2)}{(5+2x)}$$

$$= 10 - \frac{18}{(5+2x)} \frac{1}{(5+2x)^{\frac{1}{2}}} = 10 - \frac{18}{(5+2x)^{\frac{3}{2}}}$$

$$g'(x) = 10 - \frac{18}{(5+2x)^{\frac{3}{2}}}$$

$$g'(2) = 10 - \frac{18}{(5+4)^{\frac{3}{2}}} = 10 - \frac{18}{(9)^{\frac{3}{2}}} = 10 - \frac{3^2.2}{(3^2)^{\frac{3}{2}}}$$

$$= 10 - \frac{3^2.2}{3^3} = 10 - \frac{2}{3} = \frac{28}{3}$$



**Do the following at your home:**

**Question No. 24**: $\text{Find } h'(3) \text{ if } h(x) = 7x + \frac{4}{(x^2-1)^{1/3}}$

**Problem Set 7-7:**

Find $f'(x)$. Do not leave negative exponents in answers.

**Question No. 1**:

$$f(x) = (3x-2)(2x+5)$$

$$f'(x) = \frac{d}{dx}[(3x-2)(2x+5)] = (3x-2)\frac{d}{dx}(2x+5) + (2x+5)\frac{d}{dx}(3x-2)$$

$$= (3x-2)(2) + (2x+5)(3) = 6x-4+6x+15 = 12x+11$$

**Question No. 2**:

$$f(x) = (7x+3)(4-3x)$$

$$f'(x) = \frac{d}{dx}[(7x+3)(4-3x)] = (7x+3)\frac{d}{dx}(4-3x) + (4-3x)\frac{d}{dx}(7x+3)$$

$$= (7x+3)(-3) + (4-3x)(7) = -21x-9+28-21x = 19-42x$$

**Question No. 3**:

$$f(x) = (x^2+2)(3x-5)$$

$$f'(x) = \frac{d}{dx}[(x^2+2)(3x-5)] = (x^2+2)\frac{d}{dx}(3x-5) + (3x-5)\frac{d}{dx}(x^2+2)$$

$$= (x^2+2)(3) + (3x-5)(2x) = 3x^2+6+6x^2-10x = 9x^2-10x+6$$

**Question No. 4**:

$$f(x) = (3-x^2)(5x+6)$$

$$f'(x) = \frac{d}{dx}[(3-x^2)(5x+6)] = (3-x^2)\frac{d}{dx}(5x+6) + (5x+6)\frac{d}{dx}(3-x^2)$$

$$= (3-x^2)(5) + (5x+6)(-2x) = 15-5x^2-10x^2-12x = -3(5x^2+4x-5)$$

**Question No. 5**:

$$f(x) = x(x-1)^4$$

$$f'(x) = \frac{d}{dx}[x(x-1)^4] = x\frac{d}{dx}(x-1)^4 + (x-1)^4\frac{d}{dx}(x)$$

$$= 4x(x-1)^3\frac{d}{dx}(x-1) + (x-1)^4(1)$$

$$= 4x(x-1)^3 + (x-1)^4 = (x-1)^3(4x + x - 1) = (x-1)^3(5x-1)$$

**Do the following at your home: Question No. 6**: $f(x) = x^2(x+5)^3$

**Question No. 7**: $f(x) = x^2(x+3)^{\frac{3}{2}}$ **Question No.8**: $f(x) = x^3(6x-1)^{\frac{2}{3}}$

**Question No.9**:

$$f(x) = 2x(3x^2+7)^{\frac{1}{3}}$$

$$f'(x) = \frac{d}{dx}[2x(3x^2+7)^{\frac{1}{3}}] = 2x\frac{d}{dx}(3x^2+7)^{\frac{1}{3}} + (3x^2+7)^{\frac{1}{3}}\frac{d}{dx}(2x)$$

$$= 2x\{\frac{1}{3}(3x^2+7)^{\frac{1}{3}-1}\frac{d}{dx}(3x^2+7) + (3x^2+7)^{\frac{1}{3}}(2)$$

$$= \frac{2}{3}x\{(3x^2+7)^{\frac{1}{3}-1}(6x)\} + 2(3x^2+7)^{\frac{1}{3}}$$

$$= \frac{2}{3}x(6x)\{(3x^2+7)^{\frac{1}{3}}(3x^2+7)^{-1}\} + 2(3x^2+7)^{\frac{1}{3}}$$

$$= 4x^2\{\frac{(3x^2+7)^{\frac{1}{3}}}{(3x^2+7)}\} + 2(3x^2+7)^{\frac{1}{3}} = 2(3x^2+7)^{\frac{1}{3}}\{\frac{2x^2}{(3x^2+7)} + 1\}$$

$$= 2(3x^2+7)^{\frac{1}{3}}\{\frac{2x^2+3x^2+7}{(3x^2+7)}\} = 2(3x^2+7)^{\frac{1}{3}}\{\frac{5x^2+7}{(3x^2+7)}\}$$

$$= 2(5x^2+7)(3x^2+7)^{\frac{1}{3}}(3x^2+7)^{-1} = 2(5x^2+7)(3x^2+7)^{\frac{1}{3}-1}$$

$$= 2(5x^2+7)(3x^2+7)^{-\frac{2}{3}} = \frac{2(5x^2+7)}{(3x^2+7)^{\frac{2}{3}}}$$

**Do this your home: Question No. 10**: $f(x) = 3x(2x^3 + 5)^{\frac{1}{2}}$

**Question No. 11**:

$$f(x) = \frac{x}{x-1}$$

$$f'(x) = \frac{d}{dx}[\frac{x}{x-1}] = \frac{(x-1)\frac{d}{dx}(x) - x\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

**Question No. 12**:

$$f(x) = \frac{x-2}{x+1}$$

$$f'(x) = \frac{d}{dx}[\frac{x-2}{x+1}] = \frac{(x+1)\frac{d}{dx}(x-2) - (x-2)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(1) - (x-2)(1)}{(x+1)^2} = \frac{x+1-x+2}{(x+1)^2} = \frac{3}{(x+1)^2}$$

**Do these at your home: Question No. 13**: $f(x) = \frac{x^2}{2x+3}$

**Question No. 14**: $f(x) = \frac{x^3}{3x+5}$ **Question No. 15**: $f(x) = \frac{x}{3+2x^2}$

**Question No. 16**: $f(x) = \frac{3x}{1-2x^2}$

**Question No. 17**:

$$f(x) = \frac{x}{(3x+2)^{\frac{1}{2}}}$$

$$f'(x) = \frac{d}{dx}[\frac{x}{(3x+2)^{\frac{1}{2}}}] = \frac{(3x+2)^{\frac{1}{2}}\frac{d}{dx}(x) - (x)\frac{d}{dx}(3x+2)^{\frac{1}{2}}}{\{(3x+2)^{\frac{1}{2}}\}^2}$$

$$= \frac{(3x+2)^{\frac{1}{2}}(1) - (x)\frac{1}{2}(3x+2)^{\frac{1}{2}-1}\frac{d}{dx}(3x+2)}{(3x+2)}$$

$$= \frac{(3x+2)^{\frac{1}{2}} - \frac{x}{2}(3x+2)^{-\frac{1}{2}}(3)}{(3x+2)} = \frac{(3x+2)^{\frac{1}{2}} - \frac{3x}{2(3x+2)^{\frac{1}{2}}}}{(3x+2)}$$

$$= \frac{\frac{2(3x+2)-3x}{2(3x+2)^{\frac{1}{2}}}}{(3x+2)} = \frac{6x+4-3x}{2(3x+2)^{\frac{1}{2}}(3x+2)} = \frac{3x+4}{2(3x+2)^{\frac{1}{2}+1}} = \frac{3x+4}{2(3x+2)^{\frac{3}{2}}}$$

**Question No. 18**:

$$f(x)=\frac{2x}{(2x+3)^{\frac{1}{2}}}$$

$$f'(x)=\frac{d}{dx}[\frac{2x}{(2x+3)^{\frac{1}{2}}}]=\frac{(2x+3)^{\frac{1}{2}}\frac{d}{dx}(2x)-(2x)\frac{d}{dx}(2x+3)^{\frac{1}{2}}}{\{(2x+3)^{\frac{1}{2}}\}^2}$$

$$=\frac{(2x+3)^{\frac{1}{2}}(2)-(2x)\frac{1}{2}(2x+3)^{\frac{1}{2}-1}\frac{d}{dx}(2x+3)}{(2x+3)}$$

$$=\frac{2(2x+3)^{\frac{1}{2}}-x(2x+3)^{-\frac{1}{2}}(2)}{(2x+3)}=\frac{2(2x+3)^{\frac{1}{2}}-2x\frac{1}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}$$

$$=\frac{\frac{2(2x+3)-2x}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}=\frac{\frac{4x+6-2x}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}=\frac{\frac{2x+6}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}=\frac{2(x+3)}{(2x+3)(2x+3)^{\frac{1}{2}}}$$

$$=\frac{2(x+3)}{(2x+3)^{\frac{3}{2}}}$$

Another Method

Let $Y = \dfrac{2x}{(2x+3)^{\frac{1}{2}}}$

$$or \;\; \log Y = \log\{\frac{2x}{(2x+3)^{\frac{1}{2}}}\}; \text{ [taking log in both side]}$$

$$or \;\; \log Y = \log(2x) - \log\{(2x+3)^{\frac{1}{2}}\}$$

$$or \;\; \log Y = \log(2x) - \frac{1}{2}\log(2x+3)$$

$$or \;\; \frac{1}{Y}\frac{dy}{dx} = \frac{1}{2x}\frac{d}{dx}(2x) - \frac{1}{2}\frac{1}{(2x+3)}\frac{d}{dx}(2x+3)$$

$$or \;\; \frac{1}{Y}\frac{dy}{dx} = \frac{1}{2x}(2) - \frac{1}{2}\frac{1}{(2x+3)}(2)$$

$$or \;\; \frac{1}{Y}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{(2x+3)}$$

$$or \;\; \frac{dy}{dx} = Y(\frac{2x+3-x}{x(2x+3)})$$

$$or \;\; \frac{d}{dx}(\frac{2x}{(2x+3)^{\frac{1}{2}}}) = \frac{2x}{(2x+3)^{\frac{1}{2}}}(\frac{x+3}{x(2x+3)})$$

$$or \;\; \frac{d}{dx}(\frac{2x}{(2x+3)^{\frac{1}{2}}}) = \frac{2(x+3)}{(2x+3)(2x+3)^{\frac{1}{2}}}$$

$$or \;\; \frac{d}{dx}(\frac{2x}{(2x+3)^{\frac{1}{2}}}) = \frac{2(x+3)}{(2x+3)^{\frac{3}{2}}}$$

**Do these: Question No. 19**: $f(x) = \dfrac{2x+1}{(x^2+5)^{1/3}}$ **20**: $f(x) = \dfrac{3-5x}{(x^3+2)^{1/3}}$

**Additional Questions:**

$$f(x)=\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}$$

$$\text{Let Y}=\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}$$

$$or\ \ \log Y=\log\{\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}\};\ \text{[taking log in both side]}$$

$$or\ \ \log Y=\log(4x+1)^{1/4}-\log\{(2x+3)^{1/2}(5x-1)^{1/5}\}$$

$$or\ \ \log Y=\log(4x+1)^{1/4}-\log(2x+3)^{1/2}-\log(5x-1)^{1/5}$$

$$or\ \ \log Y=\frac{1}{4}\log(4x+1)-\frac{1}{2}\log(2x+3)-\frac{1}{5}\log(5x-1)$$

$$or\ \ \frac{1}{Y}\frac{dy}{dx}=\frac{1}{4}\frac{1}{(4x+1)}\frac{d}{dx}(4x+1)-\frac{1}{2}\frac{1}{(2x+3)}\frac{d}{dx}(2x+3)-\frac{1}{5}\frac{1}{(5x-1)}\frac{d}{dx}(5x-1)$$

$$or\ \ \frac{1}{Y}\frac{dy}{dx}=\frac{1}{4}\frac{1}{(4x+1)}(4)-\frac{1}{2}\frac{1}{(2x+3)}(2)-\frac{1}{5}\frac{1}{(5x-1)}(5)$$

$$or\ \ \frac{1}{Y}\frac{dy}{dx}=\frac{1}{(4x+1)}-\frac{1}{(2x+3)}-\frac{1}{(5x-1)}$$

$$or\ \ \frac{dy}{dx}=Y\{\frac{1}{(4x+1)}-\frac{1}{(2x+3)}-\frac{1}{(5x-1)}\}$$

$$or\ \ \frac{dy}{dx}=\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}\{\frac{(2x+3)(5x-1)-(4x+1)(5x-1)-(4x+1)(2x+3)}{(4x+1)(2x+3)(5x-1)}\}$$

$$or\ \ \frac{dy}{dx}=\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}\{\frac{10x^2+13x-3-20x^2-x+1-8x^2-14x-3}{(4x+1)(2x+3)(5x-1)}\}$$

$$or\ \ \frac{dy}{dx}=\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}\{\frac{-18x^2-2x-5}{(4x+1)(2x+3)(5x-1)}\}$$

$$or\ \ \frac{dy}{dx}=\frac{-(18x^2+2x+5)}{(2x+3)^{3/2}(5x-1)^{6/5}(4x+1)^{3/4}}$$

$$i.e.\frac{d}{dx}\{\frac{(4x+1)^{1/4}}{(2x+3)^{1/2}(5x-1)^{1/5}}\}=\frac{-(18x^2+2x+5)}{(2x+3)^{3/2}(5x-1)^{6/5}(4x+1)^{3/4}}$$

## Problem Set 9-1:

Q.6) $f(x) = 2.5e^{-0.4x}$ ; $\quad [\frac{d}{dx}(e^x) = e^x]$

$$f'(x) = \frac{d}{dx}[2.5e^{-0.4x}] = 2.5\frac{d}{dx}(e^{-0.4x})$$

$$= 2.5e^{-0.4x}\frac{d}{dx}(-0.4x) = 2.5e^{-0.4x}(-0.4) = -e^{-0.4x}$$

Q.8) $f(x) = 10e^{0.3x-6}$

$$f'(x) = \frac{d}{dx}[10e^{0.3x-6}] = 10\frac{d}{dx}(e^{0.3x-6})$$

$$= 10e^{0.3x-6}\frac{d}{dx}(0.3x-6) = 10e^{0.3x-6}(0.3) = 3e^{0.3x-6}$$

Q.12) $f(x) = (0.4)^{-0.3x}$; $\quad [\frac{d}{dx}(a^x) = a^x \ln a]$

$$f'(x) = \frac{d}{dx}[(0.4)^{-0.3x}] = (0.4)^{-0.3x}.\ln(0.4)\frac{d}{dx}(-0.3x)$$

$$= (0.4)^{-0.3x}.\ln(0.4)(-0.3) = 0.2749(0.4)^{-0.3x}$$

Q.14) $f(x) = 500(1.08)^{-x}$; $\quad [\frac{d}{dx}(a^x) = a^x \ln a]$

$$f'(x) = \frac{d}{dx}[500(1.08)^{-x}] = 500\frac{d}{dx}[(1.08)^{-x}] = 500.(1.08)^{-x}.\ln(1.08)\frac{d}{dx}(-x)$$

$$= 500.(1.08)^{-x}.\ln(1.08)(-1) = -38.48(1.08)^{-x}$$

## Problem Set 9-2:

Q.6) $f(x)=\ln(2x+5)^{1/3}=\frac{1}{3}\ln(2x+5);\ \ [\frac{d}{dx}(\ln x)=\frac{1}{x}]$

$$f'(x)=\frac{d}{dx}[\frac{1}{3}\ln(2x+5)]=\frac{1}{3}.\frac{1}{(2x+5)}\frac{d}{dx}(2x+5)$$

$$=\frac{1}{3}.\frac{1}{(2x+5)}(2)=\frac{2}{3(2x+5)}$$

Q.8) $f(x)=\ln(2x^3-6x);\ \ [\frac{d}{dx}(\ln x)=\frac{1}{x}]$

$$f'(x)=\frac{d}{dx}[\ln(2x^3-6x)]=\frac{1}{(2x^3-6x)}\frac{d}{dx}(2x^3-6x)$$

$$=\frac{1}{2x(x^2-3)}(6x^2-6)=\frac{1}{2x(x^2-3)}.6(x^2-1)=\frac{3(x^2-1)}{x(x^2-3)}$$

Q.9 $f(x)=\ln(3x+2)^{\frac{1}{2}}=\frac{1}{2}\ln(3x+2);\ \ [\frac{d}{dx}(\ln x)=\frac{1}{x}]$

$$f'(x)=\frac{d}{dx}[\frac{1}{2}\ln(3x+2)]=\frac{1}{2}\frac{d}{dx}[\ln(3x+2)]$$

$$=\frac{1}{2}.\frac{1}{(3x+2)}\frac{d}{dx}(3x+2)=\frac{1}{2}.\frac{1}{(3x+2)}(3)=\frac{3}{2(3x+2)}$$

**Applied Mathematics**

**Additional Problems from Differentiation**

Q.1) $f(x) = (x^3 - 3x^2 + 5)^{\frac{1}{3}}$

$$f'(x) = \frac{d}{dx}[(x^3 - 3x^2 + 5)^{\frac{1}{3}}] = \frac{1}{3}(x^3 - 3x^2 + 5)^{\frac{1}{3}-1}\frac{d}{dx}(x^3 - 3x^2 + 5)$$

$$= \frac{1}{3}(x^3 - 3x^2 + 5)^{-\frac{2}{3}}\{\frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2) + \frac{d}{dx}(5)\}$$

$$= \frac{1}{3}.\frac{1}{(x^3 - 3x^2 + 5)^{\frac{2}{3}}}\{3x^2 - 3.2x + 0\}$$

$$= \frac{1}{3}.\frac{1}{(x^3 - 3x^2 + 5)^{\frac{2}{3}}}.3(x^2 - 2x)$$

$$= \frac{(x^2 - 2x)}{(x^3 - 3x^2 + 5)^{\frac{2}{3}}}$$

Q.2) $f(x) = \dfrac{1}{4(8x-6)}$

$$f'(x) = \frac{d}{dx}[\frac{1}{4(8x-6)}] = \frac{1}{4}\frac{d}{dx}\{\frac{1}{(8x-6)}\}$$

$$= \frac{1}{4}\frac{d}{dx}(8x-6)^{-1} = \frac{1}{4}(-1)(8x-6)^{-1-1}\frac{d}{dx}(8x-6)$$

$$= -\frac{1}{4}(8x-6)^{-2}(8) = -\frac{2}{(8x-6)^2}$$

Q.3) $f(x)=\frac{2}{27(10-3x)^{\frac{3}{2}}}$

$$f'(x)=\frac{d}{dx}[\frac{2}{27(10-3x)^{\frac{3}{2}}}]=\frac{2}{27}\frac{d}{dx}\{(10-3x)^{-\frac{3}{2}}\}$$

$$=\frac{2}{27}(-\frac{3}{2})(10-3x)^{-\frac{3}{2}-1}\frac{d}{dx}(10-3x)$$

$$=-\frac{1}{9}(10-3x)^{-\frac{5}{2}}(-3)=\frac{1}{3}.\frac{1}{(10-3x)^{\frac{5}{2}}}$$

$$=\frac{1}{3\sqrt{(10-3x)^5}}$$

Q.4) $f(x)=x-\frac{1}{(5-0.3x)^{\frac{1}{3}}}$

$$f'(x)=\frac{d}{dx}[x-\frac{1}{(5-0.3x)^{\frac{1}{3}}}]=\frac{d}{dx}(x)-\frac{d}{dx}\{\frac{1}{(5-0.3x)^{\frac{1}{3}}}\}$$

$$=\frac{d}{dx}(x)-\frac{d}{dx}(5-0.3x)^{-\frac{1}{3}}=1-(-\frac{1}{3})(5-0.3x)^{-\frac{1}{3}-1}\frac{d}{dx}(5-0.3x)$$

$$=1+\frac{1}{3}(5-0.3x)^{-\frac{4}{3}}(-0.3)=1-\frac{0.3}{3}.\frac{1}{(5-0.3x)^{\frac{4}{3}}}$$

$$=1-\frac{1}{10\sqrt[3]{(5-0.3x)^4}}$$

Q.5) $f(x)=2x(x^3-5x)^{\frac{1}{2}}$

$$f'(x)=\frac{d}{dx}[2x(x^3-5x)^{\frac{1}{2}}]=2\frac{d}{dx}[x(x^3-5x)^{\frac{1}{2}}]$$

$$=2[x\frac{d}{dx}(x^3-5x)^{\frac{1}{2}}+(x^3-5x)^{\frac{1}{2}}\frac{d}{dx}(x)]$$

$$=2[x.\frac{1}{2}(x^3-5x)^{\frac{1}{2}-1}\frac{d}{dx}(x^3-5x)+(x^3-5x)^{\frac{1}{2}}.1]$$

$$=2[\frac{x}{2}(x^3-5x)^{-\frac{1}{2}}(3x^2-5)+(x^3-5x)^{\frac{1}{2}}]$$

$$=x\frac{(3x^2-5)}{(x^3-5x)^{\frac{1}{2}}}+2(x^3-5x)^{\frac{1}{2}}=\frac{3x^3-5x}{(x^3-5x)^{\frac{1}{2}}}+2(x^3-5x)^{\frac{1}{2}}$$

$$=\frac{3x^3-5x+2(x^3-5x)}{(x^3-5x)^{\frac{1}{2}}}=\frac{3x^3-5x+2x^3-10x}{(x^3-5x)^{\frac{1}{2}}}$$

$$=\frac{5x^3-15x}{(x^3-5x)^{\frac{1}{2}}}=\frac{5(x^3-3x)}{(x^3-5x)^{\frac{1}{2}}}$$

Q.6) $f(x)=\dfrac{2x-1}{(2x+3)^{1/2}}$

$$f'(x)=\frac{d}{dx}[\frac{2x-1}{(2x+3)^{1/2}}]=\frac{(2x+3)^{\frac{1}{2}}\frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(2x+3)^{\frac{1}{2}}}{\{(2x+3)^{\frac{1}{2}}\}^2}$$

$$=\frac{(2x+3)^{\frac{1}{2}}(2)-(2x-1).\frac{1}{2}.(2x+3)^{\frac{1}{2}-1}\frac{d}{dx}(2x+3)}{(2x+3)}$$

$$=\frac{2(2x+3)^{\frac{1}{2}}-\frac{1}{2}(2x-1)(2x+3)^{-\frac{1}{2}}(2)}{(2x+3)}=\frac{2(2x+3)^{\frac{1}{2}}-\dfrac{(2x-1)}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}$$

$$=\frac{\dfrac{2(2x+3)-(2x-1)}{(2x+3)^{\frac{1}{2}}}}{(2x+3)}=\frac{4x+6-2x+1}{(2x+3)^{\frac{1}{2}}(2x+3)}=\frac{2x+7}{(2x+3)^{\frac{3}{2}}}=\frac{2x+7}{\sqrt{(2x+3)^3}}$$

**Question No. 1: When x gallons of antifreeze are produced, the average cost per gallon is A(x) where,** $A(x)=\frac{100}{x}+0.04x+1$

**a) How many gallons should be produced if average cost per gallon is to be minimized?**

$$A(x)=\frac{100}{x}+0.04x+1$$

We need to find the minimum point (stationary point) to find out the minimum average cost

$$A'(x)=\frac{d}{dx}[\frac{100}{x}+0.04x+1]=100\frac{d}{dx}(\frac{1}{x})+0.04\frac{d}{dx}(x)+\frac{d}{dx}(1)$$

$$=-100\frac{1}{x^2}+0.04$$

At stationary point $A'(x)=0$

$$i.e. -100\frac{1}{x^2}+0.04=0$$

$$or, 100\frac{1}{x^2}=0.04$$

$$or, 0.04x^2=100$$

$$or, x^2=\frac{100}{0.04}=2500$$

$$or, x=50 \text{ gallon}$$

When 50 gallons of antifreeze are produced, the average cost per gallon will be minimized.

**b) Prove (a) is minimum**

We find the stationary point where $x=50$

If $x=49$ then $A'(49)=-100\frac{1}{(49)^2}+0.04=-0.001, (-\text{ve})$

The function is decreasing before the point

If $x=51$ then $A'(51)=-100\frac{1}{(51)^2}+0.04=0.001, (+\text{ve})$

The function is increasing after the point

Therefore we can conclude that the stationary point where $x=50$ gallon makes the average cost minimum.

**c) Compute the minimum average cost per gallon.**

When $x = 50$ gallon, the minimum average cost per gallon

$$A(50) = \frac{100}{50} + 0.04(50) + 1 = \$5 \text{ per gallon}$$

**Question No. 2: The profit realized when y gallons of distilled water are made and sold is**

$$P(y) = 20y - 0.005y^2$$

**a) Find the number of gallons that should be made to maximize profit.**

$$P(y) = 20y - 0.005y^2$$

We need to find the maximum point (stationary point) to find out the maximum profit

$$P'(y) = \frac{d}{dy}[20y - 0.005y^2] = 20\frac{d}{dy}(y) - 0.005\frac{d}{dy}(y^2)$$

$$= 20 - 0.01y$$

At stationary point $P'(y) = 0$

$i.e. 20 - 0.01y = 0 \qquad or, 0.01y = 20$

$or, y = \frac{20}{0.01}$ =2000 gallons

**b) Prove (a) is a maximum.**

We find the stationary point where $y = 2000$

If $y = 1999$ then $P'(1999) = 20 - 0.01(1999) = 0.01$, $(+\text{ve})$

The function is increasing before the point

If $p = 2001$ then $P'(2001) = 20 - 0.01(2001) = -0.01$, $(-\text{ve})$

The function is decreasing after the point

Therefore we can conclude that the stationary point where $y = 2000$ gallons makes the profit maximum.

**c) Compute the maximum profit**

$$P(2000) = 20(2000) - 0.005(2000)^2 = \$20,000$$

**Question 3: When x gallons of alcohol are produced, the average cost per gallon is A(x) dollars, where,**

$$A(x) = \frac{200}{0.1x+5} + 0.05x \quad ; x > 0$$

a) **Find the value of x where A(x) has a stationary point.**

At stationary point,

$A'(x) = 0$

$or, \frac{d}{dx}\left(\frac{200}{0.1x+5} + 0.05x\right) = 0$

$or, 200\frac{d}{dx}(0.1x+5)^{-1} + \frac{d}{dx}(0.05x) = 0$

$or, 200(-1)(0.1x+5)^{-1-1}\frac{d}{dx}(0.1x+5) + 0.05 = 0$

$or, -200(0.1x+5)^{-2}(0.1) + 0.05 = 0$

$or, -\frac{20}{(0.1x+5)^2} = -0.05$

$or, \frac{20}{0.05} = (0.1x+5)^2$

$or, (0.1x+5)^2 = 400$

$or, (0.1x+5) = \pm 20$

Either, $(0.1x+5) = 20$, $or, x = \frac{15}{0.1} = 150$

or, $(0.1x+5) = -20$, $or, x = \frac{-25}{0.1} = -250$

$x$ gallons can not be negative, hence x= 150 gallons

b)

$A''(x) = \frac{d}{dx}\{-20(0.1x+5)^{-2} + 0.05\} = \frac{d}{dx}\{-20(0.1x+5)^{-2}\} + \frac{d}{dx}(0.05)$

$= -20(-2)(0.1x+5)^{-2-1}\frac{d}{dx}(0.1x+5) + 0$

$= 40\frac{1}{(0.1x+5)^3}(0.1) = \frac{4}{(0.1x+5)^3}$

$A''(150) = \frac{4}{\{0.1(150)+5\}^3} = 0.0005;$ [+ve, concave up]

The value of $x = 150$ gallons occurs at local minimum of A($x$)

**c)**

$$A(150)=\frac{200}{0.1(150)+5}+0.05(150)=\$17.5 \text{ per gallon}$$

**Question 4: Based on historical data, a bus company uses the function**

$P(x)=(3+0.6x)^{\frac{1}{2}}-0.1x \quad ; x>0$ **to estimate the net weekly profit, in hundreds of dollars, if a particular bus route is x miles long. How long should the route be to maximize the net profit, and what is the maximum net profit?**

For the profit to be maximum,

$P'(x)=0$

$$or, \frac{d}{dx}\{(3+0.6x)^{\frac{1}{2}}-0.1x\}=0$$

$$or, \frac{d}{dx}\{(3+0.6x)^{\frac{1}{2}}\}-\frac{d}{dx}(0.1x)=0$$

$$or, \frac{1}{2}(3+0.6x)^{\frac{1}{2}-1}\frac{d}{dx}(3+0.6x)-0.1=0$$

$$or, \frac{1}{2}(3+0.6x)^{-\frac{1}{2}}(0.6)-0.1=0$$

$$or, \frac{0.3}{(3+0.6x)^{\frac{1}{2}}}=0.1$$

$$or, \frac{0.30}{0.1}=(3+0.6x)^{\frac{1}{2}}$$

$$or, (3+0.6x)^{\frac{1}{2}}=3$$

$$or, \{(3+0.6x)^{\frac{1}{2}}\}^2 = (3)^2$$
$$or, 3+0.6x = 9$$
$$or, x = \frac{9-3}{0.6} = 10 \text{ miles}$$
$$P''(x) = \frac{d}{dx}\{0.3(3+0.6x)^{-\frac{1}{2}} - 0.1\} = 0.3(-\frac{1}{2})(3+0.6x)^{-\frac{1}{2}-1}\frac{d}{dx}(3+0.6x)$$
$$= -0.15(3+0.6x)^{-\frac{3}{2}}(0.6) = -\frac{0.09}{(3+0.6x)^{\frac{3}{2}}}$$
$$P''(10) = -\frac{0.09}{\{3+0.6(10)\}^{\frac{3}{2}}}; \quad [-\text{ve, concave down}]$$

Therefore $x = 10$ miles makes the profit maximum and

the maximum net profit, $P(10) = \{3+0.6(10)\}^{\frac{1}{2}} - 0.1(10)$

$= 2(100) = \$200$ per week

**Question 5: When y gallons of crude oil are produced the average cost per barrel is A(y), where,**

$$A(y) = \frac{2500}{0.04y+9} + 0.16y$$

a) **Find the value of y that minimizes average cost per barrel.**

For the average cost to be minimum,

$$A'(y) = 0$$
$$or, \frac{d}{dy}\{\frac{2500}{0.04y+9} + 0.16y\} = 0$$
$$or, \frac{d}{dy}\{2500(0.04y+9)^{-1}\} + \frac{d}{dy}(0.16y) = 0$$
$$or, 2500(-1)(0.04y+9)^{-1-1}\frac{d}{dy}(0.04y+9) + 0.16 = 0$$
$$or, -2500(0.04y+9)^{-2}(0.04) + 0.16 = 0$$
$$or, -2500(0.04y+9)^{-2}(0.04) + 0.16 = 0$$
$$or, -\frac{100}{(0.04y+9)^2} = -0.16$$
$$or, \frac{100}{0.16} = (0.04y+9)^2$$
$$or, (0.04y+9)^2 = 625$$
$$or, (0.04y+9) = \pm 25$$

Either, or,

$(0.04y+9)=25$ $\quad$ $(0.04y+9)=-25$

$or, y=\dfrac{25-9}{0.04}$ $\quad$ $or, y=\dfrac{-25-9}{0.04}$

$or, y=400$ $\quad$ $or, y=-850$

we discard $y=-850$ because $y>0$

$$A''(y)=\frac{d}{dy}\{-100(0.04y+9)^{-2}+0.16\}=-100(-2)(0.04y+9)^{-2-1}(0.04)$$

$$=\frac{8}{(0.04y+9)^3}$$

$$A''(400)=\frac{8}{\{0.04(400)+9)^3}=0.000512; \quad \text{[+ve, concave up]}$$

$y=400$ gallons will make the average cost per barrel minimum

**b) Compute the minimum average cost per barrel.**

Minimum average cost per barrel,

$$A(400)=\frac{2500}{0.04(400)+9}+0.16(400)=\$164 \text{ per barrel}$$

**Do this at home: Question 6: When x gallons of olive oil are produced the average cost per barrel is A(x), where,**

$$A(x)=\frac{4,000}{0.1x+20}+0.25x; \quad x>0$$

**a) Find the value of x that minimizes average cost per barrel.**

**b) Compute the minimum average cost per barrel.**

**Question 7: Profit realized when x thousand gallons of antifreeze are produced and sold is P(x) thousand dollars, where,**

$$P(x)=(100+10x)^{\frac{1}{2}}-0.2x$$

**a) Find the value of x that leads to maximum profit.**

For the profit to be maximum,

$P'(x)=0$

$$or, \frac{d}{dx}\{(100+10x)^{\frac{1}{2}}-0.2x\}=0$$

$$or, \frac{d}{dx}(100+10x)^{\frac{1}{2}}-\frac{d}{dx}(0.2x)=0$$

$$or, \frac{1}{2}(100+10x)^{\frac{1}{2}-1}\frac{d}{dx}(100+10x)-\frac{d}{dx}(0.2x)=0$$

$$or, \frac{1}{2}(100+10x)^{-\frac{1}{2}}(10)-0.2=0$$

$$or, \frac{5}{(100+10x)^{\frac{1}{2}}}=0.2$$

$$or, \frac{5}{0.2}=(100+10x)^{\frac{1}{2}}$$

$$or, (100+10x)^{\frac{1}{2}}=25$$

$$or, (100+10x)=625$$

$$or, x=\frac{625-100}{10}=52.5 \text{ thousand gallons}$$

$$P''(x)=\frac{d}{dx}\{5(100+10x)^{-\frac{1}{2}}-0.2\}=5(-\frac{1}{2})(100+10x)^{-\frac{1}{2}-1}(10)=-25(100+10x)^{-\frac{3}{2}}$$

$$P''(52.5)=-25\{100+10(52.5)\}^{-\frac{3}{2}}=-0.0016; \quad [-\text{ve, concave down}]$$

Thus $x=52.5$ thousand gallons leads to maximum profit

**b) Compute the maximum profit.**

$$P(52.5)=\{100+10(52.5)\}^{\frac{1}{2}}-0.2(52.5)=14.5 \text{ thousand dollars}=\$14,500$$

**Question 8: Profit per tree grown and sold, by a tree grower depends upon the height of a tree at the time of sale. Taking h as tree height in inches, the profit per tree, in dollars, is approximated by,**

$$P(h)=(10+2h)^{\frac{1}{2}}-0.1h$$

a) **What tree height provides maximum profit per tree?**

For $P(h)$ to be maximum, $P'(h)=0$

$$or, \frac{d}{dh}\{(10+2h)^{\frac{1}{2}}-0.1h\}=0$$

$$or, \frac{d}{dh}(10+2h)^{\frac{1}{2}}-\frac{d}{dh}(0.1h)=0$$

$$or, \frac{d}{dh}(10+2h)^{\frac{1}{2}}-\frac{d}{dh}(0.1h)=0$$

$$or, \frac{1}{2}(10+2h)^{\frac{1}{2}-1}\frac{d}{dh}(10+2h)-0.1=0$$

$$or, \frac{1}{2}(10+2h)^{-\frac{1}{2}}(2)-0.1=0$$

$$or, \frac{1}{(10+2h)^{\frac{1}{2}}}=0.1$$

$$or, \frac{1}{0.1}=(10+2h)^{\frac{1}{2}}$$

$$or, 100=10+2h$$

$$or, h=\frac{100-10}{2}=45 \text{ inches}$$

$$P''(h)=\frac{d}{dh}\{(10+2h)^{-\frac{1}{2}}-0.1\}=-\frac{1}{2}(10+2h)^{-\frac{3}{2}}(2)=-(10+2h)^{-\frac{3}{2}}$$

$$P''(45)=-\{10+2(45)\}^{-\frac{3}{2}}=-0.001; \quad [-\text{ve, concave down}]$$

Thus, tree height, $h=45$ inches provides maximum profit per tree

b) **What is the maximum profit per tree?**

$$P(45)=\{10+2(45)\}^{\frac{1}{2}}-0.1(45)=\$5.5 \text{ per tree}$$

**Question 9: Unilever Bangladesh estimates the total potential number of customers for a new product is 1,000,000. It plans to operate a promotional campaign to sell the product and uses the response function,**

$$r(t) = 0.25 - 0.25e^{-0.01t}$$

**as a measure of the proportion of total customer potential responding to the promotion after it has been in operation for t days. On the average, one response generates $5 in revenue. Campaign costs consist of a fixed cost of $15,000 plus a variable cost of $1,000 per day of operation.**

**a) How long should the campaign continue if profit is to be maximized?**

$$\text{Revenue} = 5(1{,}000{,}000)\{r(t)\} = (5{,}000{,}000)\{0.25 - 0.25e^{-0.01t}\}$$

$$\text{Total cost} = \text{Fixed cost} + \text{Variable cost} = 15000 + 1000t$$

$$\text{Profit} = \text{Revenue} - \text{Total cost} = (5{,}000{,}000)\{0.25 - 0.25e^{-0.01t}\} - (15000 + 1000t)$$

$$= 1250000 - 1250000e^{-0.01t} - 15000 - 1000t$$

$$\text{Profit } P(t) = 1235000 - 1000t - 1250000e^{-0.01t}$$

For profit to be maximum, $P'(t) = 0$

$$or, \frac{d}{dt}\{1235000 - 1000t - 1250000e^{-0.01t}\} = 0$$

$$or, -1000 - 1250000e^{-0.01t}(-0.01) = 0$$

$$or, -1000 + 12500e^{-0.01t} = 0$$

$$or, 12500e^{-0.01t} = 1000$$

$$or, e^{-0.01t} = \frac{1000}{12500} = 0.08$$

$$or, \ln e^{-0.01t} = \ln(0.08); \quad [\text{taking } \log_e \text{ or Ln in both sides}]$$

$$or, -0.01t(\ln e) = \ln(0.08); \quad [\ln e = \log_e e = 1]$$

$$or, -0.01t = \ln(0.08)$$

$$or, t = \frac{\ln(0.08)}{-0.01} = 252.573 \text{ days}$$

$$P''(t) = \frac{d}{dt}(-1000 + 12500e^{-0.01t}) = 12500e^{-0.01t}(-0.01) = -125e^{-0.01t}$$

$$P''(252.573) = -125e^{-0.01(252.573)} = -125e^{-2.52573} = -10; \quad [-\text{ve, concave down}]$$

Thus the campaign should continue for $t = 252.573$ days for the profit to be maximum

**b) What is the maximum profit?**

$$P(252.573) = 1235000 - 1000(252.573) - 1250000e^{-0.01(252.573)} = \$882{,}427.13$$

**Question 10: An oil deposit contains 1,000,000 barrels of oil, which after being pumped from the deposit, yields revenue of $12 per barrel. The proportion of the deposit that will have been pumped out after t years of pumping is**

$$0.9-0.9e^{-0.16t}$$

**Operating costs are $345,600 per year.**

**a**) How long should pumping be continued to maximize profit?

Revenue = $12(1,000,000)\{0.9-0.9e^{-0.16t}\}$

Operating cost = 345600t

Profit = Revenue − cost = $12(1,000,000)\{0.9-0.9e^{-0.16t}\}-345600t$

Profit $P(t) = 10800000-10800000e^{-0.16t}-345600t$

For profit to be maximum, $P'(t)=0$

$or, \frac{d}{dt}\{10800000-10800000e^{-0.16t}-345600t\}=0$

$or, -10800000e^{-0.16t}(-0.16)-345600=0$

$or, 1728000e^{-0.16t}=345600$

$or, e^{-0.16t}=\frac{345600}{1728000}$

$or, e^{-0.16t}=0.2$

$or, \ln e^{-0.16t}=\ln(0.2);$ [taking $\log_e$ or Ln in both sides]

$or, -0.16t(\ln e)=\ln(0.2);$ $[\ln e=\log_e e=1]$

$or, -0.16t=\ln(0.2)$

$or, t=\frac{\ln(0.2)}{-0.16}=10.06$ years

$P''(t)=\frac{d}{dt}(1728000e^{-0.16t}-345600)=1728000e^{-0.16t}(-0.16)=-276480e^{-0.16t}$

$P''(10.06)=-276480e^{-0.16(10.06)}=-276480e^{-1.6096}=-55287.03;$ [−ve, concave down]

Thus the pumping should be continued for $t=10.06$ years for the profit to be maximum

*b*) What is the maximum profit?

$P(10.06)=10800000-10800000e^{-0.16(10.06)}-345600(10.06)$=$5,163,614.081

**Question 11: When x ounces of seed costing $2 per ounce are sown on a plot of land the crop yield is ln(2x+1) bushels worth $25 per bushels. How many ounces should be sown if the worth of the crop minus the cost of the seed is to be maximized?**

$$\text{Profit, P}(x) = 25\{\ln(2x+1)\} - 2x$$

For profit to be maximum, $\text{P}'(x) = 0 =$

$$or, \frac{d}{dx}[25\{\ln(2x+1)\} - 2x] = 0$$

$$or, 25\frac{d}{dx}\{\ln(2x+1)\} - \frac{d}{dx}(2x) = 0$$

$$or, 25\frac{1}{2x+1}(2) - 2 = 0$$

$$or, \frac{50}{2x+1} = 2$$

$$or, 2x+1 = 25$$

$$or, x = 12 \text{ ounces of seed}$$

$$\text{P}''(x) = \frac{d}{dx}(\frac{50}{2x+1} - 2) = 50\frac{d}{dx}(2x+1)^{-1} = 50(-1)(2x+1)^{-1-1}\frac{d}{dx}(2x+1)$$

$$= -50(2x+1)^{-2}(2) = -\frac{100}{(2x+1)^2}$$

$$\text{P}''(12) = -\frac{100}{(24+1)^2} = -0.16; \quad [(-\text{ve}), \text{ concave down}]$$

Thus $x = 12$ ounces of seed will make the profit maximized.

<u>**Do the followings at home**</u>

**Question 12: The revenue from, and the cost of, operating an undertaking for t years are, respectively, in millions of dollars,**

$$R(t) = 3e^{0.05t} \text{ and } C(t) = 1.5e^{0.08t}$$

a) **How long should operations continue if profit is to be maximized?**
b) **Compute maximum profit.**

**Question 13: The total potential audience for a promotional campaign is 10,000 customers. Revenue averages $3 per response to the campaign. Campaign costs are a fixed amount of $500, plus $300 per day the campaign continues. The proportion of the total audience responding by time t days is**

$$1 - e^{-0.25t}$$

a) **How long should the campaign continue if profit is to be maximized?**
b) **Compute maximum profit.**

**Question 14: The total potential audience for a promotional campaign is 2,000 customers. Revenue averages \$5 per response to the campaign. Campaign costs are a fixed amount of \$100, plus \$105.36 per day the campaign continues. The proportion of the total audience responding by time t days is**

$$1-(0.9)^t$$

a) **How long should the campaign continue if profit is to be maximized?**
b) **Compute maximum profit.**