BUS 172 Section 5, Spring 2013

Assignment # 7

Deadline: Your assignment must be submitted/ emailed on or before 1:00 PM, 14th April, 2013. Late submission will be penalized by 10% for every day after due date.

1) **Suppose that the annual percentage salary increases for the chief executive officers of all midsize corporations are normally distributed with mean 12.2% and standard deviation 3.6%. A random sample of nine observations is obtained from this population and the sample mean computed. What is the probability that the sample mean will be less than 10%?**
   Solution:

$$P(\bar{x} \le 10) = P\left(z \le \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(z \le \frac{10 - 12.2}{\frac{3.6}{\sqrt{9}}}\right)$$

$$= P(z \le -1.83) = 0.0336$$

2) **A spark plug manufacturer claims that the lives of its plugs are normally distributed with mean 36,000 miles and standard deviation 4000 miles. A random sample of 16 plugs had an average life of 34,500 miles. If the manufacturer's claim is correct, what is the probability of finding a sample mean of 34,500 or less?**
   Solution:

$$P(\bar{x} \le 34{,}500) = P\left(z \le \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(z \le \frac{34{,}500 - 36{,}000}{\frac{4000}{\sqrt{16}}}\right)$$

$$= P(z \le -1.50) = 0.0668$$

3) **The mean selling price of new homes in a city over a year was \$115,000. The population standard deviation was \$25,000. A random sample of 100 new home sales from this city was taken.**
   a. **What is the probability that the sample mean selling price was more than \$110,000?**
   b. **What is the probability that the sample mean selling price was between \$113,000 and \$117,000?**
   c. **What is the probability that the sample mean selling price was between \$114,000 and \$116,000?**

Solution:

a)

$$P(\bar{x} > 110{,}000) = P\left(z > \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(z > \frac{110{,}000 - 115{,}000}{\frac{25{,}000}{\sqrt{100}}}\right)$$

$$= P(z > -2.00) = 1 - P(z < -2.00) = 1 - 0.0228 = 0.9772$$

b)

$$P(113{,}000 < \bar{x} < 117{,}000) = P\left(\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < z < \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(\frac{113{,}000 - 115{,}000}{\frac{25{,}000}{\sqrt{100}}} < z < \frac{117{,}000 - 115{,}000}{\frac{25{,}000}{\sqrt{100}}}\right)$$

$$= P(-0.8 < z < +0.8)$$

$$= P(z < 0.80) - P(z < -0.80) = 0.7881 - 0.2119 = 0.5762$$

c)

$$P(114{,}000 < \bar{x} < 116{,}000) = P\left(\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < z < \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(\frac{114{,}000 - 115{,}000}{\frac{25{,}000}{\sqrt{100}}} < z < \frac{116{,}000 - 115{,}000}{\frac{25{,}000}{\sqrt{100}}}\right)$$

$$= P(-0.4 < z < +0.4)$$

$$= P(z < 0.40) - P(z < -0.40) = 0.6554 - 0.3446 = 0.3108$$

4) **Assume that the standard deviation of monthly rents paid by students in a particular town is \$40. A random sample of 100 students was taken to estimate the mean monthly rent paid by the whole student population**
   a. **What is the standard error of the sample mean monthly rent?**
   b. **What is the probability that the sample mean exceeds the population mean by more than \$5?**
   c. **What is the probability that the sample mean is more than \$4 below the population mean?**

Solution:

a) Standard error of the sample mean,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40}{\sqrt{100}} = 4$$

b)

$$P\{(\bar{x} - \mu) > 5\} = P\{\bar{x} > (5 + \mu)\} = P(z > \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}})$$

$$= P(z > \frac{5 + \mu - \mu}{\frac{40}{\sqrt{100}}}) = P(z > \frac{5}{4}) = P(z > 1.25)$$

$$= 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$$

c)

$$P\{(\mu - \bar{x}) > 4\} = P\{(\mu - 4) > \bar{x}\} = P\{\bar{x} < (\mu - 4)\}$$

$$= P(z < \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}})$$

$$= P(z < \frac{\mu - 4 - \mu}{\frac{40}{\sqrt{100}}}) = P(z < \frac{-4}{4}) = P(z < -1.00) = 0.1587$$

5) **A random sample of 250 homes was taken from a large population of older homes to estimate the proportion of homes with unsafe wiring. If, in fact, 30% of the homes have unsafe wiring, what is the probability that the sample proportion will be between 25% and 35% of homes with unsafe wiring?**

Solution:

$$Here,\ \hat{p} = 0.30,\ \text{n} = 250,\ \text{np} = 75,$$

$$P(25\% < \hat{p} < 35\%)$$

$$= P(\frac{o.25 - \hat{p}}{\sqrt{\frac{\hat{p}\hat{q}}{n}}} < z < \frac{0.35 - \hat{p}}{\sqrt{\frac{\hat{p}\hat{q}}{n}}})$$

$$= P(\frac{o.25 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{250}}} < z < \frac{0.35 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{250}}})$$

$$= P(-1.72 < z < 1.72) = 0.9146$$

6) It has been estimated that 43% of business graduates believe that a course in business ethics is very important for imparting ethical values to students. Find the probability that more than one-half of a random sample of 80 business graduates have this belief.

Solution:

$$Here,\ p = 0.43,\ n = 80,$$

$$P(\hat{p} > 50\%) == P(z > \frac{0.50 - p}{\sqrt{\frac{pq}{n}}})$$

$$= P(z > \frac{0.50 - 0.43}{\sqrt{\frac{(0.43)(0.57)}{80}}})$$

$$= P(z > 1.27) = 1 - 0.8980 = 0.1020$$

The probability of having one-half of the sample believing in the value of business ethics courses is 10.20%

7) An administrator for a large group of hospitals believes that of all patients 30% will generate bills that become at least 2 months overdue. A random sample of 200 patients is taken.

a. What is the standard error of the sample proportion that will generate bills that become at least 2 months overdue?

b. What is the probability that the sample proportion is less than 0.25?

c. What is the probability that the sample proportion is more than 0.33?

d. What is the probability that the sample proportion is between 0.27 and 0.33?

Solution:

a)

$$Here,\ p = 0.30,\ n = 200,$$

$$\text{Standard error},\ \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.30)(0.70)}{200}} = 0.0324$$

b)

$$Here,\ \mathrm{p} = 0.30,\ \mathrm{n} = 200,$$

$$P(\hat{p} < 0.25) = P(z < \frac{0.25 - p}{\sqrt{\frac{pq}{n}}})$$

$$= P(z < \frac{0.25 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{200}}})$$

$$= P(z < -1.54) = 0.0618$$

c)

$$Here,\ \mathrm{p} = 0.30,\ \mathrm{n} = 200,$$

$$P(\hat{p} > 0.33) = P(z > \frac{0.33 - p}{\sqrt{\frac{pq}{n}}})$$

$$= P(z > \frac{0.33 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{200}}})$$

$$= P(z > 0.92) = 1 - P(z < 0.92) = 1 - 0.8212 = 0.1788$$

d)

$$Here,\ \mathrm{p} = 0.30,\ \mathrm{n} = 200,$$

$$P(0.27 < \hat{p} < 0.33) = P(\frac{0.27 - p}{\sqrt{\frac{pq}{n}}} < z < \frac{0.33 - p}{\sqrt{\frac{pq}{n}}})$$

$$= P(\frac{0.27 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{200}}} < z < \frac{0.33 - 0.30}{\sqrt{\frac{(0.30)(0.70)}{200}}})$$

$$= P(-0.92 < z < 0.92)$$

$$= P(z < 0.92) - P(z < -0.92) = 0.8212 - 0.1788 = 0.6424$$

8) **A process produces bags of refined sugar. The weights of the contents of these bags are normally distributed with standard deviation 1.2 ounces. The contents of a random sample of 25 bags had a mean weight of 19.8 ounces. Find the 99% confidence interval for the true mean weight for all bags of sugar produced by the process.**

Solution:

Here, 99% confidence interval = confidence coefficient, 1- α = 0.99, i.e. α = 0.01

And $\bar{x}$ = 19.8 ounces, n = 25 bags, σ = 1.2 ounces

Hence,

$$\left|z_{\alpha/2}\right| = \left|z_{0.01/2}\right| = \left|z_{0.005}\right| = \left|-2.575\right| = 2.575$$

$$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}} = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

99% confidence interval,

$$\bar{x} \pm 2.575\frac{\sigma}{\sqrt{n}} = 19.8 \pm 2.575\frac{1.2}{\sqrt{25}}$$

$$= 19.18 \text{ to } 20.42 \text{ ounces}$$

9) **A personnel manager has found that historically the scores on aptitude tests given to applicants for entry level positions follow a normal distribution with a standard deviation of 32.4 points. A random sample of nine test scores from the current group of applicants had a mean score of 187.9 points**

   a. **Find an 80% confidence interval for the population mean score of the current group of applicants.**
   b. **Based on these sample results, a statistician found for the population mean a confidence interval extending from 165.8 to 210.0 points. Find the confidence level of this interval.**

Solution: Here known standard deviation, σ = 32.4 points, sample size, n = 9 test scores

a) Here, 80% confidence interval = confidence coefficient, 1- α = 0.80, i.e. α = 0.20 and $\bar{x}$ = 187.9 points

Hence,

$$\left|z_{\alpha/2}\right| = \left|z_{0.20/2}\right| = \left|z_{0.10}\right| = \left|-1.28\right| = 1.28$$

$$\bar{x} \pm z_{\alpha/2}\sigma_{\bar{x}} = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

80% confidence interval,

$$\bar{x} \pm 1.28\frac{\sigma}{\sqrt{n}} = 187.9 \pm 1.28\frac{32.4}{\sqrt{9}}$$

$$= 174.07 \text{ to } 201.72 \text{ points}$$

b) Confidence coefficient, (1- α) is not known. i.e. $z_{\alpha/2}$ is not known

Confidence interval = 165.8 to 210 points = 187.9 ± 22.1

$$i.e.\ z_{\alpha/2}\frac{32.4}{\sqrt{9}} = 22.1$$

$$i.e.\ z_{\alpha/2} = \frac{22.1x\sqrt{9}}{32.4}$$

$$i.e.\ z_{\alpha/2} = 2.04$$

Hence,

$$\left|z_{\alpha/2}\right| = 2.04 = \left|-2.04\right| = \left|z_{0.02}\right| = \left|z_{0.04/2}\right|$$

i.e. α = 0.04
confidence coefficient, 1- α = 1— 0.04 = 0.96 = 96%,

Therefore the confidence level is 96%

10) Management wants an estimate of the proportion of the corporation's employees who favour a modified bonus plan. From a random sample of 344 employees it was found that 261 were in favour of this particular plan. Find a 90% confidence interval estimate of the true population proportion that favours this modified bonus plan.

Solution:

Here, 90% confidence interval = confidence coefficient, 1- α = 0.90, i.e. α = 0.10

And $\hat{p} = \frac{261}{344} = 0.759$; $\hat{q} = 1 - 0.759 = 0.241$; n= 344

$$|z_{\alpha/2}| = |z_{0.10/2}| = |z_{0.05}| = |-1.645| = 1.645$$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.759 \pm 1.645\sqrt{\frac{(0.759)(0.241)}{344}} = 0.721 < P < 0.797$$

11) **In a random sample of 95 manufacturing firms 67 indicated that their company attained ISO certification within the last two years. Find a 99% confidence interval for the population proportion of companies that have been certified within the last 2 years.**

Solution:

Here, 99% confidence interval = confidence coefficient, 1- α = 0.99, i.e. α = 0.01

And $\hat{p} = \frac{67}{95} = 0.705$; $\hat{q} = 1 - 0.705 = 0.295$; n= 95

$$|z_{\alpha/2}| = |z_{0.01/2}| = |z_{0.005}| = |-2.575| = 2.575$$

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}} = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.705 \pm 2.575\sqrt{\frac{(0.705)(0.295)}{95}} = 0.584 < P < 0.825$$

12) **A clinic offers a weight reduction program. A review of its records found the following weight losses, in pounds, for a random sample of 10 of its clients at the conclusion of the program:**

18 25 6 11 15 20 16 19 12 17

a. **Find a 99% confidence interval for the population mean.**
b. **Find a 92% confidence interval for the population mean.**

Solution:

a) Here population standard deviation, σ unknown, therefore we will use sample standard deviation, s = 5.3 ; $\bar{x}$ = 15.9; n = 10

Confidence interval for population mean when σ unknown,

$$\bar{x} \pm t_{\underset{(n-1)D.F}{\alpha/2}} \sigma_{\bar{x}} = \bar{x} \pm t_{\underset{(n-1)D.F}{\alpha/2}} \frac{s}{\sqrt{n}}$$

Confidence coefficient, 1- α = 0.99, i.e. α = 0.01

Hence, $t_{\underset{(n-1)D.F}{\alpha/2}} = t_{\underset{(10-1)D.F}{0.01/2}} = t_{\underset{(9)D.F}{0.005}} = 3.25$

99% confidence interval,

$$\bar{x} \pm t_{\underset{(9)D.F}{0.005}} \frac{s}{\sqrt{n}} = 15.9 \pm 3.25 \frac{5.3}{\sqrt{10}} = 10.45 \text{ to } 21.34$$

b)

Confidence coefficient, 1- α = 0.92, i.e. α = 0.08

Hence, $t_{\underset{(n-1)D.F}{\alpha/2}} = t_{\underset{(10-1)D.F}{0.08/2}} = t_{\underset{(9)D.F}{0.04}} = 1.918$

92% confidence interval,

$$\bar{x} \pm t_{\underset{(9)D.F}{0.04}} \frac{s}{\sqrt{n}} = 15.9 \pm 1.918 \frac{5.3}{\sqrt{10}} = 12.68 \text{ to } 19.11$$

13) A car rental company is interested in the amount of time its vehicles are out of operation for repair work. Find a 90% confidence interval for the mean number of days in a year that all vehicles in the company's fleet are out of operation if a random sample of nine cars showed the following number of days that each had been inoperative:

16 10 21 22 8 17 19 14 19

Solution:

Here population standard deviation, σ unknown, therefore we will use sample standard deviation, s = 4.79 ; $\bar{x}$ = 16.22; n = 9

Confidence interval for population mean when σ unknown,

$$\bar{x} \pm t_{\alpha/2_{(n-1)D.F}} \sigma_{\bar{x}} = \bar{x} \pm t_{\alpha/2_{(n-1)D.F}} \frac{s}{\sqrt{n}}$$

Confidence coefficient, 1- α = 0.90, i.e. α = 0.10

Hence, $t_{\alpha/2_{(n-1)D.F}} = t_{0.10/2_{(9-1)D.F}} = t_{0.05_{(8)D.F}} = 1.86$

90% confidence interval,

$$\bar{x} \pm t_{0.05_{(8)D.F}} \frac{s}{\sqrt{n}} = 16.22 \pm 1.86 \frac{4.79}{\sqrt{9}} = 13.25 \text{ to } 19.19$$