<u>**BUS 172 Section 5, Spring 2013**</u>

<u>**Assignment # 6**</u>

Deadline: Your assignment must be submitted/ emailed on or before 1:00 PM, 14th March, 2013. Late submission will be penalized by 10% for every day after due date.

1) **Let the random variable Z follow a standard normal distribution.**
   a. **Find P(z<1.2)**
   b. **Find P(z>1.33)**
   c. **Find P(z<-1.70)**
   d. **Find P(z> -1)**
   e. **Find P(1.20<Z<1.33)**
   f. **Find P(-1.70<Z<1.20)**
2) **If x~N(15, 16), find the probability that x is larger than 18**
   i.e. here, $\mu = 15$ **and** $\sigma^2 = 16$
   Solution:

$$P(x > 18) = P(z > \frac{18-\mu}{\sigma})$$
$$= P(z > \frac{18-15}{4}) = P(z > 0.75)$$
$$= 1 - P(z < 0.75) = 1 - 0.7734 = 0.2266$$

3) **A client has an investment portfolio whose mean value is equal to $500,000 with a standard deviation of $15,000. She has asked you to determine the probability that the value of her portfolio is between $485,000 and $530,000?**
   Solution:

$$P(485000 \le x \le 530000)$$
$$= P(\frac{485000-\mu}{\sigma} \le z \le \frac{530000-\mu}{\sigma})$$
$$= (\frac{485000-500000}{15000} \le z \le \frac{530000-500000}{15000})$$
$$= (-1 \le z \le +2)$$
$$= P(z \le +2) - P(z \le -1)$$
$$= 0.9772 - 0.1587 = 0.8185$$

4) **A company produces lightbulbs whose life follows a normal distribution, with mean 1200 hours and standard deviation 250 hours. If we choose a lightbulb at random, what is the probability that its lifetime will be between 900 and 1300 hours?**
   Solution:

$$P(900 \le x \le 1300)$$
$$= P\left(\frac{900-\mu}{\sigma} \le z \le \frac{1300-\mu}{\sigma}\right)$$
$$= \left(\frac{900-1200}{250} \le z \le \frac{1300-1200}{250}\right)$$
$$= (-1.2 \le z \le +0.4)$$
$$= P(z \le +0.4) - P(z \le -1.2)$$
$$= 0.6554 - 0.1151 = 0.5403$$

5) **A very large group of students obtains test scores that are normally distributed, with mean 60 and standard deviation 15.**
   a. **What proportion of the students obtained scores between 85 and 95?**
   b. **Find the cutoff point for the top 10% of all students.**

Solution:

a)

$$P(85 \le x \le 95)$$
$$= P\left(\frac{85-\mu}{\sigma} \le z \le \frac{95-\mu}{\sigma}\right)$$
$$= \left(\frac{85-60}{15} \le z \le \frac{95-60}{15}\right)$$
$$= (1.67 \le z \le 2.33)$$
$$= P(z \le 2.33) - P(z \le 1.67)$$
$$= 0.9901 - 0.9525 = 0.0376$$

That is, 3.76% of the students obtained scores in the range 85 to 95.

b) Suppose b is the cutoff point. i.e. 10% students get marks more than b

$$P(x \ge b) = 10\% = 0.10$$
$$i.e. P\left(z \ge \frac{b-\mu}{\sigma}\right) = 0.10$$
$$i.e. P\left(z < \frac{b-\mu}{\sigma}\right) = 1 - 0.10 = 0.90$$
$$i.e. \frac{b-60}{15} = 1.28$$
$$i.e. b = (1.28 * 15) + 60 = 79.2$$

Thus, we conclude that 10% of the students obtain scores above 79.2.

6) Let the random variable Z follow a standard normal distribution.

   a. The probability is 0.70 that Z is less than what number?
   b. The probability is 0.25 that Z is less than what number?
   c. **The probability is 0.2 that Z is greater than what number**?

Solution:

a)

$$P(z < b) = 0.70$$

$$b = 0.525$$

b)

$$P(z < b) = 0.25$$

$$b = -0.675$$

c)

$$P(z > b) = 0.20$$

$$P(z < b) = 0.80$$

$$b = 0.84$$

7) Let the random variable X follow a normal distribution with $\mu = 50$ **and** $\sigma^2 = 64$

   a. Find the probability that X is greater than 60.
   b. Find the probability that X is greater than 35 and less than 62.
   c. Find the probability that X is less than 55.
   d. The probability is 0.2 that X is greater than what number?

Solution:

a)

$$P(x > 60) = P(z > \frac{60 - \mu}{\sigma}) = 1 - P(z < \frac{60 - 50}{8})$$

$$= 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$$

b)

$$P(35 < x < 62) = P(\frac{35-\mu}{\sigma} < z < \frac{62-\mu}{\sigma})$$
$$= P(\frac{35-50}{8} < z < \frac{62-50}{8})$$
$$= P(-1.875 < z < 1.5)$$
$$= P(z < 1.5) - P(z < -1.875)$$
$$= 0.9332 - 0.0304 = 0.9028$$

c)

$$P(x < 55) = P(z < \frac{55-\mu}{\sigma}) = P(z < \frac{55-50}{8})$$
$$= P(z < 0.625) = 0.7340$$

d)

$$P(x > b) = 0.2$$
$$i.e. P(z > \frac{b-50}{8}) = 0.2$$
$$i.e. P(z < \frac{b-50}{8}) = 0.8$$
$$i.e. \frac{b-50}{8} = 0.84$$
$$i.e. b = 56.72$$

8) It is known that amounts of money spent on text books in a year by students on a particular campus follow a normal distribution with mean \$380 and standard deviation \$50.

a. What is the probability that a randomly chosen student will spend less than \$400 on textbooks in a year?

b. What is the probability that a randomly chosen student will spend more than \$360 on textbooks in a year?

c. What is the probability that a randomly chosen student will spend between \$300 and \$400 on textbooks in a year?

d. Find the range of dollar spending on textbooks in a year that includes 80% of all students on this campus.

Solution:

a)

$$P(x < 400) = P(z < \frac{400-\mu}{\sigma}) = P(z < \frac{400-380}{50})$$
$$= P(z < 0.40) = 0.6554$$

b)

$$P(x > 360) = P(z > \frac{360 - \mu}{\sigma}) = P(z > \frac{360 - 380}{50})$$
$$= P(z > -0.40) = 1 - P(z < -0.40) = 1 - 0.3446 = 0.6554$$

c)

$$P(300 < x < 400) = P(\frac{300 - \mu}{\sigma} < z < \frac{400 - \mu}{\sigma})$$
$$= P(\frac{300 - 380}{50} < z < \frac{400 - 380}{50})$$
$$= P(-1.6 < z < 0.40) = P(z < 0.40) - P(z < -1.6)$$
$$= 0.6554 - 0.0548 = 0.6006$$

d) The area under the normal curve is equal to 0.80 for an infinite number of ranges.

9) A contractor regards the cost of fulfilling a particular contract as a normally distributed random variable with mean \$500,000 and standard deviation \$50,000.

a. What is the probability that the cost of fulfilling the contract will be between \$460,000 and \$540,000?

b. The probability is 0.2 that the contract will cost less than how much to fulfil?

c. Find the shortest range such that the probability is 0.95 that the cost of fulfilling the contract will fall in this range.

Solution:

a)

$$P(460,000 < x < 540,000)$$
$$= P(\frac{460,000 - \mu}{\sigma} < z < \frac{540,000 - \mu}{\sigma})$$
$$= P(\frac{460,000 - 500,000}{50,000} < z < \frac{540,000 - 500,000}{50,000})$$
$$= P(-0.80 < z < 0.80) = P(z < 0.80) - P(z < -0.80)$$
$$= 0.7881 - 0.2119 = 0.5762$$

b)

$$P(x < b) = 0.2$$
$$i.e. P(z < \frac{b-\mu}{\sigma}) = 0.2$$
$$i.e. P(z < \frac{b-500{,}000}{50{,}000}) = 0.2$$
$$i.e. \frac{b-500{,}000}{50{,}000} = -0.84$$
$$i.e.\ b = 458{,}000$$

c) The shortest range will be the one that is centered on the Z of zero. The Z that corresponds to an area of 0.95 centered on the mean is a Z of ±1.96. This yields an interval of the mean ± 1.96 standard deviation, i.e. \$402,000 to \$598,000

10) A saleswoman makes initial telephone contract with potential customers in an effort to assess whether a follow-up visit to their homes is likely to be worthwhile. Her experience suggests that 40% of the initial contracts lead to follow-up visits. If she contacts 100 people by telephone, what is the probability that between 45 and 50 home visits will result?

Solution:
Let X be the number of follow up visits. Then X has a binomial distribution with n= 100 and p= 0.40. Approximating the required probability gives

$$P(45 \le x \le 50)$$
$$= P(\frac{45-\mu}{\sigma} < z < \frac{50-\mu}{\sigma})$$
$$= P(\frac{45-(100)(0.40)}{\sqrt{(100)(0.40)(0.60)}} < z < \frac{50-(100)(0.40)}{\sqrt{(100)(0.40)(0.60)}})$$
$$= P(1.02 < z < 2.04)$$
$$= P(z < 2.04) - P(z < 1.02)$$
$$= 0.9793 - 0.8461 = 0.1332$$

11) A car rental company has determined that the probability a car will need service work in any given month is 0.2. The company has 900 cars.

a. What is the probability that more than 200 cars will require service work in a particular month?

b. What is the probability that fewer than 175 cars will need service work in a given month?

Solution:

a)

P= 0.20, n= 900 cars

$$P(x > 200)$$
$$= P(z > \frac{(200 - 0.5) - \mu}{\sigma})$$
$$= P(z > \frac{199.5 - (900)(0.20)}{\sqrt{(900)(0.20)(0.80)}})$$
$$= P(z > \frac{199.5 - 180}{12})$$
$$= P(z > 1.625) = 1 - P(z < 1.625) = 1 - 0.9479 = 0.0521$$

b)

$$P(x < 175)$$
$$= P(z < \frac{(175 + 0.5) - \mu}{\sigma})$$
$$= P(z < \frac{175.5 - (900)(0.20)}{\sqrt{(900)(0.20)(0.80)}})$$
$$= P(z < \frac{175.5 - 180}{12})$$
$$= P(z < -0.375) = 0.3538$$

12) The tread life of a brand of tire can be represented by a normal distribution with mean 35,000 miles and standard deviation 4000 miles. A sample of 100 of these tires is taken. What is the probability that more than 25 of them have tread lives of more than 38,000 miles?

Solution:

$$P(x > 38{,}000) = P(z > \frac{38{,}000 - \mu}{\sigma})$$
$$= P(z > \frac{38{,}000 - 35{,}000}{4000})$$
$$= P(z > 0.75) = 1 - P(z < 0.75) = 1 - 0.7734 = 0.2266$$

Here, Proportion P= 0.2266; q= 1-0.2266 = 0.7734; n= 100

$$P(x > 25)$$
$$= P\left(z > \frac{(25 - 0.5) - \mu}{\sigma}\right)$$
$$= P\left(z > \frac{24.5 - (100)(0.2266)}{\sqrt{(100)(0.2266)(0.7734)}}\right)$$
$$= P\left(z > \frac{24.5 - 22.66}{4.18}\right)$$
$$= P(z > 0.44) = 1 - P(z < 0.44) = 1 - 0.67 = 0.33$$