

BUS 172

Descriptive Statistics

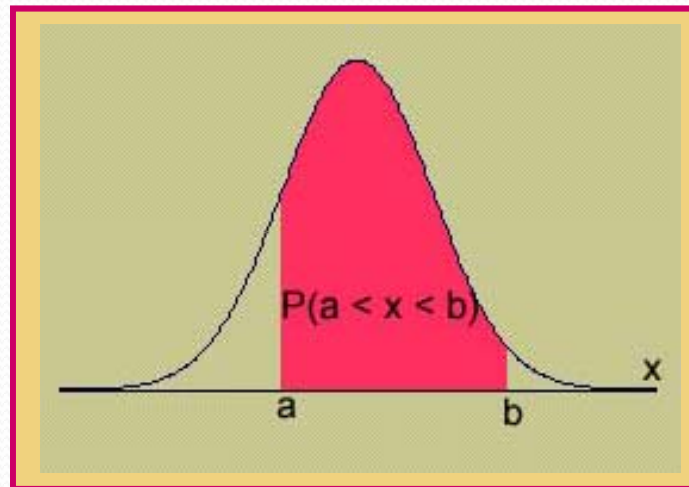
Lecture 15

Continuous Random Variables

- Continuous random variables can assume the infinitely many values corresponding to points on a line interval.
- **Examples:**
 - Heights, weights
 - length of life of a particular product
 - experimental laboratory error

Properties of Continuous Probability Distributions

- The area under the curve is equal to 1.
- $P(a \leq x \leq b)$ = **area under the curve** between a and b.



- There is no probability attached to any single value of x . That is, $P(x = a) = 0$.
- One important continuous random variable is the normal random variable.

The Normal Distribution

- The formula that generates the normal probability distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ for } -\infty < x < \infty$$

$$e = 2.7183 \quad \pi = 3.1416$$

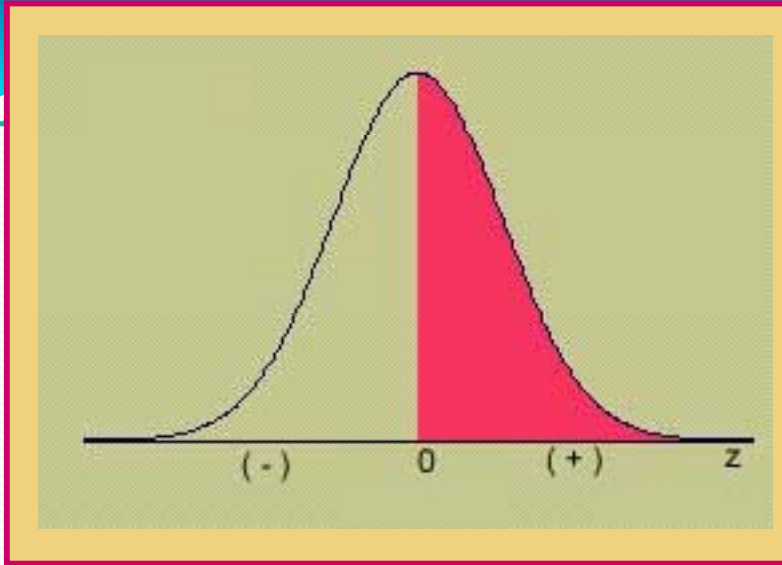
μ and σ are the population mean and standard deviation.

- The shape and location of the normal curve changes as the mean and standard deviation change.

The Standard Normal Distribution

- To find $P(a < x < b)$, we need to find the area under the appropriate normal curve.
- To simplify the tabulation of these areas, we **standardize** each value of x by expressing it as a z-score, i.e. we need to find that the position of x is how many standard deviations σ away from the mean μ .

$$z = \frac{x - \mu}{\sigma}$$



The Standard Normal (z) Distribution

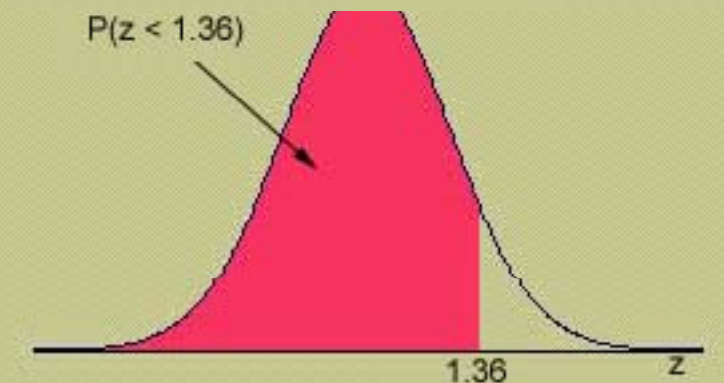
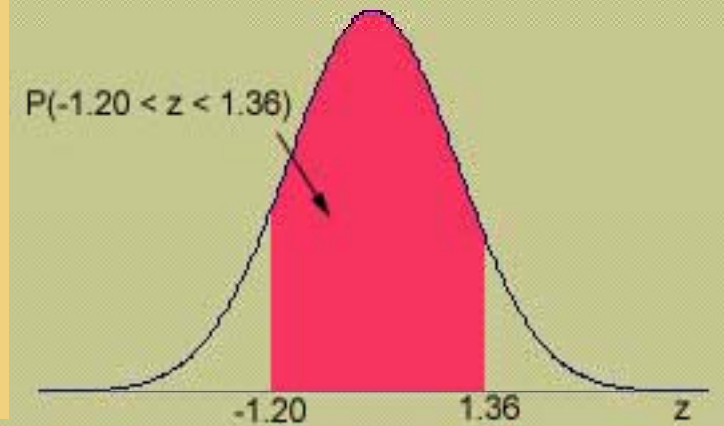
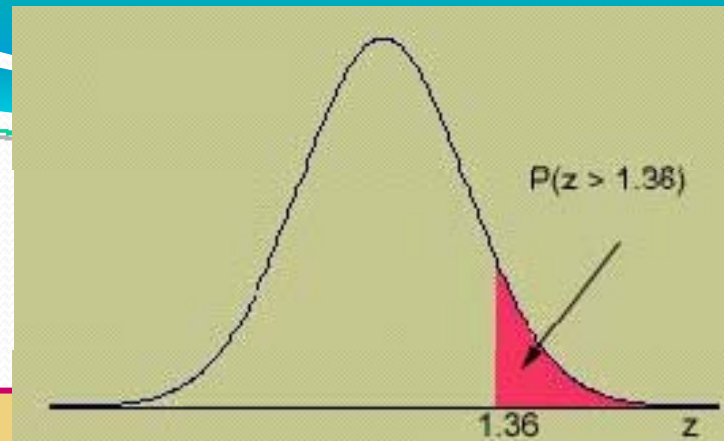
- Mean = 0; Standard deviation = 1
- When $x = \mu$, $z = 0$
- Values of z to the left of center are negative
- Values of z to the right of center are positive
- Total area under the curve is 1.

Example

$$P(z \leq 1.36) = 0.9131$$

$$P(z > 1.36) \\ = 1 - 0.9131 = 0.0869$$

$$P(-1.20 \leq z \leq 1.36) = \\ 0.9131 - 0.1151 = 0.7980$$

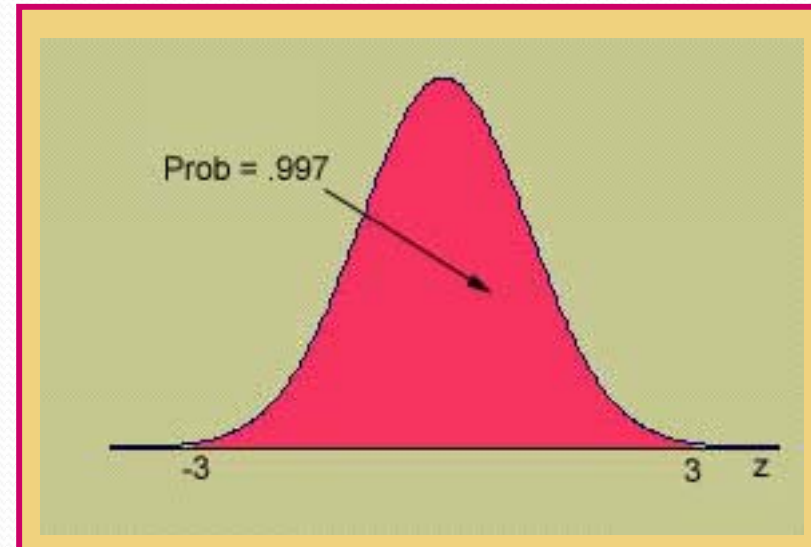


- ✓ To find an area to the left of a z-value, find the area directly from the table.
- ✓ To find an area to the right of a z-value, find the area in Table 3 and subtract from 1.
- ✓ To find the area between two values of z, find the two areas in Table 3, and subtract one from the other.

Remember the Empirical Rule:
Approximately 99.7% of the
measurements lie within 3
standard deviations of the mean.

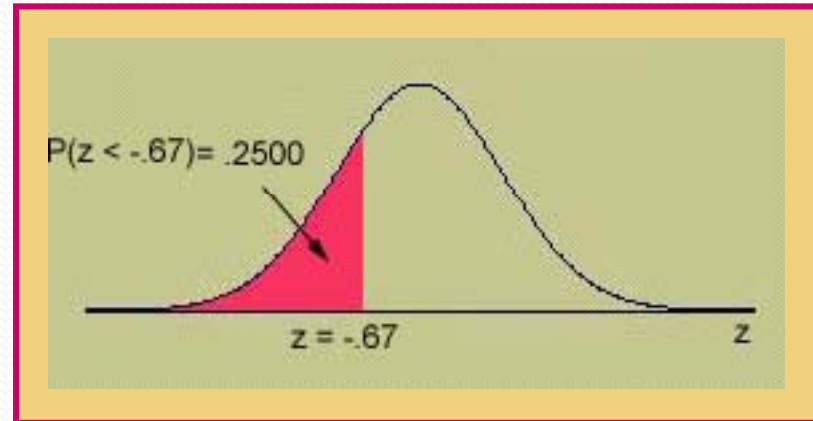
$$P(-1.96 \leq z \leq 1.96) = .9750 - .0250 = .9500$$

$$P(-3 \leq z \leq 3) = .9987 - .0013 = .9974$$



Find the value of z that has area .25 to its left.

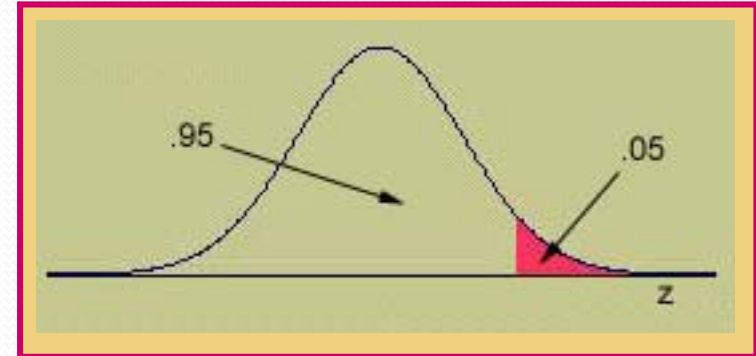
1. Look for the four digit area closest to .2500 in Table 3.
2. What row and column does this value correspond to?
3. $z = -.67$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810

Find the value of z that has area .05 to its right.

1. The area to its left will be $1 - .05 = .95$
2. Look for the four digit area closest to .9500 in Table 3.
3. Since the value .9500 is halfway between .9495 and .9505, we choose z halfway between 1.64 and 1.65.
4. $z = 1.645$



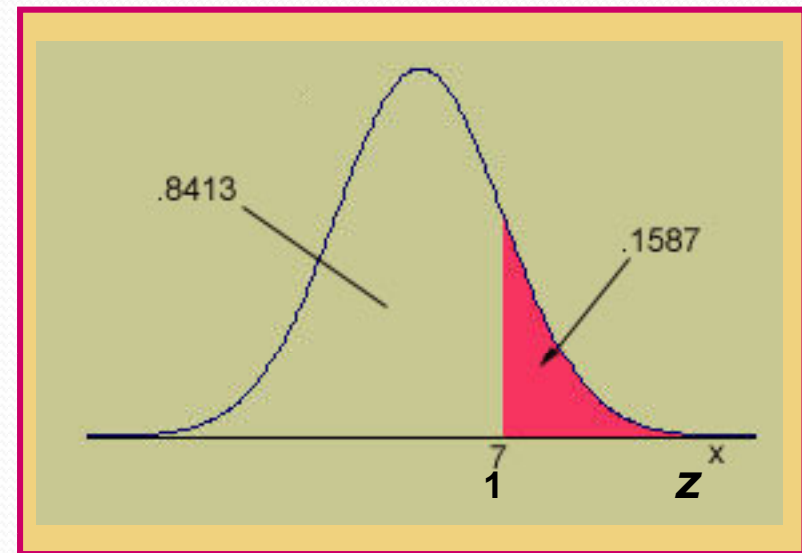
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Finding Probabilities for the General Normal Random Variable

- ✓ To find an area for a normal random variable x with mean μ and standard deviation σ , *standardize or rescale* the interval in terms of z .
- ✓ Find the appropriate area using Table

Example: x has a normal distribution with $\mu = 5$ and $\sigma = 2$. Find $P(x > 7)$.

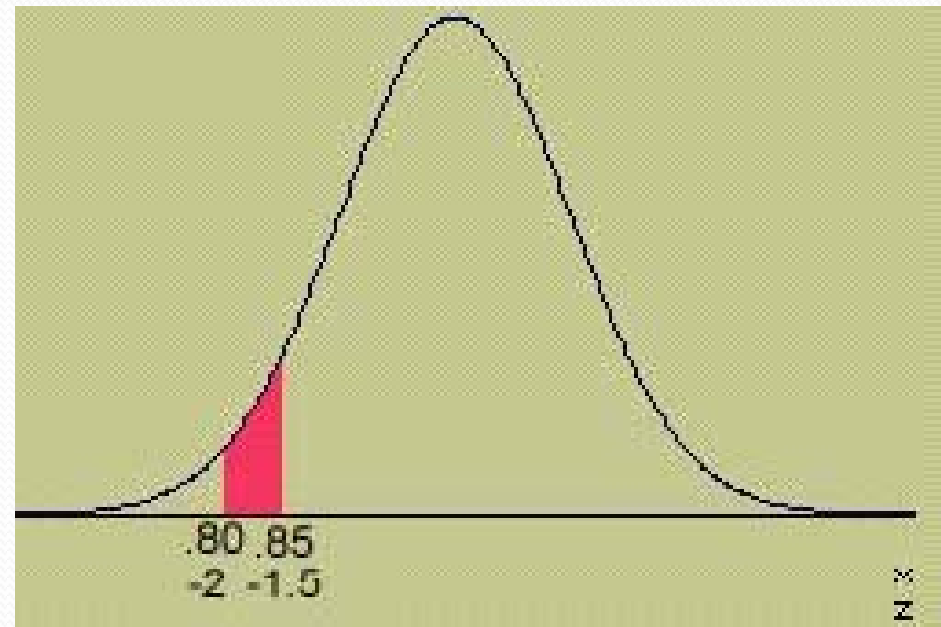
$$\begin{aligned} P(x > 7) &= P\left(z > \frac{7-5}{2}\right) \\ &= P(z > 1) = 1 - .8413 = .1587 \end{aligned}$$



Example

The weights of packages of ground beef are normally distributed with mean 1 pound and standard deviation 0.10. What is the probability that a randomly selected package weighs between 0.80 and 0.85 pounds?

$$\begin{aligned}P(.80 < x < .85) &= \\P(-2 < z < -1.5) &= \\ .0668 - .0228 &= .0440\end{aligned}$$



Example

What is the weight of a package such that only 1% of all packages exceed this weight?

$$P(x > ?) = .01$$

$$P(z > \frac{?-1}{.1}) = .01$$

$$\text{From Table, } \frac{?-1}{.1} = 2.33$$

$$? = 2.33(.1) + 1 = 1.233$$

