BUS 173
Applied Statistics

Lecture 9

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Inference Concerning a Population Variance

- The sample variance s^2 can be used in its standardized form:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

- which has a Chi-Square distribution with n - 1 degrees of freedom.

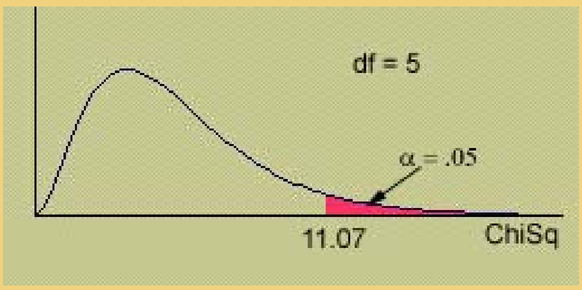


TABLE 5 (continued)

$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$
2.70554	3.84146	5.02389	6.63490
4.60517	5.99147	7.37776	9.21034
6.25139	7.81473	9.34840	11.3449
7.77944	9.48773	11.1433	13.2767
9.23635	11.0705	12.8325	15.0863
10.6446	12.5916	14.4494	16.8119
12.0170	14.0671	16.0128	18.4753

For example, the value of chi-square that cuts off .05 in the upper tail of the distribution with *df* = 5 is χ^2 =11.07.

TABLE SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION VARIANCE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \sigma^2 \geq \sigma_0^2$ $H_a: \sigma^2 < \sigma_0^2$	$H_0: \sigma^2 \leq \sigma_0^2$ $H_a: \sigma^2 > \sigma_0^2$	$H_0: \sigma^2 = \sigma_0^2$ $H_a: \sigma^2 \neq \sigma_0^2$
Test Statistic	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$
Rejection Rule: *p*-value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $\chi^2 \leq \chi^2_{(1-\alpha)}$	Reject H_0 if $\chi^2 \geq \chi^2_{\alpha}$	Reject H_0 if $\chi^2 \leq \chi^2_{(1-\alpha/2)}$ or if $\chi^2 \geq \chi^2_{\alpha/2}$

Example

•A cement manufacturer claims that his cement has a compressive strength with a standard deviation of 10 kg/cm2 or less. A sample of n = 10 measurements produced a mean and standard deviation of 312 and 13.96, respectively. Do these data produce sufficient evidence to reject the manufacturer's claim? Use a = .05.

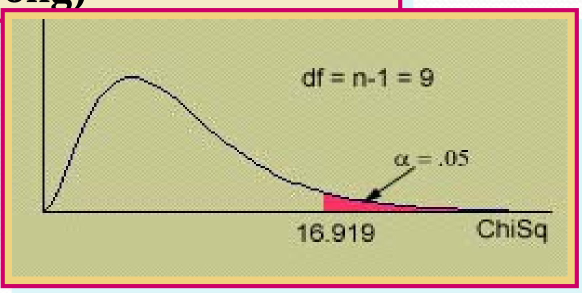
A test of hypothesis:

H_0: $\sigma^2 \leq 10^2$ (claim is correct)

H_a: $\sigma^2 > 10^2$ (claim is wrong)

uses the test statistic:

$$\chi^2 = \frac{(n-1)s^2}{10^2} = \frac{9(13.96^2)}{100} = 17.5$$



<u>Rejection region</u>: Reject H_0 if $\chi^2 > 16.919$ ($\alpha = .05$).

<u>Conclusion:</u> Since χ^2= 17.5, H_0 is rejected. The standard deviation of the cement strengths is more than 10.

- **Inference Concerning Two Population Variances**

- We can make inferences about the ratio of two population variances in the form a ratio. We choose two independent random samples of size n_1 and n_2 from normal distributions.
- If the two population variances are equal, the statistic

$$F = \frac{s_1^2}{s_2^2}$$

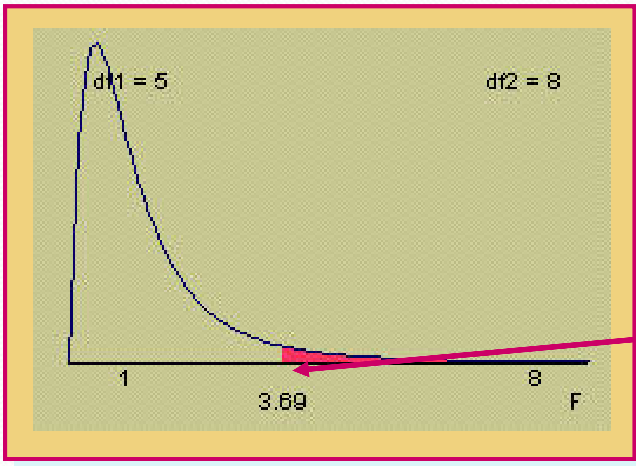
- has an F distribution with (numerator) $df_1 = n_1 - 1$ and (denominator) $df_2 = n_2 - 1$ degrees of freedom.

- A one-tailed hypothesis test about two population variances can always be formulated as an upper tail test. This approach eliminates the need for lower tail F values.

TABLE SUMMARY OF HYPOTHESIS TESTS ABOUT TWO POPULATION VARIANCES

	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \sigma_1^2 \le \sigma_2^2$ $H_a: \sigma_1^2 > \sigma_2^2$	$H_0: \sigma_1^2 = \sigma_2^2$ $H_a: \sigma_1^2 \neq \sigma_2^2$ Note: Population 1 has the larger sample variance
Test Statistic	$F = \frac{s_1^2}{s_2^2}$	$F = \frac{s_1^2}{s_2^2}$
Rejection Rule: *p*-value	Reject H_0 if $p\text{-value} \le \alpha$	Reject H_0 if $p\text{-value} \le \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $F \ge F_\alpha$	Reject H_0 if $F \ge F_{\alpha/2}$

•F Table gives only upper critical values of the F statistic for a given pair of df_1 and df_2.



For example, the value of F that cuts off at α= .05 in the upper tail of the distribution with df_1 = 5 and df_2 = 8 is F =3.69.

Example

•An experimenter has performed a lab experiment using two groups of rats. He wants to test that the population variances are equal.

	Standard (2)	**Experimental (1)**
Sample size	10	11
Sample Std Dev	2.3	5.8

Preliminary test:

$$H_0 : \sigma_1^2 = \sigma_2^2 \text{ versus } H_a : \sigma_1^2 \neq \sigma_2^2$$

Test statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{5.8^2}{2.3^2} = 6.36$$

We designate the sample with the larger standard deviation as sample 1, to force the test statistic into the upper tail of the *F* distribution.

Example

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_a: \sigma_1^2 \neq \sigma_2^2$$

Test statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{5.8^2}{2.3^2} = 6.36$$

The rejection region is two-tailed, with $\alpha = .05$, but we only need to find the upper critical value, which has $\alpha/2 = .025$ to its right.

From Table 6, with df_1=10 and df_2 = 9, we reject H_0 if $F_{stat} > 3.96$.

CONCLUSION: Reject H_0. There is sufficient evidence to indicate that the variances are **unequal.**

Example

•A sample of 26 arrival times for the Milbank service provides a sample variance of 48 and a sample of 16 arrival times for the Gulf Park service provides a sample variance of 20. Because the Milbank sample provided the larger sample variance, we will denote Milbank as population 1.

Preliminary test:

$$H_0: \sigma_1^2 = \sigma_2^2 \text{ versus } H_a: \sigma_1^2 \neq \sigma_2^2$$

Test statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{48}{20} = 2.40$$

The corresponding F distribution has (n_1-1) = 26-1=25 numerator degrees of freedom and (n_2-1)= 16-1=15 denominator degrees of freedom.

The rejection region is two-tailed, with $\alpha = 0.05$, but we only need to find the upper critical value, which has $\alpha/2 = .025$ to its right.

From F Table , with df_1=25 and df_2 = 15,

we reject H_0 if F_{stat} =2.40> 2.69.

CONCLUSION: Fail to Reject H_0 at 5% significance level.

with $\alpha = 0.10$; $\alpha/2 = .05$

From F Table , with df_1=25 and df_2 = 15,

we reject H_0 if F_{stat} =2.40> 2.28.

CONCLUSION: Reject H_0 at 10% significance level.

There is sufficient evidence to indicate that the variances are **unequal at 10% significance level.**

Example

In a public opinion survey samples of 31 men and 41 women will be used to study attitudes about current political issues. The researcher conducting the study wants to test to see whether the sample data indicate that women show a greater variation in attitude on political issues than men. The survey results provide a sample variance of 120 for women and a sample variance of 80 for men.

$$H_0: \sigma^2_{\text{women}} \le \sigma^2_{\text{men}}$$
$$H_a: \sigma^2_{\text{women}} > \sigma^2_{\text{men}}$$

Test statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{120}{80} = 1.50$$

The corresponding F distribution has (n_1-1) = 41-1=40 numerator degrees of freedom and (n_2-1)= 31-1=30 denominator degrees of freedom.

The rejection region is upper-tailed, with $\alpha = 0.05$

From F Table , with df_1=40 and df_2 = 30,

we reject H_0 if F_{stat} =1.50≥ 1.79.

CONCLUSION: Fail to Reject H_0 at 5% significance level.

with $\alpha = 0.10$, From F Table , with df_1=25 and df_2 = 15,

we reject H_0 if F_{stat} =1.50 ≥ 1.57.

CONCLUSION: Fail to Reject H_0 at 10% significance level.

There is not sufficient evidence to reject that the women show a lesser variation in attitude on political issues than men. Therefore there is a chance that men show a greater variation in attitude on political issues than women.